UNIT - I

1

Steam Nozzle

Syllabus

Types and Shapes of nozzles, Flow of steam through nozzles, Critical pressure ratio, Variation of mass flow rate with pressure ratio. Effect of friction. Metastable flow.

Contents

1.1 Introduction

- A **nozzle** is a pipe or tube of varying cross-sectional area and it can be used to direct or modify the flow of fluid (liquids or gases).
- It is a duct by flowing through which the velocity of a fluid increases at the expense of pressure drop.
- If a duct decreases the velocity of fluid and causes corresponding increase in pressure then it is called as **diffuser**.
- If the cross-section of a duct decreases continuosly from inlet to outlet, it is said to be *convergent* and if it increases from inlet to outlet, it is said to be *divergent*. Refer Fig. 1.1 (a) and (b).
- If the cross-section decreases initially and then increases, the duct is known as *convergent - divergent*. Also the portion of minimum cross-section is known as *throat*. Refer Fig. 1.1 (c).

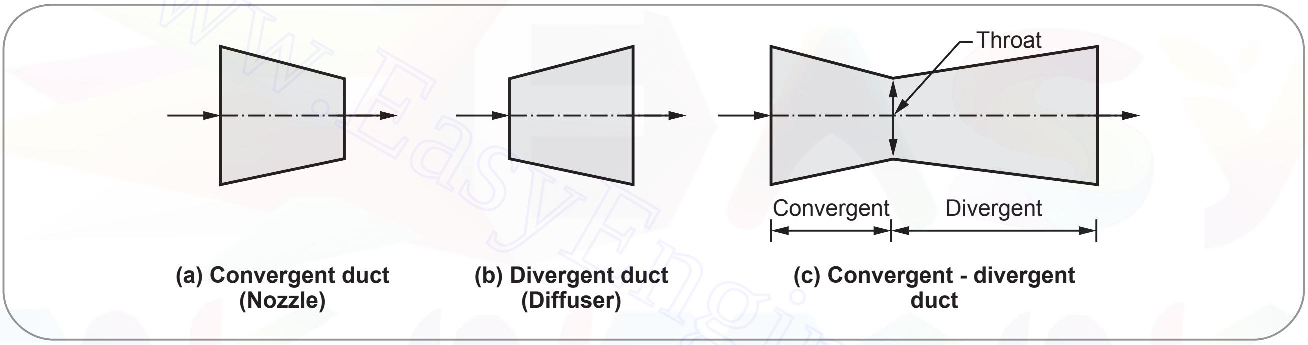


Fig. 1.1

- The **steam nozzle** is a passage of varying cross-section by means of which a part of the enthalpy of steam is converted into kinetic energy as the steam expands from high to low pressure.
- The amount of energy converted depends upon the type of expansion and pressure ratio.
- Mainly, isentropic expansion provides the maximum conversion of energy. Hence, nozzles are so shaped that isentropic expansion is obtained.

Applications

- Steam nozzles are used to produce a high velocity jet of steam for steam turbines.
- They are used in injectors of boiler for pumping feed water.
- They are used in injectors to maintain high vacuum in power plant condensers or steam jet refrigeration condensers.
- High velocity steam is also used in cleaning of wide range of surfaces, for moisturization in the production of paper, etc.

1.2 Types of Nozzle

- There are mainly following two types of nozzles :

(i) Convergent nozzle	ii) Convergent-divergent nozzle

(i) Convergent nozzle

- The cross-sectional area of these nozzles is defined by the expansion process and the conditon of steam at inlet and outlet. Refer Fig. 1.2 (a).
- In these nozzles, the cross-section area reduces from inlet to outlet section.
- This type of nozzle is used when pressure ratio (p_2/p_1) is upto 0.58 with saturated steam. This value is called as critical pressure ratio.

(ii) Convergent - Divergent nozzle

- When the pressure ratio has a value less than the critical value, a divergent part is required in addition to convergent portion.
- The divergent section is a long pipe and the divergent angle is limited to 7°, to avoid separation from the wall. Refer Fig. 1.2 (b).
- The least cross-section of the convergent-divergent nozzle is called as **throat**.
- These nozzles are always designed to discharge maximum mass for a given set of conditions.

1.3 Static and Stagnation Properties of Nozzle

- The stagnation values are useful reference conditions in compressible flow. If the properties of flow (such as $T, \rho, p,$ etc) are known at a point. The stagnation properties at a point are defined as those which are to be obtained when the local flow were imagined to goes down to zero velocity isentropically. The stagnation properties are denoted by a subscript zero. Hence, the stagnation enthalpy is defined as.

$$h_0 = h + \frac{V^2}{2}$$

For a perfect gas, above equation can be written as

$$C_p T_0 = C_p T + \frac{V^2}{2}$$

which defines stagnation temperature

Then,

$$\frac{T_0}{T} = 1 + \frac{V^2}{2C_p T} = 1 + \left(\frac{\gamma - 1}{2}\right)\frac{\gamma^2}{\gamma RT}$$

$$\frac{T_0}{T} = 1 + \left(\frac{\gamma - 1}{2}\right) M^2 \quad \ldots(1.1)$$

where, M = Mach number

Iostropic relations can be used to obtain stagnation pressure and stagnation density as

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{\gamma}{\gamma-1}} \quad \ldots(1.2)$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{\gamma}{\gamma-1}} \quad \ldots(1.3)$$

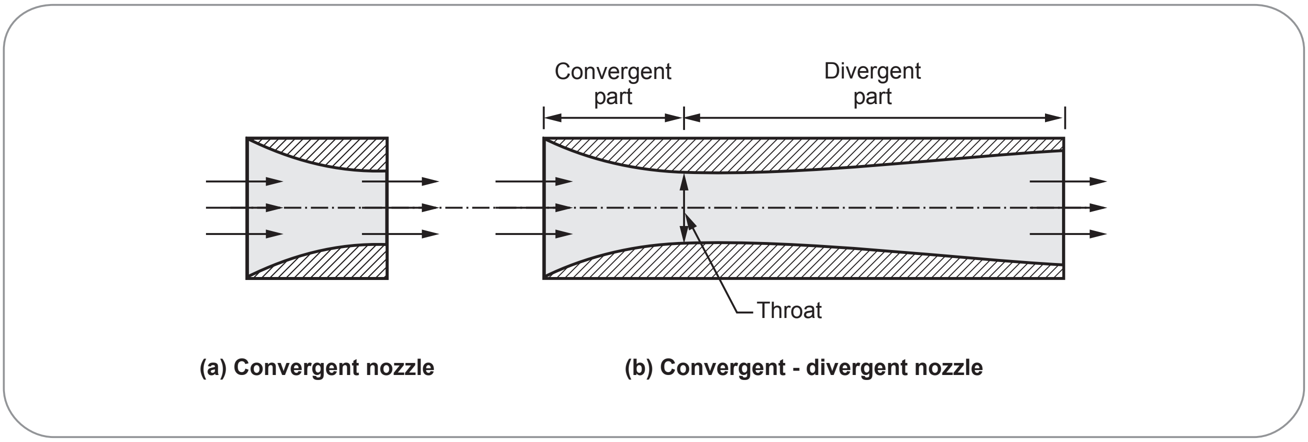


Fig. 1.2 : Types of nozzles

Functions of steam nozzle

1) To control the direction or characteristics of fluid flow(to increase the velocity) as it exits from the pipe.
2) These are used to control the rate of flow, speed, dirction, mass as well as pressure of the steam that emerges from the nozzle.

1.4 Flow of Steam through Nozzle

AU : June-16, May-18

- Consider an expansion of steam in a nozzle under steady state condition between the inlet section (1-1) to outlet section (2-2). Refer Fig. 1.3.
- Consider a unit mass of steam flows through a nozzle as shown in Fig. 1.3.

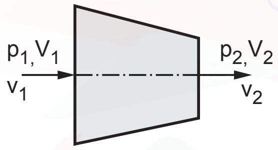


Fig. 1.3 : Flow through a nozzle

- Let,

p_1 and p_2 = Inlet and outlet pressure of steam through a nozzle.

V_1 and V_2 = Inlet and outlet velocity of steam through a nozzle.

v_1 and v_2 = Inlet and outlet specific volume of steam through a nozzle.

- The steady flow energy equation for a unit mass of steam is given as,

$$\left[h_1 + Z_1 g + \frac{V_1^2}{2}\right] + Q = \left[h_2 + Z_2 g + \frac{V_2^2}{2}\right] + W \quad \ldots (1.4)$$

- But, in case of nozzles, workdone W = 0, heat transfer Q = 0 and change in potential energy $(Z_1 g - Z_2 g)$ is also negligible. Therefore, the above equation (1.4) becomes,

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$\frac{V_2^2}{2} = (h_1 - h_2) + \frac{V_1^2}{2}$$

$$V_2^2 = 2\,(h_1 - h_2) + V_1^2$$

$$V_2 = \sqrt{2\,(h_1 - h_2) + V_1^2} \quad \ldots (1.5)$$

- In the above equation (1.5), if the inlet velocity of steam to the nozzle is negligible then $V_1 \approx 0$.

$$\therefore \quad V_2 = \sqrt{2\,(h_1 - h_2)}$$

In the above equation enthalpy is in kJ/kg. Converting this enthalpy in J/kg,

$$\therefore \quad V_2 = \sqrt{2 \times 1000 \times (h_1 - h_2)}$$

$$\therefore \quad V_2 = 44.72\,\sqrt{(h_1 - h_2)} \quad \ldots (1.6)$$

Now, in the above equation (1.6), **enthalpy is in kJ/kg.**

- The steam flowing through the nozzle follows approximately the following equation :

During the process the work done/kg of steam is,

$$= \frac{n}{n-1}\,(p_1 v_1 - p_2 v_2)$$

But, the gain in kinetic energy must be equal to the work done during the cycle i.e.

Gain in K.E. = Work done during the process

$$\frac{V^2}{2} = \left(\frac{n}{n-1}\right)(p_1 v_1 - p_2 v_2)$$

$$= \left(\frac{n}{n-1}\right) p_1 v_1 \left(1 - \frac{p_2 v_2}{p_1 v_1}\right)$$

We know that, $p_1 v_1{}^n = p_2 v_2{}^n \quad \therefore \frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$

$$\therefore \quad \frac{V^2}{2} = \left(\frac{n}{n-1}\right) p_1 v_1 \left(1 - \frac{p_2}{p_1}\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}\right)$$

$$= \left(\frac{n}{n-1}\right) p_1 v_1 \left(1 - \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}\right)$$

$$\therefore \quad V^2 = 2\left(\frac{n}{n-1}\right) p_1 v_1 \left(1-\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}\right)$$

$$\therefore \quad V = \sqrt{2\left(\frac{n}{n-1}\right) p_1 v_1 \left(1-\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}\right)} \quad \ldots (1.7)$$

where, n = Expansion index (1.13 to 1.3)

1.5 Mass Flow Rate Through Nozzle (Critical Pressure Ratio) AU : June-16, 17

- It is already discussed that, the nozzles are always designed for maximum discharge. The mass flow rate of steam through the nozzle is given by,

$$\dot{m} = \rho AV = \frac{AV}{v}, \text{ kg/sec} \quad \ldots \left(\because \rho = \frac{1}{v}\right)$$

where, A = Cross-section area of nozzle at throat

$$\text{or} \quad \dot{m} = \frac{AV}{v_2}$$

$$\text{But,} \quad \frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} \quad \therefore v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

$$\therefore \quad \dot{m} = \frac{A \times \sqrt{2\left(\frac{n}{n-1}\right) p_1 v_1 \left(1-\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}\right)}}{v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}} \quad \ldots (1.8)$$

- From the above equation (1.8) it is clear that the flow rate or discharge through the nozzle is the function of (p_2/p_1). Therefore, for maximum discharge through the nozzle, differentiate the above equation with respect to (p_2/p_1) and equate to zero.

$$\frac{d(\dot{m})}{d(p_2/p_1)} = 0$$

After simplification we get,

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \quad \ldots (1.9)$$

Substituting this value of ratio of throat pressure (p_2) to inlet pressure (p_1) in equation (1.10) we will get the maximum value of mass flow rate through the nozzle i.e.,

$$\dot{m}_{max} = A\sqrt{n\left(\frac{p_1}{v_1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}} \quad \ldots (1.10)$$

- From the above equation (1.10) it is clear that, the maximum mass flow rate depends on the initial conditions of the steam (p_1, v_1) and throat area (A).
- It is independent of the final pressure of steam (at the exit of nozzle). It means the addition of divergent part of nozzle after the throat does not affect the mass flow rate of steam.
- If we substitute equation (1.9) [Value of p_2/p_1] in equation (1.7) [value of V] then

$$V_{max} = \sqrt{2\left(\frac{n}{n+1}\right) p_1 v_1} \quad \ldots (1.11)$$

- It means, the velocity of steam through the nozzle is also dependent on the initial conditions of the steam $(p_1 \, v_1)$.

1.6 Super-saturated Flow and its Effect in Nozzle (Wilson Line) AU : June-16

- When dry saturated steam is expanded adiabatically or isentropically, it becomes wet and is shown by vertical line on Mollier diagram.
- If steam is initially superheated, the condensation should start after it has become dry saturated. This is possible when the steam has proceeded through some distance in the nozzle and in a short interval of time. But, from practical point of view, the steam has a great velocity(sometimes sonic and ever supersonic). Thus, the phenomenon of condensation does not take place at the expected rate. As a result of this, equilibrium between the liquid of vapor phase is

delayed and the steam continues to expand in a dry state. The steam in such a set of conditions is said to be supersaturated/ in metastable state.

- It is also called supercooled steam, as its temperature at any pressure is less than the saturation temperature corresponding to the pressure. The flow of supersaturated steam, through the nozzle is called supersaturated flow or metastable flow.
- Experiments of supersaturated flow of steam have shown that there is a limit to which the supersaturared flow is possible. This limit is represente by "Wilson line" on T-S and h-s diagram as shown in Fig. 1.4 (a) and (b) respectively.
- It may be noted that the Wilson line closely follows the 0.97 dryness fraction line. Beyond this Wilson line, there is no supersaturation. The steam suddenly condenses and restores its normal equilibrium state.
- In Fig. 1.4 (b) is shown the isentropic expansion of steam in nozzle. The point 'A' represents the position of initial dry saturated steamat pressure 'p_1'. The line 'AC' represents the isentropic expansion of steam in the supersaturated region.
- The metastable state (point C) is obtained by drawing a vertical line through 'A' to meet the Wilson line. At 'C', the steam condenses sudddenly. The line CD represents the condensation of steam at constant enthalpy. The point 'D' is obtained by drawing a horizontal line through 'C' to meet the throat pressure (p_2) of the nozzle. The line DF represents the isentropic expansion of steam in the divergent portion.

Effects of supersaturation

1) Since the condensation does not take place during supersaturated expansion, so the temperature at which the supersaturation occurs will be less than the saturation temperature corresponding to pressure. Therefore, the density of supersaturated steam will be more than for the equillibrium condition, which gives the increase in mass of steam discharged.

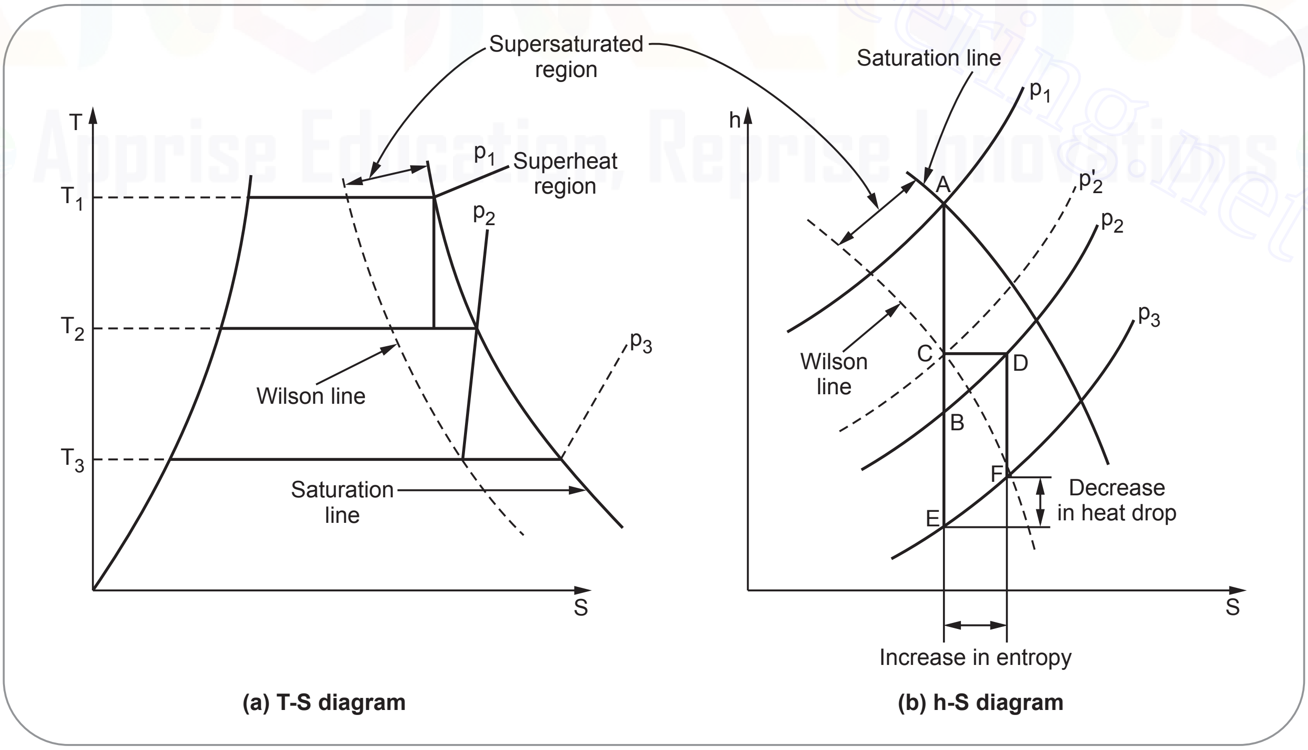


Fig. 1.4

2) The supersaturation increases the entropy and specific volume of steam.

3) The supersaturation increases dryness fraction of steam.

4) The supersaturation reduces the heat drop (for the same pressure limits) below that for thermal equilibrium. Hence, the exit velocity of steam is reduced.

1.7 Solved Examples

Ex. 1.1 : *Calculate the air fuel ratio on both mass and molar basis for the complete combustion of octane* (C_8H_{18}) *with theoretical amount of air and 150 % theoretical air*

Sol. : $p_1 = 15$ bar, $T_1 = 300$ °C, $p_3 = 2$ bar,

$V_1 = 150$ m/s, $\eta_{nozzle} = 90$ %, $m_s = 1$kg/sec,

$C_{ps} = 2.4$ kJ/kgk

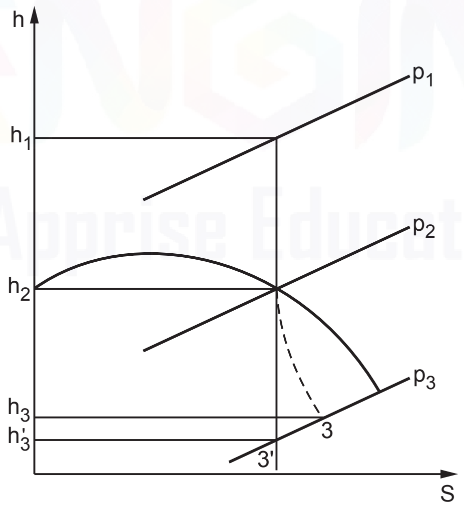


Fig. 1.5

$$\frac{p_2}{p_1} = 0.546$$

$$p_2 = p_1(0.546) = 15 \times 0.546 = \mathbf{8.19\ bar}$$

At nozzle entrance

From steam tables

At $p_1 = 15$ bar, $T_1 = 300$ °C

$h_1 = 3020$ kJ/kg

At throat, At

$p_2 = 8.19$ bar, $h_2 = 2900$ kj/kg, $V_2 = 0.3\ m^3/kg$

At nozzle exit, At

$p_3 = 2$ bar, $h'_3 = 2640$ kJ/kg

$$\therefore \quad h_{01} = h_1 + \left(\frac{V_1^2}{2 \times 1000}\right) \text{kJ/kg}$$

$$= 3020 + \left(\frac{150^2}{2000}\right)$$

$$h_{01} = \mathbf{3031.25\ kJ/kg}$$

Velocity of steam at throat,

$$V_2 = 44.72\sqrt{(h_{01} - h_2)}$$

$$= \mathbf{512.331\ m/s}$$

Area at throat,

$$A_2 = \frac{mv_2}{V_2} = \frac{1 \times 0.3}{512.33}$$

$$A_2 = 5.855 \times 10^{-4}\ m^2$$

$$\eta_{nozzle} = \left(\frac{h_{01} - h_2}{h_{01} - h'_3}\right)$$

$$(h_{01} - h_3) = \eta_{nozzle}(h_{01} - h'_3)$$

$$h_3 = (3031.25) - 0.9(3031.25 - 2640)$$

$$h_3 = \mathbf{2679.125\ kJ/kg}$$

From Mollier diagram,

At $p_3 = 2$ bar, $h'_3 = 2680$ kJ/kg

$$v_3 = 0.85\ m^3/kg$$

$\therefore$ Velocity of steam at exit

$$V_3 = 44.72\sqrt{(h_{01} - h_3)}$$

$$= 44.72\sqrt{(3031.25 - 2679.125)}$$

$$\boxed{V_3 = 839.17 \text{ m/s}}$$

Area of exit, $A_3 = \dfrac{\dot{m}_5 \, v_3}{V_3}$

$$= \left(\frac{1 \times 0.85}{839.17}\right)$$

$$\boxed{A_3 = 1.012 \times 10^{-3} \text{ m}^2}$$

Ex. 1.2 : *Dry saturated steam at a pressure of 8 bar enters a convergent divergent nozzle and leaves it at a pressure of 1.5 bar. If the flow is isentropic and the corresponding expansion index is 1.135. Find the ratio of cross sectional area at exit and throat for maximum discharge.*

Sol. :

Given, $p_1 = 8$ bar

$p_3 = 1.5$ bar

$\eta = 1.135$

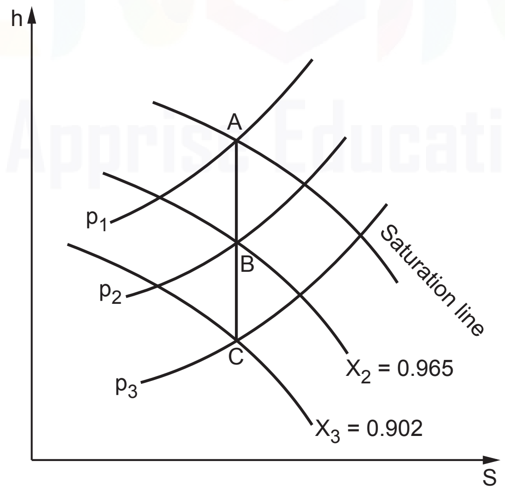


Fig. 1.6

Let, A_2 = Cross-sectional area at throat

A_3 = Cross-sectional area at exit

$\dot{m}_s$ = Mass of steam discharged/sec

When $n = 1.135$, critical pressure ratio

$$\frac{p_2}{p_1} = 0.577$$

$\therefore$ $p_2 = 0.577 \, p_1 = 0.577 \times 8$

$p_2 = \mathbf{4.616 \text{ bar}}$

From Mollier diagram, $h_1 = 2775$ kJ/kg, $h_2 = 2650$ kJ/kg, $h_3 = 2465$ kJ/kg, $X_2 = 0.965$, $X_3 = 0.902$

From steam table, we also find that the specific volume of steam at throat corresponding to 4.616 bar.

$$V_{g2} = 0.405 \text{ m}^3/\text{kg}$$

and, specific volume of steam at exit corresponding to 1.5 bar

$$V_{g3} = 1.159 \text{ m}^3/\text{kg}$$

Heat drop between entrance and throat,

$$h_{d2} = (h_1 - h_2) = 125 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at throat

$$V_2 = 44.72\sqrt{h_{d2}}$$

$$= 44.72\sqrt{125}$$

$$= 500 \text{ m/s}$$

and, $\dot{m}_s = \dfrac{A_2 V_2}{x_2 V_{g2}}$

$$A_2 = \frac{\dot{m}_s \times x_2 V_{g2}}{V_2}$$

$$A_2 = 0.000786 \, \dot{m}_s \qquad \ldots(1.12)$$

Heat drop between entrance and exit

$$h_{d3} = (h_1 - h_3) = (2775 - 2465)$$

$$= 310 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at exit

$$V_3 = 44.72\sqrt{h_{d3}}$$

$$= 44.72\sqrt{310}$$

$$= 787.4 \text{ m/s}$$

and, $$\dot{m}_s = \frac{A_3 V_3}{x_3 V_{g3}}$$

$$A_3 = \frac{\dot{m}_s x_3 V_{g3}}{V_3} = \mathbf{0.00133}\ \dot{m}_s \quad \ldots(2)$$

∴ Ratio of cross-sectional area at exit and throat is given by

$$\frac{A_3}{A_2} = \frac{0.00133\ \dot{m}_s}{0.000786\ \dot{m}_s} = \mathbf{1.692}$$

Ex. 1.3 : *Steam enters a group of nozzles of a steam turbine at 12 bar and 220 °C and leaves at 1.2 bar. The steam turbine develops 220 kW with a specific steam consumption of 13.5 kg/kWh. If the diameter of nozzles at throat is 7 mm, calculate the number of nozzles.*

Sol. :

Given, $p_1 = 12$ bar

$T_1 = 220°C$

$p_3 = 1.2$ bar

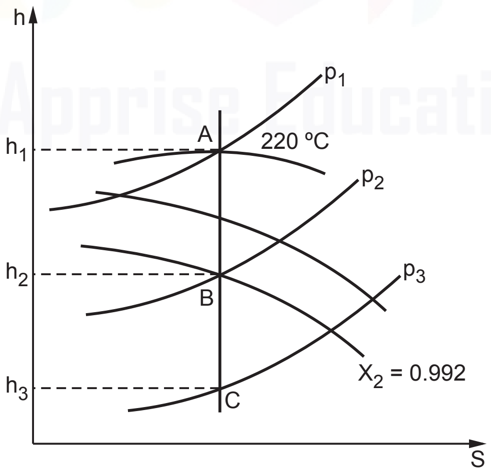


Fig. 1.7

Power developed,

$$p = 220 \text{ kW}$$

$$\dot{m}_s = 13.5 \text{ kg/kWh}$$

$$d_2 = 7 \text{ mm}$$

From Mollier diagram,

At, $p = 12$ bar and 220 °C

$$h_1 = 2860 \text{ kJ/kg}$$

Enthalpy of steam at throat (i.e. at $p_2 = 6.552$ bar)

We know that, for superheated steam,

$$p_2 = 0.546\ P_1 = 0.546 \times 12$$

$$= 6.552 \text{ bar}$$

$$h_2 = 2750 \text{ kJ/kg and } x_2 = 0.922$$

From steam table,

at $p_2 = 6.552$ bar

$$V_{g2} = 0.29 \text{ m}^3/\text{kg}$$

$$h_{d2} = (h_1 - h_2) = (2860 - 2750)$$

$$= 110 \text{ kJ/kg}$$

∴ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_{d2}} = \mathbf{470\ m/s}$$

Area of nozzle at throat,

$$A_2 = \frac{\pi}{4} d_2^2$$

$$= 38.5 \times 10^{-6} \text{ m}^2$$

∴ Mass flow Rate/Nozzle

$$\dot{m}_s = \frac{A_2 V_2}{x_2 V_{g2}}$$

$$= \left(\frac{38.5 \times 10^{-6} \times 470}{0.992 \times 0.29}\right)$$

$$= \mathbf{0.063\ kg/s}$$

Total mass flow rate,

$$\dot{m}_{total} = 13.5 \times 220 = 2970 \text{ kg/hr}$$

$$= 0.825 \text{ kg/sec}$$

$$\therefore \text{Number of Nozzles} = \left(\frac{\text{Total Mass Flow Rate}}{\text{Mass Flow Rate / Nozzle}}\right)$$

$$= 13.1$$

$$\approx \mathbf{14\ Nozzles}$$

Ex. 1.4 : *Dry saturated steam enters a nozzle at a pressure of 10 bar and with an initial velocity of 90 m/s. The outlet pressure is 6 bar and the outlet velocity is 435 m/s. The heat loss from Nozzle is 9 kJ/kg of steam flow. Calculate the dryness fraction and the area at the exit, if the area at the inlet is 1256 mm^2.*

Sol. : Let, x_3 = Dryness fraction of steam at the exit. From steam table,

at $p = 10$ bar

$$h_1 = 2776 \text{ kJ/kg},$$

$$V_{g1} = 0.1943 \text{ m}^3/\text{kg}$$

Corresponding to a pressure of 6 bar

$$h_{f3} = 670.4 \text{ kJ/kg}$$

$$h_{fg3} = 2085 \text{ kJ/kg}$$

and $V_{g3} = 0.3155 \text{ m}^3/\text{kg}$

We know that for a steady flow through the nozzle,

$$h_1 + \frac{1}{1000}\left(\frac{V_1^2}{2}\right) = h_3 + \frac{1}{1000}\left(\frac{V_3^2}{2}\right) + \text{Losses}$$

$$h_3 = h_1 + \frac{1}{2000}\left(V_1^2 - V_3^2\right) - \text{Losses}$$

$$= 2776.2 + \frac{1}{2000}\left(90^2 - 435^2\right) - 9$$

$$h_3 = \mathbf{2676.6\ kJ/kg}$$

Enthalpy of wet steam,

$$h_3 = h_{f3} + (h_{fg3} \times x_3)$$

$$2676.6 = h_{f3} + x_3 h_{fg3}$$

$$2676.6 = 670.4 + (x_3 \times 2085)$$

$$\boxed{x_3 = 0.962}$$

Area at exit,

We know that,

$$\frac{A_1 V_1}{x_1 V_{g1}} = \frac{A_3 V_3}{x_3 V_{g3}}$$

$$A_3 = \left(\frac{1256 \times 10^{-6} \times 90 \times 0.962 \times 0.3155}{1 \times 0.1943 \times 435}\right)$$

$$A_3 = 406 \times 10^{-6} \text{ m}^2$$

$$\boxed{A_3 = 406 \text{ mm}^2}$$

Ex. 1.5 : *A convergent-divergent nozzle is required to discharge 2 kg of steam per second. The nozzle is supplied with steam of 10 bar and 200 °C and discharge take place against a back pressure of 0.34 bar. Estimate the throat and exit areas. Assume isentropic flow and take the index n = 1.3. If the nozzle efficiency is assumed to be 85 %, determine the exit area.*

Sol. :

Given, $p_1 = 10$ bar $\quad \dot{m}_s = 2$ kg/sec

$T_1 = 200$ °C $\quad n = 1.3$

$= 200 + 273 \quad R = 287$ J/kgK

$= 473$ K $\quad P_3 = 0.34$ bar

$\eta_{Nozzle} = 85$ %

We know that, pressure at the throat drop between entrance and throat

$$p_2 = 0.528 \times p_2 = (0.528 \times 10)$$

$$= \mathbf{5.28\ bar}$$

$$h_{d2} = (h_1 - h_2) = \left(\frac{n}{n-1}\right) \times P_1 V_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right]$$

$$= \left(\frac{1.3}{1.3-1}\right) \times 2 \times 287 \times 473 \times \left[1 - \left(\frac{5.28}{10}\right)^{\frac{1.3-1}{1.3}}\right]$$

$$= 161224.6 \text{ J/kg}$$

$$h_{d2} = 161.224 \text{ kJ/kg}$$

$$\therefore \quad V_2 = 44.72\sqrt{h_{d2}} = \mathbf{567.83\ m/s}$$

We know that,

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \mathbf{1.1588}$$

$$T_2 = \mathbf{408.18\ k}$$

and,

$$pv_2 = mRT_2$$

$$v_g = \frac{mRT_2}{p_2} = \left(\frac{2\times 287\times 408.16}{5.28\times 10^5}\right)$$

$$= 0.443\ m^3/kg$$

We know that,

$$\dot{m}_s = \frac{A_2V_2}{v_2}$$

$$A_2 = \frac{\dot{m}_s v_2}{V_2} = \left(\frac{2\times 0.44}{567.83}\right)$$

$$= 1.55\times 10^{-3}\ m^2$$

$$A_2 = \mathbf{1550\ mm^2}$$

Exit Area

Heat drop between entrance and exit,

$$h_{d3} = (h_1 - h_3) = \left(\frac{n}{n-1}\right)\times \dot{m}_s RT_1\left[1-\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}}\right]$$

$$= \left(\frac{1.3}{1.3-1}\right)\times 2\times 287\times 473\times\left[1-\left(\frac{0.34}{10}\right)^{\frac{1.3-1}{1.3}}\right]$$

$$= 637363.58\ kJ/kg$$

$$\boxed{h_{d3} = 637.36\ kJ/kg}$$

$\therefore$ Velocity at exit of nozzle,

$$V_3 = 44.72\sqrt{h_{d3}} = \mathbf{1129\ m/s}$$

We know that,

$$\frac{T_1}{T_3'} = \left(\frac{p_1}{p_3}\right)^{\frac{n-1}{n}} = \left(\frac{10}{0.34}\right)^{\frac{1.3-1}{1.3}} = \mathbf{2.1822}$$

$$T_3' = \frac{T_1}{2.182} = \mathbf{216.76\ K}$$

Since nozzle efficiency, $\eta = 85\ \%$

$\therefore$ Heat lost due to friction

$$= (1-\eta_{nozzle})\times h_{d3} = \mathbf{95.6\ kJ/kg}$$

We know that, heat drop lost in friction

$= \dot{m}_s \times$ Specific heat $\times$ Increase in temperature

$$\therefore \text{ Increase in temperature} = \frac{\text{Heat drop lost in friction}}{\dot{m}_s\times \text{specific heat}}$$

$$= \left(\frac{95.6}{2\times 1.005}\right)$$

$$= \mathbf{47.56\ K}$$

$\therefore$ $T_3 = T_3'$ + [Increase in temperature due to friction]

$$= (216.76 + 47.56)$$

$$= \mathbf{264.32\ K}$$

And,

$$p_3v_3 = \dot{m}_s RT_3$$

$$v_3 = \frac{\dot{m}_s RT_3}{P_3} = \left(\frac{2\times 287\times 264.32}{0.34\times 10^5}\right)$$

$$= \mathbf{4.462\ m^3/kg}$$

We know that,

$$\dot{m}_s = \frac{A_3V_3}{v_3}$$

$$A_3 = \frac{\dot{m}_s v_3}{V_3} = \left(\frac{2\times 4.462}{1129}\right)$$

$$= 7.904\times 10^{-3}\ m^2$$

$$\boxed{A_4 = 7904\ mm^2} \text{ - Exit area}$$

Ex. 1.6 : *Dry saturated steam at a pressure of 11 bar enters a convergent-divergent nozzle and leaves at a pressure of 2 bar. If the flow is adiabatic and frictionless, determine :*

i) The exit velocity of steam.

ii) Ratio of cross section of exit and that at throat.

AU : May-15, Marks 16

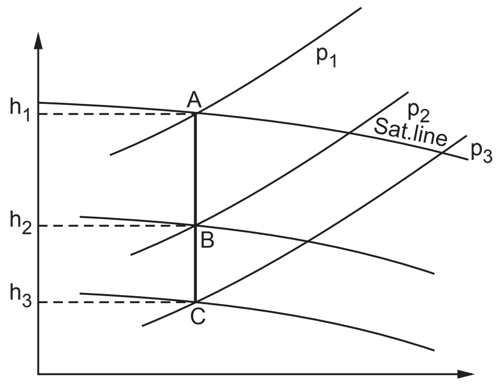


Fig. 1.8

Sol. :

Dry saturated steam, $P_1 = 11$ bar, $P_3 = 2$ bar

Let,

A_2 = c/s Area at throat

A_3 = c/s Area at exit

We know that, for dry saturated steam, critical pressure ratio,

$$\frac{P_2}{P_1} = 0.577$$

$$\therefore \quad P_2 = 0.577 \times P_1$$

$$= 0.577 \times 11 = 6.347 \text{ bar}$$

From Mollier diagram,

$$h_1 = 2770 \text{ kJ/kg}, \quad h_2 = 2680 \text{ kJ/kg}$$

$$h_3 = 2460 \text{ kJ/kg}$$

$$x_2 = 0.97, \quad x_3 = 0.89$$

From steam table, at

$$P_2 = 6.347 \text{ bar}, \quad V_{g2} = 0.295 \text{ m}^3/\text{kg}$$

And, at $\quad P_3 = 2$ bar, $\quad V_{g3} = 0.8854 \text{ m}^3/\text{kg}$

Heat drop between entrance and throat,

$$h_{d_2} = (h_1 - h_2) = (2770 - 2680)$$

$$= 90 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at throat

$$V_2 = 44.72\sqrt{h_{d_2}} = 44.72\sqrt{90}$$

$$= 424.25 \text{ m/s}$$

And, $\quad m = \dfrac{A_2 V_2}{x_2 V_{g2}}$

$$A_2 = \frac{m x_2 V_{g2}}{V_2} = \frac{m \times 0.97 \times 0.295}{424.25}$$

$$A_2 = 6.745 \times 10^{-4} \text{ m}$$

Heat drop between entrance and exit

$$h_{d_3} = (h_1 - h_3) = (2770 - 2460)$$

$$= 310 \text{ kJ/kg}$$

i) Velocity of steam at exit

$$V_3 = 44.72\sqrt{h_{d_3}} = 44.72\sqrt{310}$$

$$\mathbf{V_3 = 787.4 \text{ m/s}}$$

And, $\quad m = \dfrac{A_3 V_3}{x_3 V_{g3}}$

$$A_3 = \frac{m x_3 V_{g3}}{V_3} = \left(\frac{m \times 0.89 \times 0.8854}{787.4}\right)$$

$$A_3 = 1.00076 \times 10^{-3} \text{ m}$$

ii) Ratio and cross-section of exit and that at throat

$$\frac{A_3}{A_2} = \left[\frac{1.00076 \times 10^{-3} \text{ m}}{6.745 \times 10^{-4} \text{ m}}\right]$$

$$\boxed{\frac{A_3}{A_2} = 1.484}$$

Ex. 1.7 : *In a stage of impulse reaction turbine operating with 50 % degree of reaction, the blades are identical in shape. The outlet angle of the moving blade is 19° and the absolute discharge velocity of steam is 100 m/s in the direction 70° to the motion of the blades. If the rate of flow through the turbine is 15000 kg/hr, calculate the power developed by the turbine.*

AU : Dec.-16, Marks 16

Ans. : 50 % Degree of Reaction (Parson's turbine)

Blades are identical in shape.

$$\phi = 19° = \alpha, \qquad V_2 = 100 \text{ m/s}$$

$$\theta = 70° = \beta \qquad \dot{m} = 15000 \text{ kg/hr}$$

$$= \frac{15000}{3600} \text{ kg/s}$$

$$\dot{m} = \mathbf{4.167} \text{ kg/s}$$

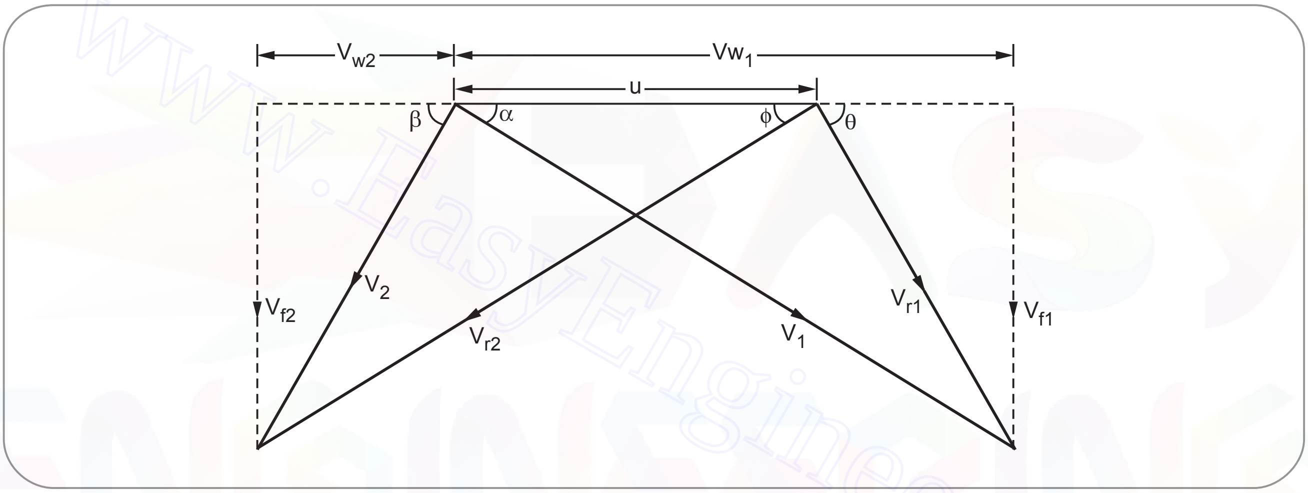


Fig. 1.9

From inlet velocity triangle.

$$V_{f_1} = V_1 \sin\alpha = V_{r_1} \sin\theta$$

$$V_1 = V_{r_1}\left(\frac{\sin(70°)}{\sin(19°)}\right)$$

$$\therefore \qquad V_1 = 2.886\ V_{r_1} \qquad \ldots()$$

Also, $(V_{w_1} - u) = V_{r_1} \cos\theta$

Consider outlet velocity triangle

$$\cos\beta = \frac{V_{w_2}}{V_2}$$

$$V_{w_2} = V_2 \cos\beta = 100 \cos 70° = 34.2 \text{ m/s}$$

$$V_{f_2} = V_2 \sin\beta = 100 \sin 70° = 93.97 \text{ m/s}$$

$$\text{and } \tan\phi = \left(\frac{V_{f_2}}{V_{w_2} + u}\right)$$

$$\tan(19°) = \frac{(93.97)}{(34.2 + u)}$$

$$u = 238.7 \text{ m/s}$$

$$\sin\phi = \frac{V_{f_2}}{V_{r_2}}$$

$$V_{r_2} = \frac{V_{f_2}}{\sin\phi} = \frac{93.97}{\sin(19°)} = 288.634 \text{ m/s}$$

Consider

$$(V_{w_1} - u) = V_{r_1}\cos\theta$$

$$V\cos\alpha - u = V_{r_1}\cos\theta$$

$$(2.886\, V_{r_1})\cos(19°) - 238.71 = V_{r_1}[\cos(70°)]$$

$$(2.728\, V_{r_1} - 0.342\, V_{r_1}) = 238.71$$

$$2.386\; V_{r_1} = 238.71$$

$$\boxed{V_{r_1} = 100.05 \text{ m/s}}$$

$\therefore \quad V_1 = 2.886\, V_{r_1} = \mathbf{288.73\ m/s}$

$\therefore \quad V_{w_1} = V_1\cos\alpha = \mathbf{273\ m/s}$

Hence, power developed,

$$P = m(V_{w_1} - V_{w_2})u$$

$$= (4.167)(273 - 34.2)\,(238.7)$$

$$\boxed{P = 237.53 \text{ kW}}$$

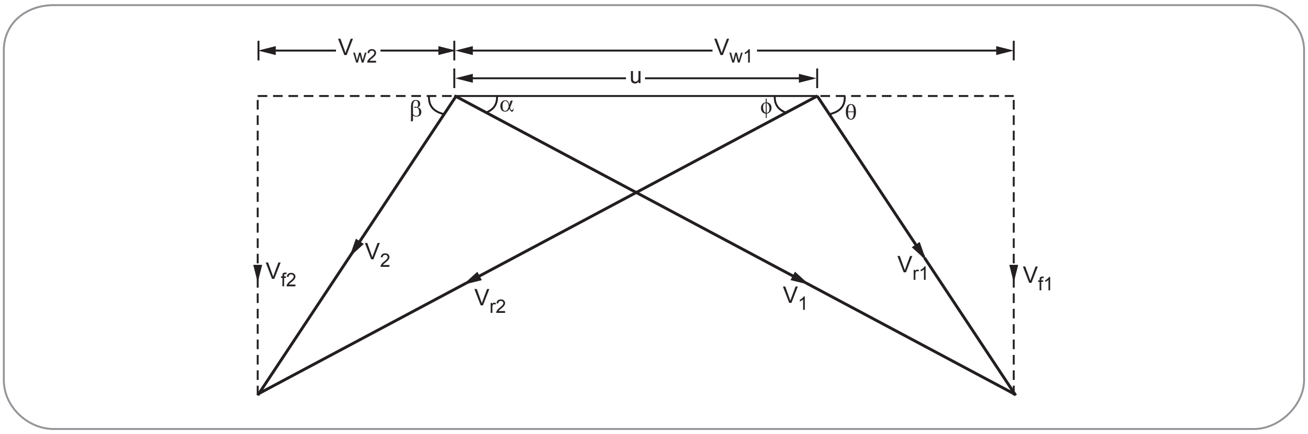


Fig. 1.10

1.8 Two Marks Questions with Answers

Q.1 When nozzle is said to be a convergent nozzle ?

Ans. : When the cross-section of the nozzle decreases continuously from entrance to exit

Q.2 What is internal efficiency ?

Ans. : The ratio of total useful heat drop to the total isentropic heat drop, is called internal efficiency.

Q.3 What is back pressure ?

Ans. : The pressure at which the steam leaves the nozzle is known as back pressure

Q.4 In a nozzle, where is whole frictional loss is assumed to occur ?

Ans. : Between throat and exit

Q.5 Define degree of reaction.

Ans. : It is defined as the ratio of the static pressure drop in the rotor to the static pressure drop in the stage.

Q.6 Define reheat factor.

Ans. : Reheat factor is the ratio of the cumulative heat to the adiabatic drop from initial condition to exhaust pressure.

Q.7 What are the assumptions made in the analysis of air standard cycles ?

Ans. : The working fluid is air, Air is considered as ideal gas.

All the processes in (ideal) power cycles are internally reversible.

Combustion process is modeled by a heat-addition process from an external source.

Q.8 When the nozzle is said to be underdamping ?

Ans. : When the back pressure of a nozzle is below the designed value of pressure at exit of nozzle, the nozzle is said to be under damping.

Q.9 Define blading efficiency.

Ans. : The ratio of the workdone on the blades to the energy supplied to the blades, is called blading efficiency.

Q.10 What is the condition of steam when it leaves the nozzle ?

Ans. : The condition of steam when it leaves the nozzle low pressure and a high velocity.

Q.11 What is the critical pressure ratio ?

Ans. : $\dfrac{p_2}{p_1} = \left[\dfrac{2}{n+1}\right]^{n/n-1}$

Q.12 In a De-Laval nozzle expanding superheated steam from 10 bar to 0.1 bar, the pressure at the minimum cross-section will be

Ans. : Pressure ratio ($p_2 / p_1 = 0.546$),

while $p_1 = 10$ bar, $p_2 = 10 \times 0.546$,

$p_2 = 5.46$ bar

Q.13 At which section the flow of steam in a nozzle is subsonic ?

Ans. : Convergent portion

Q.14 State the types of steam nozzle.

Ans. : 1. Convergent nozzle.

2. Divergent nozzle. 3. Convergent - divergent nozzle

Q.15 Why velocity of steam gets reduced when it moves through nozzle ?

Ans. :

- The friction between steam and walls of nozzle.
- Internal friction of steam itself.
- Shock losses

Q.16 What is the effect of friction in nozzle ? **AU : May - 18**

OR What is the effect of friction on the flow through a steam nozzle ? **AU : Dec.-15**

Ans. :

- The enthalpy drop is reduced and hence the final velocity.
- The kinetic energy gets converted into heat due to friction and is absorbed by the steam. Due to this, the final dryness fraction of steam increases.
- Steam becomes more dry due to increased dryness fraction and hence specific volume of steam increases and mass flow rate decreases.

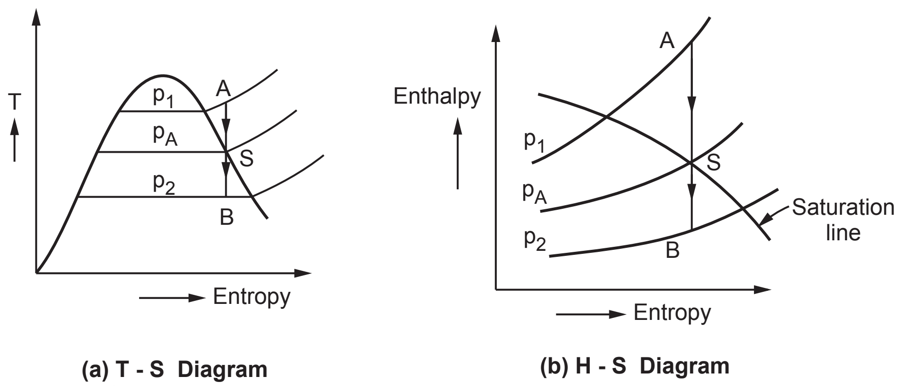


Fig. 1.11

Q.17 Draw T-S & H-S diagram super saturated or meta stable flow.

Ans. : Refer Fig. 1.11

Q.18 What is Meta stable flow ?

Ans. : When the supersaturated steam is expanded in the nozzle, the condensation should occur in the nozzle. Since the steam has a great velocity, the condensation does not take place at the expected rate. So the equilibrium between the liquid and vapour phase is delayed and the steam continues to expand in a dry state. The steam in such set of condition is said to be supersaturated or meta stable.

Q.19 What are the conditions that produce super saturation of steam in nozzles ?

Ans. : When the superheated steam expands in the nozzle, the condensation will occur in the nozzle. Since, the steam has more velocity, the condensation will not take place at the expected rate. So, the equilibrium between the liquid and vapour phase is delayed and the steam continues to expand in a dry state. The steam in such set of condition is said to be supersaturated or meta stable flow.

Q.20 What are the differences between supersaturated flow and isentropic flow through steam nozzles ?

Ans. :

Sr. No.	Supersaturated flow	Isentropic flow
1.	Entropy is not constant	Entropy is constant
2.	Reduce in enthalpy drop	No reduce in enthalpy drop
3.	We cannot use mollier diagram	We can use mollier diagram

Review Questions

1. *Explain various types of nozzles.*
2. *What is stagnation properties of nozzle ?*
3. *Explain with neat sketch Wilson line.*
4. *Derive the experssion for critical pressure ratio.*
5. *State the effect of super saturation.*

1.9 University Questions with Answers

June - 2016

Q.1 *Define critical pressure ratio of a nozzle and discuss why attainment of sonic velocity determines the maximum discharge through steam nozzle.* **(Refer sections 1.5 and 1.4)** **[10]**

Q.2 *Explain the metastable expansion of steam in a nozzle with help of h-s diagram.*
(Refer section 1.6) **[6]**

June - 2017

Q.3 *Derive the equation for crictical pressure ratio in steam nozzle.* **(Refer section 1.5)** **[8]**

May - 2018

Q.4 *Derive the condition for maximum flow rate in steam nozzle.* **(Refer section 1.4)** **[13]**

❑❑❑

Notes

UNIT - II

2

Boilers

Syllabus

Types and comparison. Mountings and Accessories. Fuels - Solid, Liquid and Gas. Performance calculations, Boiler trial.

Contents

2.1 Introduction

- **Boiler** or **steam generator** is a closed pressure vessel used for generation of steam under high pressure.
- This steam is mainly used for power generation, process heating and space heating purposes.
- A boiler is commonly made of steel in which the chemical energy of fuel is converted into heat by the combustion process. This heat energy is transferred to water so as to produce steam.
- The design of boilers is very complicated and depends on the type of fuel used and its power (capacity).

2.2 Classification of Boiler

The boilers may be classified as follows :

1. According to the relative position of water and flue gases

a) **Water tube boilers :** In these boilers, the water passes through the tubes and flue gases pass through the external surface of the tubes.

For example : Babcock-Wilcox, La-mont, Benson and Package boilers.

b) **Fire tube boilers :** In these boilers, the flue gases are passed through the tubes and the tubes are surrounded by the water.

For example : Cochran, Lancashire and Locomotive boilers.

2. According to the method of furnace

a) **Externally fired boilers :** In these boilers, the furnace is placed outside the boiler shell.

For example : All water tube boilers.

b) **Internally fired boilers :** In these boilers, the furnace is placed inside the boiler shell.

For example : All fire tube boilers.

3. According to the method of water circulation

a) **Natural circulation boilers :** In these boilers, the water is circulated by natural convection which is set up due to heating of water. Due to temperature gradient water flows from high density region to low density region.

For example : Babcock and Wilcox boiler.

b) **Forced circulation boilers :** In these boilers, the water is circulated by using a pump driven by motor.

For example : La-mont and Benson boilers.

4. According to the use

a) **Stationary boilers :** These type of boilers are commonly used in industries, power plants, etc. for power generation.

For example : Lancashire and Babcock-Wilcox boilers.

b) **Mobile boilers :** These boilers are used in ships, locomotives, etc. because they continuously move from one place to another place.

For example : Locomotive boilers.

5. According to the axis of shell

a) **Vertical boilers :** In these boilers the axis of shell is vertical.

For example : Cochran boilers.

b) **Horizontal boilers :** In these boilers, the axis of shell is horizontal.

For example : Lancashire and Locomotive boilers.

6. According to the pressure of steam generated

a) **Low pressure boilers :** When the pressure of generated steam is below 25 bar, then it is called as low pressure boiler.

For example : Cochran, Lancashire and Locomotive boilers.

b) **High pressure boilers :** When the pressure of generated steam is above 25 bar and upto 160 bar then it is called as high pressure boiler.

For example : La-mont and Loeffler boilers.

7. According to the heat source

a) Heat is generated due to combustion of solid, liquid and gaseous fuels.

b) Waste heat from other processes or nuclear energy.

2.3 Requirements of Good Boiler

A good boiler must possess the following qualities :

- The boiler should be capable to generate steam at the desired pressure and quantity in minimum possible time with minimum fuel consumption.
- The boiler should be light in weight and it should occupy less floor area.
- The boiler must be able to meet the fluctuating demands without fluctuations in pressure.
- All the boiler parts should be easily accessible or approachable for cleaning and inspecation.
- The boiler should be leak proof.
- It should have simple installation with minimum time and manpower.
- It should start quickly.
- There should be no deposition of mud and foreign materials on heated surface because it affect the efficiency of boiler.
- The design of boiler should allow high heat transfer rates with minimum pressure drop by providing high velocity of water and flue gases.
- The initial cost, installation cost and maintenance cost of boiler should be minimum.
- The boiler should confirm to the safety regulations as per the *Indian Boiler Act (IBA) 1923.*

2.4 Selection Criteria of Boiler

While selecting a boiler for a particular application the following factors should be considered :

- Pressure and steam generation rates.
- Available floor space.
- Quality of generated steam.
- The availability of fuel and water.
- The requirement of power.
- Erection facilities.
- The cost of operation, installation and maintenance.

2.5 Fire Tube and Water Tube Boiler

1) Fire tube package boiler

- A fire tube boiler is a boiler in which hot gases from a fire pass through one or more tubes running through a sealed container of water.
- The heat of these gases is transferred through the walls of the tubes by thermal conduction hence heating the water and ultimately creating the steam. Refer Fig. 2.1.

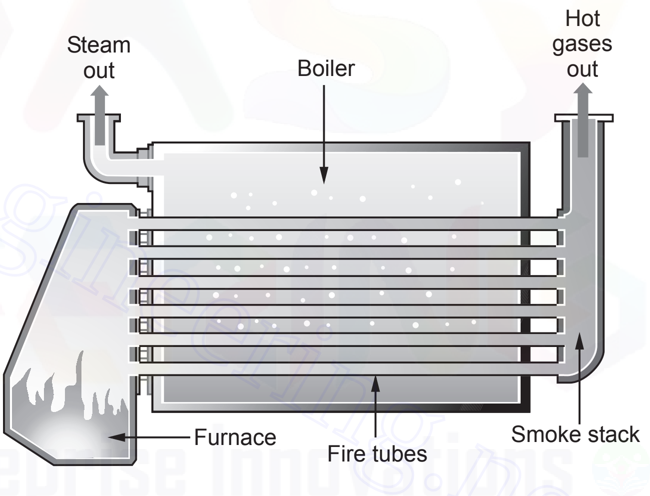


Fig. 2.1 : Fire tube package boiler

- These type of boilers are commonly used in locomotives, marine applications, rockets, etc.
- Cornish boiler, Lancashire boiler, Locomotive boiler, Vertical boiler are the types of fire tube boiler.
- In these boilers, the fuel is burnt in a firebox to produce hot combustion gases.
- The firebox is surrounded by a cooling jacket of water connected to the long, cylindrical boiler shell.
- The hot gases are directed along a series of fire tubes that penetrate the boiler and heat the water, in this way generating wet steam.
- Sometimes, this steam is passed through the superheater to dry the steam or superheat the steam.

2) Water tube package boiler

- A water tube boiler is a type of boiler in which water circulates in tubes heated externally by the fire.
- In these boilers, fuel is burned inside the furnace, creating hot gas which heats water in the steam generating tubes.
- In small boilers additional generating tubes are separate in the furnace while in large boilers water filled tubes make up the walls of the furnace to generate the steam. Refer Fig. 2.2.
- The heated water then rises into the steam drum where saturated steam is drawn off the top of the drum.
- In some cases, the steam will pass through the superheater to become superheated.
- Cool water at the bottom of the steam drum returns to the feedwater drum through down-comer tubes, where it pre-heats the feedwater supply.
- These boilers are used for power generation, sugar industry, textile industry, food processing plant, marine applications, locomotives, etc.
- Babcock-Wilcox boiler, La-mont boiler, Benson boiler are the types of water tube boiler.

2.6 Low Pressure Boiler

- The boiler which generates steam below 20 bar pressure is called low pressure boiler.
- The following boilers are low pressure boilers
  1) Cochran boiler
  2) Babcock and Wilcox boiler
  3) Lancashire boiler

2.6.1 Cochran Boiler

Working principle

- It's a vertical shell type with hemispherical crown. Due to this less material is required for construction for same volumes.
- Also the strength increases to withstand high steam pressures.
- The fire box strength increases to withstand high steam pressures.
- The fire box is kept at the bottom. Due to hemispherical shape it absorbs all the radiant heat.
- As fire enters the tube water circulated in the tube starts generating steam.
- The generated steam contains water particle.

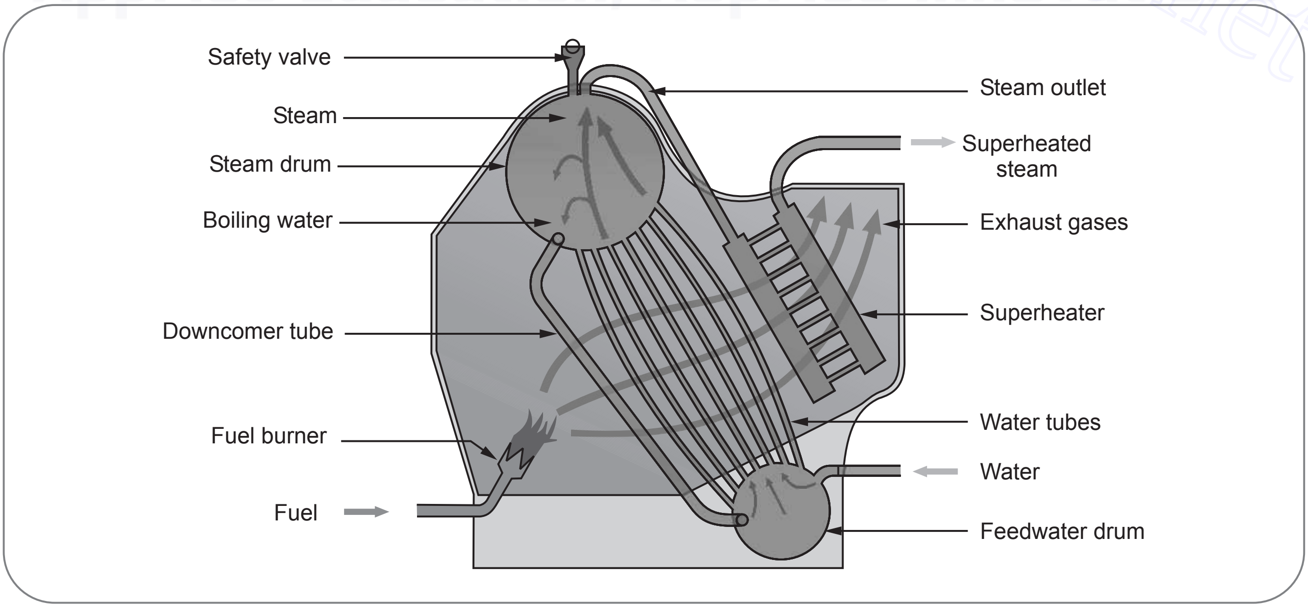


Fig. 2.2 : Water tube package boiler

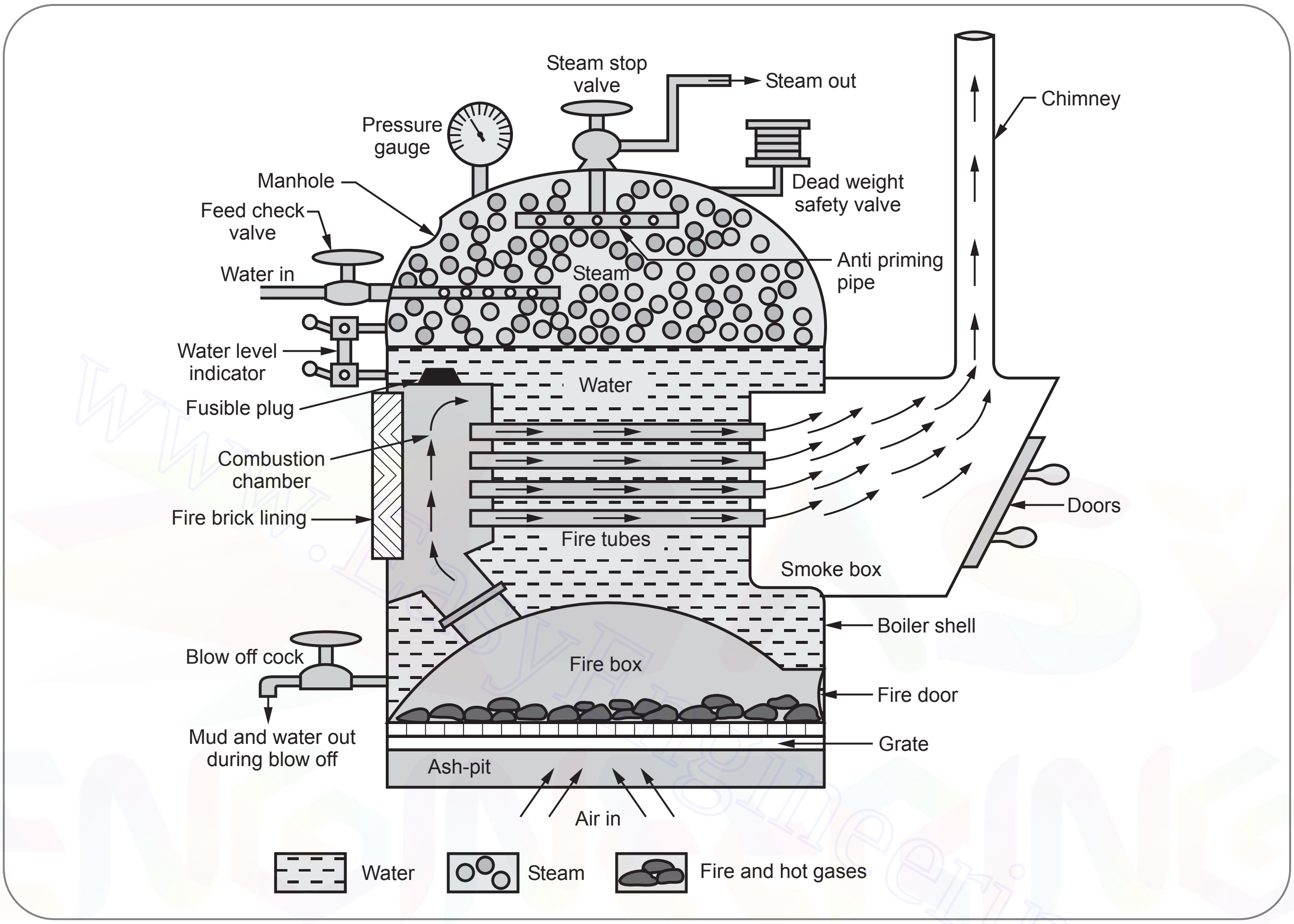


Fig. 2.3 : Cochran boiler

- To convert superheated steam it is need transfer to super heater.
- The boiler mountings are used to control the boiler.
- The boiler accessories are used to improve the efficiency of boiler.
- **Merits :** Less floor area, No joints to fire box, Any type of fuel can be used.
- **Demerits :** i) Interior is not easily accessible for cleaning, insepection and repairing purpose.

  ii) Overheating of crown of firebox will take place if thick scale or deposits of mud form on the waterside surface.

2.6.2 Babcock and Wilcox Boiler

- Babcock and Wilcox boiler is a water tube boiler as the water is inside the tubes and hot flue gases flows over the tubes.
- Fig. 2.4 shows the schematic diagram of Babcock and Wilcox boiler with its different functional parts.
- The boiler shell consist of high quality of steel and is placed longitudinally. It is known as water and steam drum. The water level in the drum should be kept slightly above the center.
- The drum is connected by short tubes with uptake header and by long tubes with downtake header.

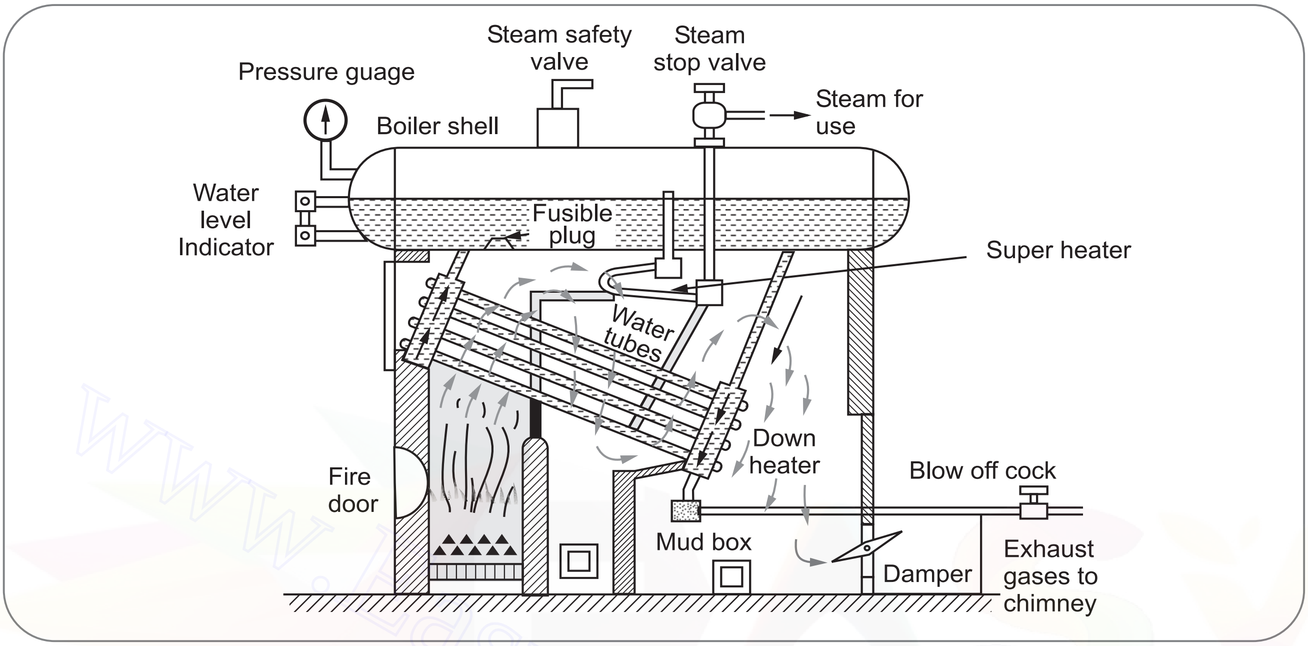


Fig. 2.4 : Babcock and Wilcox boiler

- A series of water tubes are connected to the uptake and downtake header at an angle of 15° to horizontal. This inclined position helps to make water flow.
- The grate is arranged below the uptake header. The fuel is supplied to grate through fire door. The fuel is burnt and forms hot flue gases.
- These flue gases are force to move in a specific path over the water tube due to baffle arrangement.
- The heat is transferred from hot flue gases to the water tubes and bottom cylindrical surface of the drum.
- The water flow is set due to the density difference in the water.
- The portion of water tubes at uptake header is subjected to the high temperature hot gases so the temperature of the water in this section rises due to decreased density and it flows to the drum via uptake header.
- Simultenously the water enters into tubes through downtake header and flow of water sets.
- The water and steam are seperated in the drum and as the steam is lighter it is collected in the upper portion of drum.
- To improve the quality of steam the superheater is placed between water drum and tubes.
- The steam formed in the drum is passed through the superheater and it becomes superheated.
- This superheated steam is then supplied to the turbine for electricity generation.
- The evaporative capacity for this boiler is upto 40000 kg/hr and operating pressure is 11 to 17 bar.

2.6.3 Lancashire Boiler

- Lancashire boiler is a fire tube boiler as the hot flue gases flows inside the tube and the water flows around these tubes.
- It is stationary, internally fired and natural circulation type of boiler.
- Its shell consist of the brick work as shown in Fig. 2.5.
- This boiler consist of a cylindrical shell and two large tubes are passed through this shell.

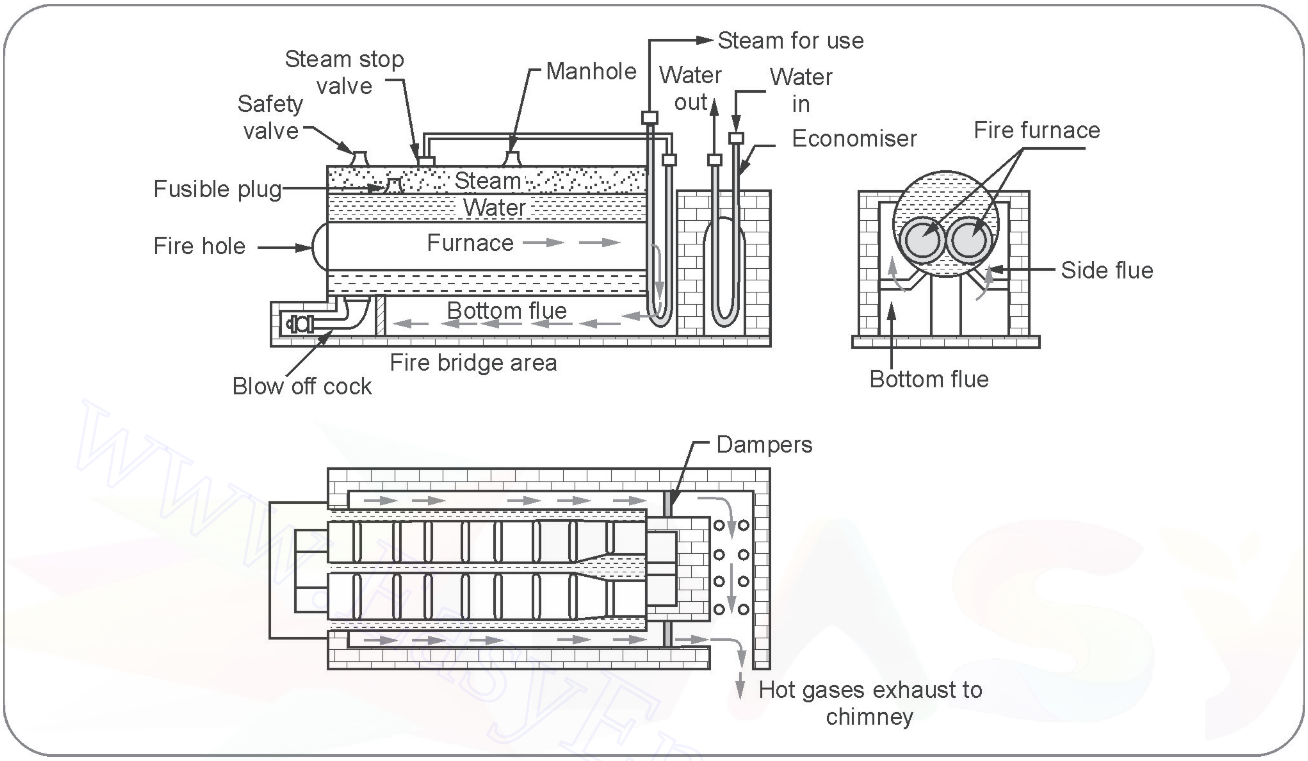


Fig. 2.5 : Lancashire boiler

- The brick work forms the channels for flow of flue gases known as bottom flue and side flue passage.
- The grate is provided at the front end of the flue tube and the coal is fed to grate through the fire doors.
- A brick bridge arch is provided at the end of grate to avoid the coal and ash particle to pass through the fire tubes.
- This helps in preventing deposition of particles on the wall of fire tubes and enhance the heat transfer rate.
- The hot flue gases passed through the flow channels from the side and bottom of the boiler and transfers the heat to the water which converts it into the steam.
- Dampers are provided at the end of the fire tubes to control the flow of gases and to regulate the combustion.
- The superheater and the economiser are easily fitted in the lancashire boiler system.
- The superheater is placed at the end of main flue tube so that the flue gases can passed over the superheater before entering the bottom flow passage.
- The evaporative capacity of this boiler is upto 10000 kg/h and operating pressure is upto 15 bar.

Advantages of Lancashire Boiler

- Good evaporation quality.
- Inferior quality of coal can be used as a fuel.
- Heating surface area per unit volume is large.
- Easy maintenance.
- Efficiency is high (upto 80 - 85 %)
- Load fluctuations can be easily met.

Disadvantages of Lancashire Boiler

- It requires large space.
- Large amount of material and labour requires for boiler construction.
- Due to repeted heating and cooling, there is resultant expansion and contraction. This results in upsetting the brickwork and hence infiltration of air.
- Slow steam generation.

Applications of Lancashire Boiler

- Processing agent in textile, paper, sugar and chemical industries.
- To drive steam turbines, locomotives and marine applications.

2.7 High Pressure Boiler

- If the boiler generates steam more than 20 bar pressure then it is called high pressure boiler.
- The following boilers are high pressure boilers
  1) La mont boiler 2) Benson boiler

2.7.1 La Mont Boiler

- It is a high pressure, forced circulation boiler invented by scientist *La Mont*.
- This boiler produces the steam at pressure 120 bar and temperature 500 °C.
- Fig. 2.6 shows the arrangement of La Mont boiler.
- The feed water is supplied to the boiler through economiser where it is preheated before entering the boiler. The water is circulated with the help of feed pump.
- From the steam separating drum, the water is supplied to the radient evaporator and part of the vapour is seperated in the drum.
- The danger of overheating is reduced due to circulation of large quantity of water (almost 10 times the evaporation).
- The steam separated in the boiler is further passed through the superheater and finally supplied to the prime mover.
- The heat wasted from the flue gases is utilised in the economiser and air preheater.
- These boilers generate 40 - 50 tonnes of superheated steam per hour.

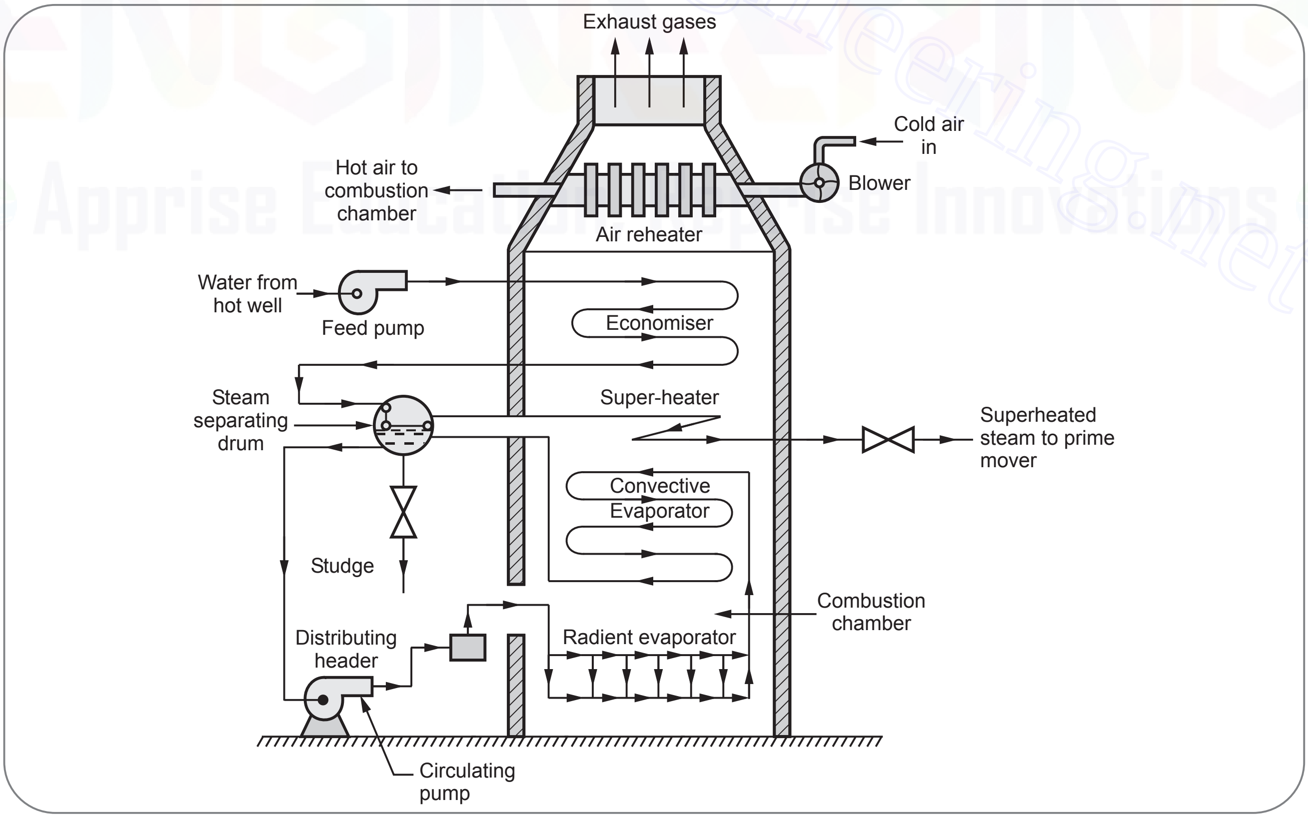


Fig. 2.6 : La Mont boiler

2.7.2 Benson Boiler

- Benson boiler is the modification of La Mont boiler. The difficulties experienced in La Mont boiler are overcame in the Benson boiler.
- The arrangement of Benson boiler is shown in Fig. 2.7.
- The feed pump circulates the water to the evaporator through economiser. The drum is eliminated in this type of boiler.
- The major portion of water is converted into the steam in radient evaporator.
- The remaining portion of water is evaporated in the convective evaporator and pressure of steam rises upto 225 bar (i.e. supercritical stage).
- The major difficulty occurs during operation of Benson boiler is deposition of salts on the tubes of convective evaporator. Hence to avoid this, boilers are flushed one after every 4000 working hours.
- This supercritical pressure steam is then passed through the superheater and then to prime mover.
- The steam generation takes place at 500 bar pressure and 650 °C temperature. Also these boilers generate upto 150 tonnes of steam per hour.

Advantages :

- Reduced weight due to elimination of drum.
- Expansion joints are not required as the forced circulation reduces overheating of parts.
- Easy and quicker erection of boiler.
- Require less floor space.
- Lower explosion hazards.

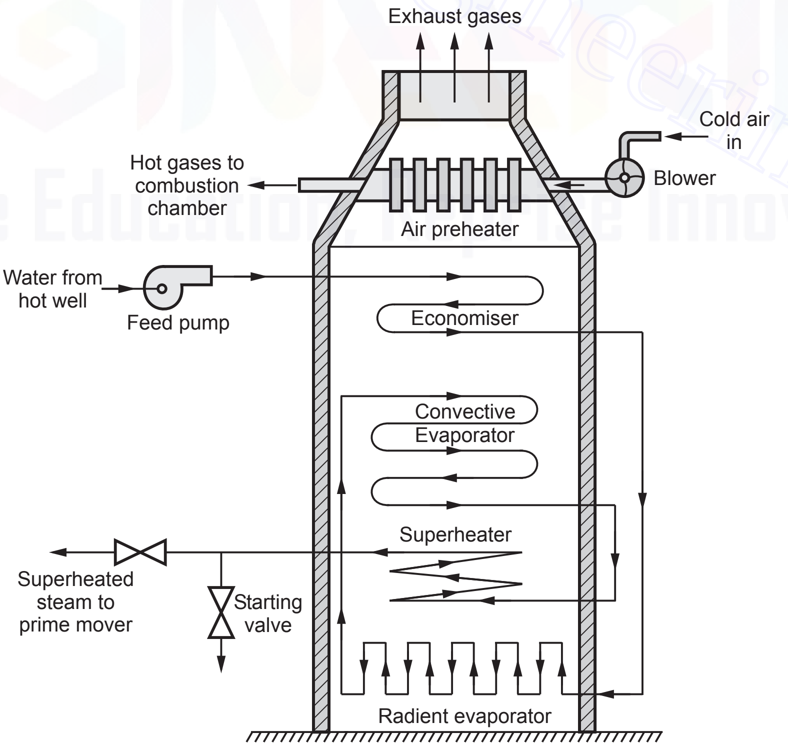


Fig. 2.7 : Benson boiler

2.7.3 Advantages of High Pressure Boilers

The advantages of high pressure boilers can be listed as follows :

1) To increase the rate of heat liberation, pressurised combustion is used which increases rate of firing of fuel.
2) The heat released in combustion is utilised more effectively by the use of multiple small diameter tubes.
3) The forced circulation of water gives positive circulation of water and increases evaporative capacity of the boiler.
4) It reduces the number of steam drums required.
5) Due to high velocity of water through the tubes, the tendency of scale formation is eliminated.
6) There is better flexibility for the arrangement of the components because high head required for natural circulation is eliminated using forced circulation. So components can be arranged horizontally.
7) Less floor space is required.
8) The efficiency of the plant is increased upto 40 to 42 % by using high pressure and temperature steam.
9) The variable load requirements can be achieved.
10) The possibility of gas and air leakages is less.

Note 1 : The construction and working of some high pressure boilers are explained below :

2.8 Comparison between Fire Tube and Water Tube Boiler

Sr. No.	Fire tube boiler	Water tube boiler
1.	Flue gases flow through tubes.	Water flow through tubes.
2.	Steam generation rate slow (up to 9000 kg/hr)	It is high (To 450000 kg/hr)
3.	Generate low pressures up to 25 bar.	Generate high pressures up to 220 bar.
4.	Steam temperature 300 ^{o}C around.	Steam temperature 550 ^{o}C.
5.	Internally fired type.	Externally fired type.
6.	Chances of explosion is less.	Chances of explosion is high due to high pressures.
7.	Larger shell diameters.	Smaller shell diameter for same steam generation rate.
8.	Parts are not accessible.	Parts are accessible for cleaning.
9.	Requires less attention.	Requires more attention.

2.9 Boiler Mountings and Accessories

Boiler mountings

- For the operation and safety of the boiler different fittings and devices are necessary. These devices are called as **boiler mountings**.

- For example : Safety valve, feed check valve, water level indicator, steam stop valve, etc.
- Table 2.1 gives function and location of various boiler mountings :

Sr. No.	Boiler mounting	Location	Function
1.	Bourdon's pressure gauge	It is attached on the upper part of the front end plate.	It is used to indicate the steam pressure in the boiler.
2.	Safety valves	It is attached on the top of front end plate.	It is used to release the excess steam when the steam pressure inside the boiler exceeds.
3.	Water level indicator	It is attached to the lower part of front end plate.	It is used to indicate the water level inside the boiler.
4.	Fusible plug	It is fitted over the crown of the furnace or over the combustion chamber.	It is used to put off the fire in the furnace of boiler when the water level falls below unsafe level.
5.	Feed check valve	It is fitted to the shell below the water level of the boiler.	It is used to allow the supply of water at high pressure to the boiler and prevent the back flow of water.
6.	Blow-off cock	It is fitted to the lowest part of the boiler shell.	It is used to empty the boiler for cleaning, repair and inspection. It is also used to discharge the mud and sediments carried with the feed water.
7.	Steam stop valve	It is fitted to the highest part of the boiler shell.	It is used to regulate the flow of steam from the boiler to the engine and shut off the steam flow when not required.

Table 2.1 Location and function of various boiler mountings

Boiler accessories

- The auxillary parts which are used to increase the overall efficiency of the plant are called as **boiler accessories.**
- For example : Economiser, air-preheater, water feeding equipment, superheater, etc.
- Table 2.2 gives the function and location of various boiler accessories :

Sr. No.	Boiler accessory	Location	Function
1.	Economiser	It is fitted at the passage of flue gases from the boiler to chimney.	It extracts the waste heat of the chimney gases to preheat the water before feeding into the boiler. This reduces fuel consumption.
2.	Air-preheater	It is placed after the economiser and before the gases enter the chimney.	It extracts the waste heat of the flue gases and pre-heat the air supplied to the combustion chamber. It reduces fuel consumption.
3.	Superheater	It is fitted in the path of flue gases flowing to the chimney.	It is used to increase the temperature of the steam above its saturation temperature by passing the steam through a small set of tubes and hot gases over them.

Table 2.2 Location and function of various boiler accessories

2.9.1 Boiler Mountings

- Following are the common boiler mountings used

1) Pressure gauge 2) Water level indicator

3) Safety valve 4) Fusible plug

5) Steam stop valve

- They are explained below

1) Pressure Gauge

- The Fig. 2.8 shows pressure gauge.
- Pressure gauge is used to read the pressure in boiler.
- It controls boiler operation.

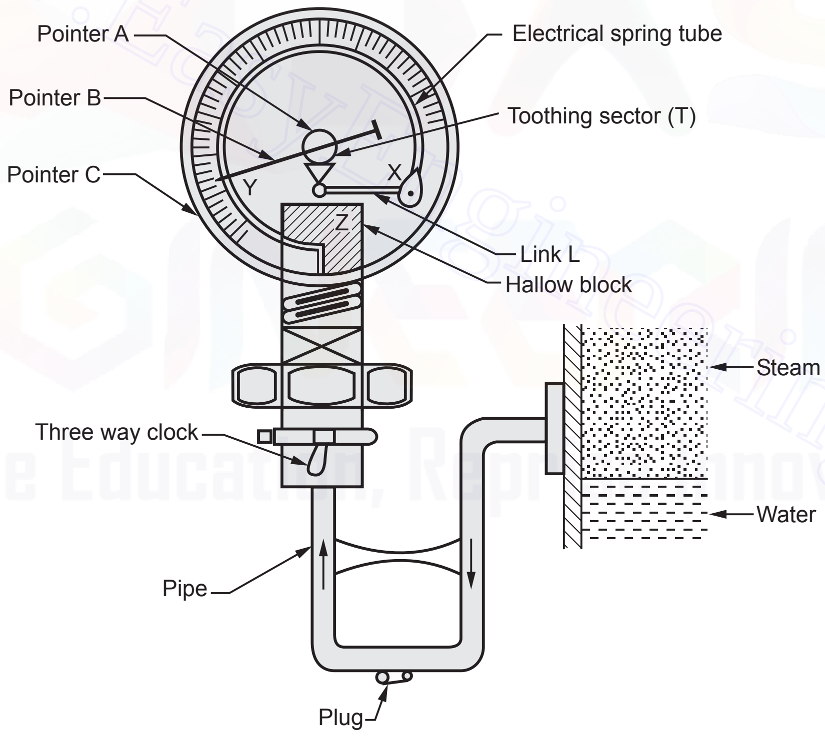


Fig. 2.8 : Pressure gauge

2) Water Level Indicator

- The Fig. 2.9 shows water level indicator.
- Water level indicator is used to measure level of water in water drum and tube.

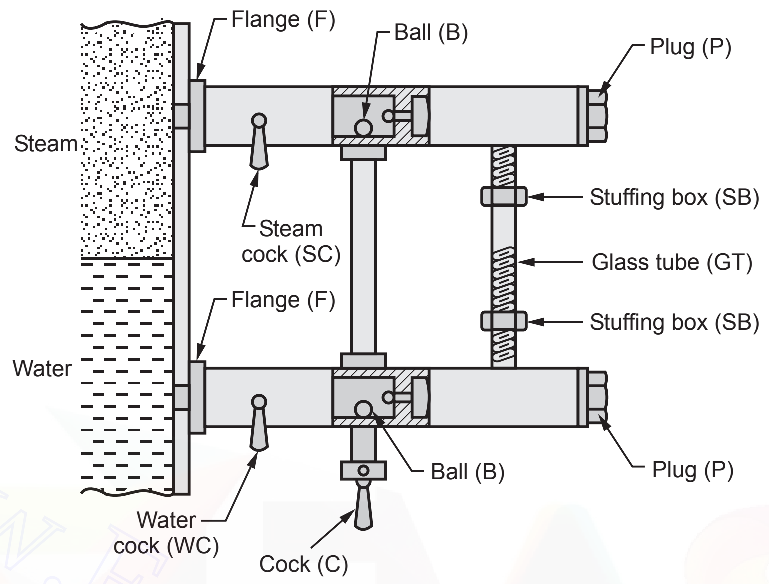


Fig. 2.9 : Water level indicator

3) Safety Valve

- Fig. 2.10 shows safety valve.

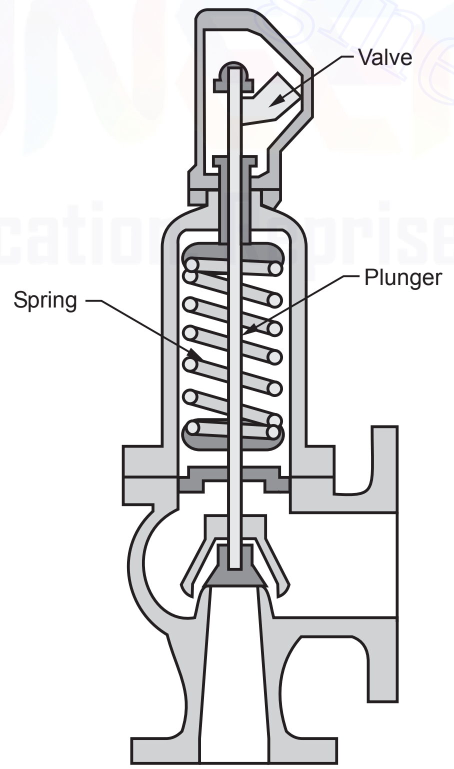


Fig. 2.10 : Safety value

- Safety valve is used to bypass the steam under critical condition.

4) Fusible Plug

- The Fig. 2.11 diagram shows fusible plug.
- Fusible plug present in furnace and measures furnace temperature.

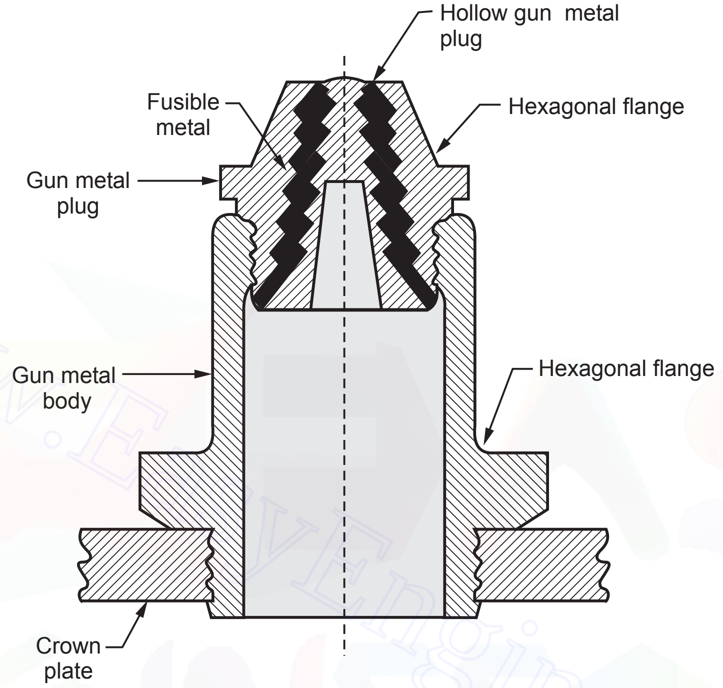


Fig. 2.11 : Fusible plug

5) Steam Stop Valve

- The Fig. 2.12 shows steam stop valve.
- It is used to control the steam under risk condition.

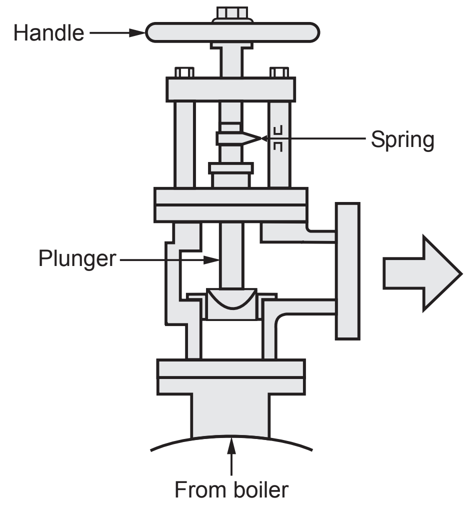


Fig. 2.12 : Steam stop value

2.9.2 Boiler Accessories

Following are the common boiler accessories used.

1) Super heater
2) Economizer
3) Air-preheater

- They are explained below.

1) Super Heater

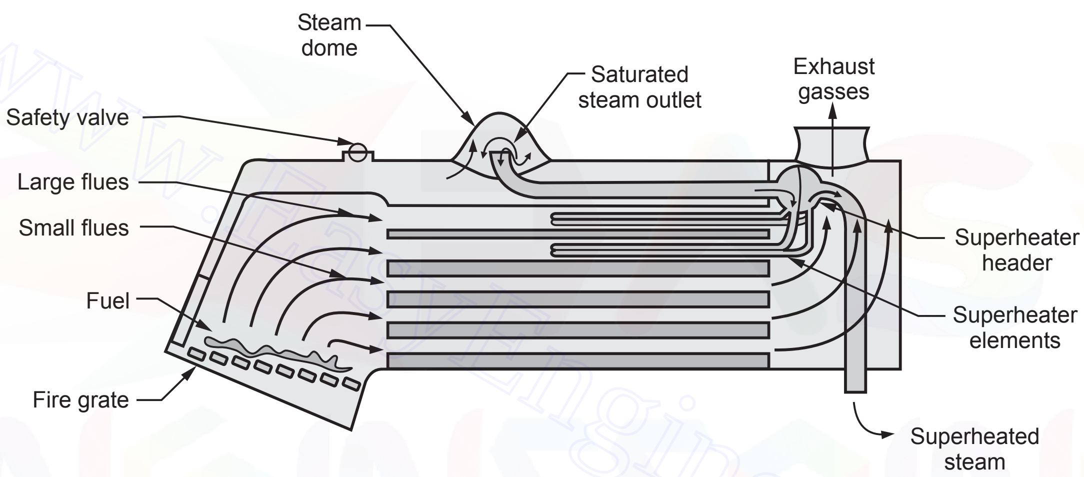


Fig. 2.13 : Super heater

- The diagram shows super heater.
- The super heaters are used to make steam to super heated with high temperature.
- Super heated steam increases quality of steam and turbine life.

2) Economizer

- The Fig. 2.14 diagram shows economizer.

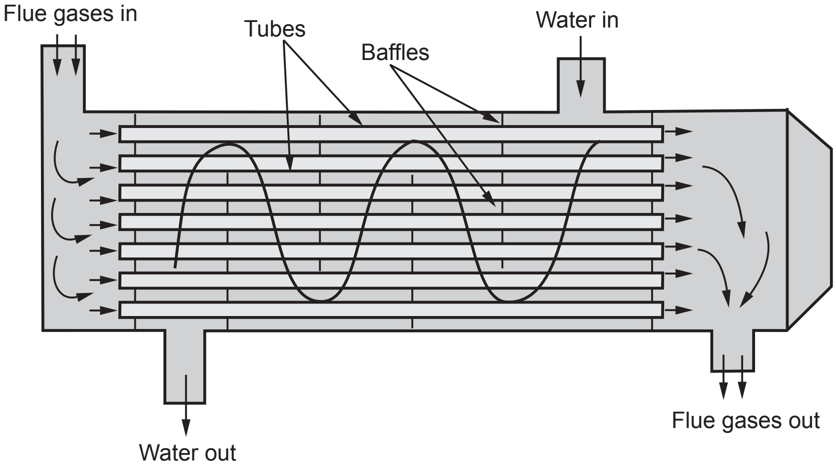


Fig. 2.14 : Economizer

- Economizer is used to pre-heat the water.
- Economizer improves the efficiency of boiler.

3) Air pre-heater

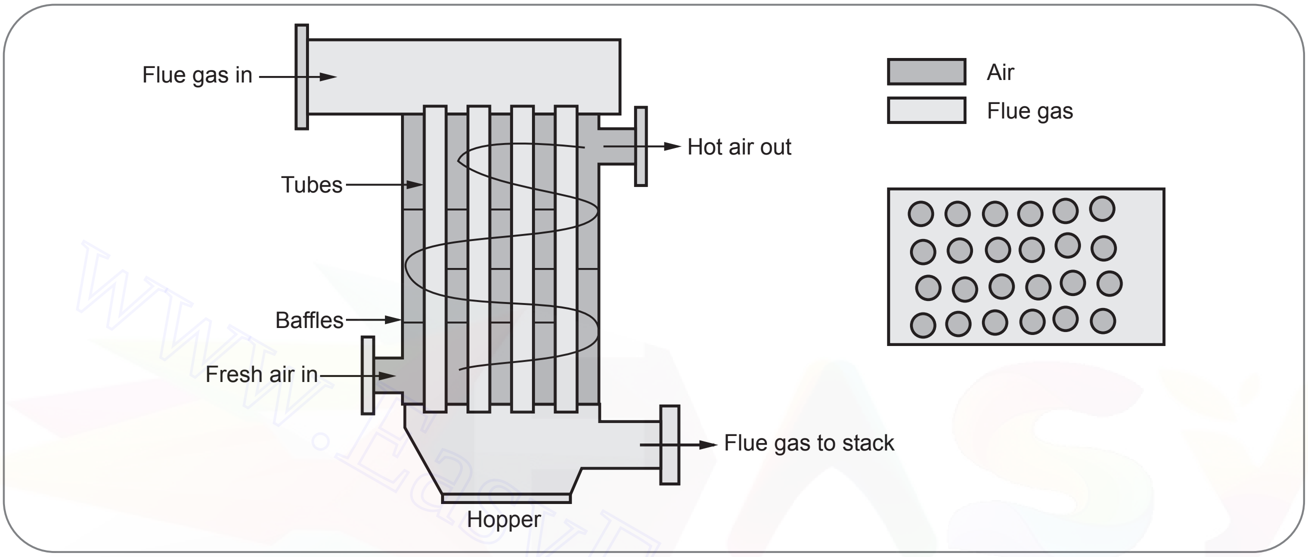


Fig. 2.15 : Air pre-heater

- The above diagram shows Air pre - heater.
- Air pre - heater is used to pre-heat the air for combustion.
- Air pre - heater improves efficiency of boiler.

2.10 Introduction to IBR

IBR is a Indian boiler regulation, the boilers are manufactured in India follows :

- The standards of IBR
- The boiler house plan and design to be approved by IBR.
- The scale of the boiler and method for manufacturing to be approved by IBR Engineer.
- Chimney height should not be exceed 30 m from floor space.
- Chimney should be printed with proper color strips.
- The required height if the chimney to be submitted to IBR.
- The emissions of chimney should not be exceeds government standard.
- Required mountings and accessories to be added to boiler.

2.11 Analysis of Boiler

- The analysis of boiler includes to study the performance of the boiler.
- The following parameters are required to study the performance of the boiler.

1) Evaporation or Evaporation capacity of the boiler

- The evaporation is also termed as capacity of boiler.
- The amount of steam generated by a boiler in kg/hr at full load condition.

2) Boiler efficiency

It is a ratio of boiler output (steam) to the heat supplied (fuel).

$$\eta_b = \frac{m_s (h - h_{f1})}{m_f \times CV}$$

where m_s = Mass of the steam in kg/hr

h = enthalpy of the boiler kJ /kg

h_{f1} = enthalpy of economizer in kJ/kg

m_f = Mass of fuel in kg/hr

CV = Calorific value kJ/kg.

To find h = ?

$$h_{wet} = h_f + x\, h_{fg}$$

$$h_{liq} = h_f + h_{fg}$$

$$h_{sup} = h_f + h_{fg} + C_{ps} [T_{sup} - T_s]$$

3) Equivalent evaporation : m_e

The amount of water evaporated from feed water at 100 °C and converted into dry saturated at same temperature and standard atmospheric pressure is known as equivalent evaporation.

$$(m_e) = \frac{\dot{m}_s (h - h_{f1})}{2257}$$

$$\dot{m}_s = \frac{m_s}{m_f} \quad \text{kg/kg of coal}$$

Solved Examples

Ex. 2.1 : *5500 kg/hr steam is produced as a pressure of 7.6 bar in a boiler with a dryness fraction of 0.98. The feed water temperature is 51 °C. The amount of coal burn is 650 kg having CV of 30500 kJ/kg. Determine the following 1) Boiler efficiency 2) Equivalent evaporation.*

Sol. : Given Data : m_s = 5500 kg/hr, P_v = 7.6 bar, x = 0.98,

Feed water temperature = transmises = 51 °C (T_W),

CV = 30500 kJ/kg, m_f = 650 kg

1) Efficiency of boiler (η_b) :

$$\eta = \frac{m_s (h - h_{f1})}{m_f \times CV}$$

m_s = 5500/3600 = 1.52 kg/sec.

m_s = 5500 kg /hr.

$h = h_{wet} = h_f + x\, h_{fg}$ … (at p = 7.6 bar)

h = 711.7 + 0.98 (2053.7)

h = 2724.32 kJ/kg = (h_{wet})

h_{f1} = 4.187 × 51 = 213.537 kJ/kg

$$\eta_b = \frac{5500(2724.32 - 213.537)}{650 \times 30500}$$

$$\eta = \mathbf{69.65\ \%}$$

2) Equivalent evaporation (m_e) :

$$\dot{m}_s = \frac{m_s}{m_f} = 8.46$$

$$m_e = \frac{\dot{m}_s (h - h_{f1})}{2257}$$

$$m_e = \frac{8.46\ (2510.78)}{2257}$$

$$m_e = \mathbf{9.41\ kg/kg\ of\ fuel}$$

Ex. 2.2 : *Water tube boiler produce 6000 kg/hr steam at a pressure of 10.5 bar and consumes a coal of 10.83 kg/min. The steam produced has a temperature of 250 °C. The CV of fuel is 30500 kJ/kg. The water initially enters into the economizer and have a temperature of 49 °C. Determine the following 1)* η_b *2)* m_e

Sol. : Given Data : P = 10.5 bar, m_s = 6000 kg/hr,

T_W = 49 °C, CV = 30500 kJ/kg, T_{sup} = 250 °C,

m_f = 10.83 kg/min, m_f = 10.83×60 = 649.8 kg/hr,

$h = h_f + h_{fg} + m \cdot C_{ps} [T_{sup} - t_s]$

$= 772 + 2006 + 2.1\ [250 - 182]$,

h = 2920.8 kJ/kg, h_{f1} = 49×4.187 = 205.163 kJ/kg

1) Boiler Efficiency (η_b)

$$\eta_b = \frac{m_s [h - h_{f1}]}{m_f \times CV}$$

$$= \frac{6000\ [2920.8 - 205.163]}{649.8 \times 30500}$$

$$\eta_b = \mathbf{82.21\%}$$

2) Equivalent evaporation (m_e)

$$m_e = \frac{\dot{m}_s (h - h_{f1})}{2257}$$

$$\dot{m}_s = \frac{m_s}{m_f} = 9.23$$

$$m_e = \mathbf{11.109 \text{ kg/kg of fuel.}}$$

Ex. 2.3 : *A boiler evaporates 3.6 kg of water per kg of coal into dry saturated steam at 10 bar. The temperature of feed water is 32 °C. Find equivalent evaporation from and at 100 °C.*

Sol. : Given Data : $\frac{m_s}{m_f} = 3.6$ kg, $P_b = 10$ bar, $T_w = 32$ °C

To find : m_e

$$\therefore \quad m_e = \frac{m_s (h - h_{f1})}{2257} \quad \frac{3.6 (h_f + h_{fg} - h_{f1})}{2257}$$

$$m_e = \frac{3.6 \; (762.6 + 2013.6 - 32 \times 4.187)}{2257}$$

$$\mathbf{m_e = 4.2 \text{ kg/kg of fuel}}$$

Ex. 2.4 : *In a boiler test 1250 kg of coal consumed in 24 hrs. Mass of water evaporated is 13000 kg and mean effective pressure is 7 bar. Feed water temperature was 40 °C and heating value of coal is 30,000 kJ/kg. Taking enthalpy of 1 kg of steam at 7 bar as 2570 kJ, find equivalent evaporation per kg of coal and boiler efficiency.*

Sol.: Given Data : $m_f = 1250$ kg, t = 24 hrs., $m_s = 13000$ kg, P_b= 7 bar, $T_w = 40$ °C, CV = 30,000 kJ/kg, h = 2570 kJ/kg.

To find : i) m_e ii) η_b

i) Equivalent evaporation (m_e)

$$\therefore \quad m_e = \frac{\dot{m}_s (h - h_{f1})}{2257}$$

$$= \frac{10.4 \; (2570 - 40 \times 4.187)}{2257}$$

$$\mathbf{m_e = 11.07 \text{ kg/kg of fuel.}} \quad \text{... Ans.}$$

ii) Boiler efficiency (η_b)

$$\eta_b = \frac{m_s (h - h_{f1})}{m_f \times CV} = \frac{13000 \; (2570 - 40 \times 4.187)}{1250 \times 30{,}000}$$

$$\mathbf{\eta_b = 0.8328 \text{ or } 83.28 \; \%} \quad \text{...Ans.}$$

Ex. 2.5 : *The following particulars refer to a steam power plant consisting of a boiler, superheater and economizer. Steam pressure = 20 bar, Mass of steam generated = 10000 kg/hr., Mass of coal used = 1300 kg/hr., CV for coal = 29000 kJ/kg. Temperature of feed water entering the economizer = 35°. temperature of feed water leaving the economizer = 105 °C., Dryness fraction of the steam leaving the boiler = 0.98. Temperature of steam leaving the superheater = 350 °C. Determine -*

1) Overall efficiency of the boiler plant.'

2) Equivalent evaporation of the given boiler from and at 100 °C . in kg of steam generated/kg of coal burnt and

3) Percentage of heat utilized in economizer, boiler and super heater.

Sol. : Given Data : P = 20 bar, m_s = 10,000 kg/hr, m_f = 1300 kg/hr, CV = 29000 kJ/kg, T_{w1} = 35 °C, T_{w2} = 105 °C, T_{sup} = 350 °C, x = 0.98

To find : i) η_R ii) m_e

iii) Percentage of heat utilized in boiler, super heater and economizer.

i) Overall efficiency of plant (η_{over}) :

Finally the boiler is producing heated steam.

$$\therefore \quad \eta_{over} = \frac{m_s (h_{sup} - h)}{m_f \times CV}$$

$$h_{sup} = h_f + h_{fg} + C_{ps} [T_{sup} - t_s]$$

$$= 908.69 + 1890.0 + 2.1 \; [350 - 212]$$

$$h_{sup} = 3088.49 \text{ kJ/kg.}$$

$$\eta_{over} = \frac{10{,}000 \; (3088.49 - 35 \times 4.187)}{1300 \times 29000}$$

$$\mathbf{\eta_{over} = 0.7803 \text{ or } 78.03 \; \%}$$

ii) Equivalent evaporation (m_e) :

$$M_e = \frac{\dot{m}_s (h_{sup} - h_{f2})}{2257}$$

$$\dot{m}_s = \frac{m_s}{m_f} = \frac{10000}{1300} = 7.692 \text{ kg/kg of fuel.}$$

$$\therefore \quad m_e = \frac{7.692\,(3088.49 - 105 \times 4.187)}{2257}$$

$$= \mathbf{9.027 \text{ kg/kg of fuel}}$$

iii) Percentage of heat utilized

- **Heat utilized in economizer :**

$$= \frac{\dot{m}_s (h_{f2} - h_{f1})}{CV}$$

$$= \frac{7.692\ (105 \times 4.187 - 35 \times 4.187)}{29000} \times 100$$

% heat in economizer = **7.77 %**

- **Percentage of heat utilized in superheater**

$$= \frac{\dot{m}_s (h_{sup} - h)}{CV}$$

$$h = h_f + x\ h_{fg}$$

$$= 908.69 + 0.98\ (1890.0)$$

$$= \mathbf{2760.89 \text{ kg/kg.}}$$

% of heat used in super heater $= \frac{7.692\ (3088.49 - 2760.89)}{29000} \times 100$

$$= \mathbf{8.68\ \%}$$

- **Percentage of heat utilized boiler**

$$= \frac{\dot{m}_s (h - h_{f_2})}{CV}$$

$$= \frac{7.692\ (2760.89 - 105 \times 4.187)}{29000} \times 100$$

$\therefore$ Percentage of heat utilized in boiler = **61.5%**

Ex. 2.6 : *A boiler working at a pressure of 1.4 MN / m^2 evaporates 8 kg of water per kg of coal fired from feed water at 39 °C. The steam at the stop valve is 95 % . Determine the equivalent evaporation from and at 100 °C in kg steam/kg coal.*

Sol. : Given Data :

$P_b = 1.4 \text{ MN / m}^2 = \frac{1.4 \times 10^6}{10^5} = 14$ bar; $x = 0.95$,

$\dot{m}_s = 8$ kg/kg of fuel, $T_w = 39$ °C

To find : i) m_e

i) Equivalent evaporation (m_e)

$$\therefore \quad m_e = \frac{\dot{m}_s (h - h_{f1})}{2257}$$

$$h = h_f + x\,h_{fg}$$

$$= 830.1 + 0.95 \times (1957.7)$$

$$= 2689.9 \text{ kJ/kg}$$

$$h_{f1} = T_w \times 4.187 = 39 \times 4.187$$

$$= 163.29 \text{ kJ/kg.}$$

$$\therefore \quad m_e = \frac{8\,(2689.9 - 163.29)}{2257}$$

$$= \mathbf{8.955 \text{ kg/kg of fuel}}$$

2.12 Solved Examples

Ex. 2.7 : *The following readings were recorded during boiler trial of 6 hour duration : Pressure of steam generated -12 bar, Mass of steam generated - 40000 kg, Dryness fraction of steam generated - 0.85, Feed water temperature - 30 °C, Coal used - 4000 kg. Calorific value of coal -*
33400 kJ/kg, Find : i) Factor of equivalent evaporation, ii) Equivalent evporation from and at 100 °C iii) Efficiency of boiler.

Sol. : Given data : $p = 12$ bar, $m_s = 40000$ kg, $x = 0.85$, $T_w = 30$ °C, $m_f = 4000$ kg,
CV = 33400 kJ/kg

To find : i) Factor of equivalent evaporation

ii) Equivalent evaporation from and at °C

iii) Efficiency of boiler

Step 1 : Calculate equivalent evaporation

$$m_e = \frac{\dot{m}_s(h - h_{f1})}{2257}$$

$$\dot{m}_s = \frac{m_s}{m_f} = \frac{40000}{4000} = 10 \text{ kg/kg of fuel}$$

$$\therefore \quad m_e = \frac{10\,(2782.7 - 30 \times 4.187)}{2257}$$

$$\therefore \quad \mathbf{m_e = 11.77 \text{ kg/kg of fuel}} \qquad \text{... Ans.}$$

Step 2 : Calculate efficiency of boiler

$$\eta_b = \frac{m_s(h - h_f)}{\dot{m}_f \times CV}$$

$$= \frac{40000\,(2782.7 - 30 \times 4.187)}{4000 \times 33400}$$

$$\therefore \quad \mathbf{\eta_b = 0.7955 \text{ or } 79.55\ \%} \qquad \text{... Ans.}$$

Step 3 : Calculate factor of equivalent evaporation :

$$f_e = \frac{(h - h_{f1})}{2257} = \frac{(2782.7 - (30 \times 4.187))}{2257}$$

$$\therefore \quad \mathbf{f_e = 1.177} \qquad \text{... Ans.}$$

Ex. 2.8 : *The following data relates to a trial on boiler using economizer air preheater and superheater : Condition of steam at exit of boiler = 20 bar, 0.96 dry, Temperature of steam at exit of superheater = 300 °C, Steam evaporation rate / kg of fuel = 12 kg, Room temperature,* t_0 *= 25 °C, Temperature of feed water at exit of economizer,* t_1 *= 50 °C, Temperature of air at exit of air preheater,* t_a *= 70 °C, The temperature of flue gases at inlet to superheater economizer, air preheater and exit of air preheater are respectively 650 °C, 430 °C, 300 °C, and 180 °C respectively. Assume that air supplied is 19 kg/kg of fuel of calorific value of 45,000 kJ/kg, find :*

i) Equivalent evaporation with and without economizer, from and at 100 °C.

ii) Thermal efficiency of the boiler with and without economizer.

Sol. : Given data, : P_b = 20 bar, x = 0.96 dry, T_{sup} = 300 °C, $\dot{m}_s$ = 12 kg/kg of fuel, t_0 = 25 °C, t_1 = 50 °C, t_α = 70 °C, T_1 = 650 °C, T_2 = 430 °C, T_3 = 300 °C, T_4 = 180 °C, CV = 45,000 kJ/kg, $\dot{m}_a$ = 19 kg/kg of fuel.

To find :

i) Equivalent evaporation with and without economizer at 100 °C.

ii) Thermal efficiency of boiler with and without economizer.

Step 1 : Calculate equivalent evaporation with and without economizer at 100 °C.

i) With economizer :

$$m_e = \frac{\dot{m}_s(h - h_{f1})}{2257}$$

$$= h_f + x h_{fg} = 908.5 + 0.96\,(1888.7)$$

$$= 2721.652 \text{ kJ/kg}$$

$$h_{f1} = 50 \times 4.187 = 209.35$$

$$\therefore \quad m_e = \frac{12\,(2721.652 - 209.35)}{2257}$$

$$\mathbf{m_e = 13.357 \text{ kJ/kg of fuel}} \qquad \text{... Ans.}$$

ii) Without economizer :

$$m_e = \frac{\dot{m}_s(h)}{2257} = \frac{12 \times 2721.652}{2257}$$

$$\therefore \quad \mathbf{m_e = 14.47 \text{ kg/kg of fuel}} \qquad \text{...Ans.}$$

Step 2 : Calculate thermal efficiency of boiler with and without economizer

$$\eta_{th} = \frac{\dot{m}_s(h - h_{f1})}{CV}$$

$$= \frac{12\,(2721.652 - 209.35)}{45{,}000}$$

$$\mathbf{\eta_{th} = 0.6699 = 66.99\ \%} \qquad \text{... Ans.}$$

ii) Without economizer

$$\eta_{th} = \frac{\dot{m}_s(h)}{CV} = \frac{12\,(2721.652)}{45{,}000}$$

$$\mathbf{\eta_{th} = 0.7257 = 72.57\ \%} \qquad \text{... Ans.}$$

Ex. 2.9 : *In a boiler test 1250 kg of coal is consumed in 24 hours, mass of water evaporated is 13000 kg and boiler pressure of 7 bar. Feed water temperature was 40 ºC and heating value of coal is 30000 kJ/kg. Find equivalent evaporation per kg of coal and boiler efficiency. (Take enthalpy of 1 kg of steam at boiler exit as 2570 kJ/kg).*

Sol. : Given Data : m_f = 1250 kg, t = 24 hrs., m_s = 13000 kg, P_b= 7 bar, T_w = 40 ºC, CV = 30,000 kJ/kg, h = 2570 kJ/kg.

To find : i) m_e ii) η_b

i) Equivalent evaporation (m_e)

$$\therefore \quad m_e = \frac{\dot{m}_s (h - h_{f1})}{2257}$$

$$= \frac{10.4 \ (2570 - 40 \times 4.187)}{2257}$$

$$\mathbf{m_e = 11.07 \text{ kg/kg of fuel.}} \quad \text{... Ans.}$$

ii) Boiler efficiency (η_b)

$$\eta_b = \frac{m_s (h - h_{f1})}{m_f \times CV}$$

$$= \frac{13000\ (2570 - 40 \times 4.187)}{1250 \times 30{,}000}$$

$$\eta_b = \mathbf{0.8328 \text{ or } 83.28\ \%} \quad \text{...Ans.}$$

2.13 Heat Balance Sheet

- Heat balanced sheet is used to determine utilization of heat in the boiler to determine steam generation.

2.13.1 Steps Involved in Heat Balance Sheet

Step - I : Heat supplied Q_A

$$Q_A = m_f \times CV$$

Step - II : Heat used to convert steam (Q_B)

$$Q_B = \dot{m}_s (h - h_{f1})$$

$$\therefore \quad \dot{m}_s = \frac{m_s}{m_f}$$

Step - III : Heat carried away by hot flue gases (Q_C)

$$Q_C = m_g \times C_{pg} \times (T_f - T_i)$$

Step - IV : Heat carried away by moisture (Q_D)

$$Q_D = M_m [h_f + h_{fg} + C_{ps} [T_{sup} - T_s] - h_{f2}]$$

where h_{f2} = Enthalpy of boiler house

Step - V : Heat un-accounted (Q_E)

$$Q_E = Q_A - [Q_B + Q_C + Q_D]$$

Heat supplied Q_A	kJ	Heat used	kJ	%
		Q_B = Heat used convert steam	Q_B.......kJ	%
Q_A = $m_f \times CV$ for 1 kg fuel	Q_A =...... kJ			
		Q_C = Heat by hot flue gases	Q_C...... kJ	%
		Q_D = Heat by moisture	Q_D...... kJ	%
		Q_E = Heat un-accounted	Q_E......kJ	%
	100 %			100 %

2.14 Solved Examples

Ex. 2.10 : *The following observation were recorded during boiler trial fuel used = 65 kg /hr mass of the steam = 540 kg/hr at 10 bar moisture in fuel is 2 % by mass. The mass of dry fuel gases is 9 kg /kg of fuel the lower calorific value is 32000 kJ/kg. The temperature of fuel gases is 325 °C temperature of boiler is 28 °C feed water temperature is 50 °C. The dryness fraction of the steam is 0.95 specific heat of gas is 1 kJ/kg K specific heat of superheated steam is 2.3 kJ/ kg K. Determine the following :*

i) Boiler efficiency ii) Equivalent evaporation

iii) Prepare the energy balance sheet

Sol. : Given data : $m_f = 65$ kg/hr, $m_s = 540$ kg/hr,

$m_m = \frac{2}{100} = 0.02$ /kg, $m_g = 9$ kg/kg of fuel,

$P_b = 10$ bar, x = 0.95, CV = 32000 kJ/kg,

$T_f = 325$ °C, $T_b = 28$ °C, $T_w = 50$ °C, (economizer)

$C_{pg} = 1$ kJ/kg K,

$C_{ps} = 2.3$ kJ/kg K

i) Boiler efficiency (η_b)

$$\eta_b = \frac{m_s (h - h_{f1})}{m_f \times CV} = \frac{540 (h - h_{f1})}{65 \times 32000}$$

$$= \frac{540 (2675.52 - 209.35)}{65 \times 32000}$$

$$h = h_f + x h_{fg} = 762.6 + (0.95)\ 2013.6$$

$$= 2675.52 \text{ kJ/kg.}$$

$$h_{f1} = 50 \times 4.187 = 209.35 \text{ kJ/kg}$$

$$\therefore \quad \eta_b = \mathbf{64.03\ \%}$$

Step - I : Heat supplied Q_A

$$Q_A = m_f \times CV = (1 - 0.02) \times 32000$$

$$Q_A = \mathbf{31360 \text{ kJ/kg}}$$

Step - II : Heat used to convert steam (Q_B)

$$Q_B = \dot{m}_s (h - h_{f1})$$

$$\therefore \quad \dot{m}_s = \frac{m_s}{m_f} = \frac{540}{65} = 8.3076 \text{ kg/kg fuel}$$

$$\therefore \quad Q_B = 8.3076\ (2675.52 - 209.35)$$

$$= 20494 \text{ kJ/kg}$$

$$M_e = \frac{\dot{m}_s (h - h_{fg})}{2257} = \frac{8.30\ (2675.52 - 209.35)}{2257}$$

$$M_e = \mathbf{9.069 \text{ kg/kg of fuel}}$$

Step - III : $Q_{t1} = 1$ Heat carried hot flue gases (Q_C)

$$Q_C = m_g \times C_{pg} \times (T_f - T_b)$$

$$= 9 \times 1 \times [325 - 28]$$

$$Q_C = \mathbf{2673 \text{ kJ/kg}}$$

Step - IV ': Heat carried away by moisture (Q_D)

$$Q_D = M_m [h_f + h_{fg} + C_{ps} [T_{sup} - T_s] - h_{f2}]$$

$$= 0.02\ [762.6 + 2013.6 + 2.3\ (325 - 179.9) - 117.236]$$

$$Q_D = \mathbf{59.85 \text{ kJ/kg}}$$

$$Q_E = Q_A - [Q_B + Q_C + Q_D]$$

$$= 31360 - [20494 + 2673 + 59.85]$$

$$Q_E = \mathbf{8133.15 \text{ kJ/kg}}$$

Heat supplied Q_A	kJ	Heat used	kJ/kg	%
$Q_A = m_f \times CV$ for 1 kg. of fuel.	31360	Q_B	20494	65.33
		Q_C	2673	8.52
		Q_D	59.85	0.19
		Q_E	8133.15	25.99
	100 %	**Total**	**31360**	**99.98 %**

Ex. 2.11 : *In the boiler trial following observations are recorded mass of fuel is 1520 kg/hr. The temperature of feed water is 30 °C dryness fraction of steam is 0.95. The pressure of steam is 8.5 bar coal burns per hr 200 kg. Calorific value of the coal is 27,300 kJ/kg. The unburned coal collected is 16 kg /hr, CV of 2000 kJ/kg. The mass of fuel is 17.73 kg/kg of the coal. The temperature of the fuel gas 330 °C. The boiler room temperature is 17 °C, Mean specific heat of the fuel gas 1 kJ/kg K estimate thermal efficiency and draw heat balance sheet.*

Sol. : Given data : $m_s = 1520$ kg/hr, $T_w = 30$ °C,

$T_b = 17$ °C, $T_f = 330$ °C, x = 0.95, $P_b = 8.5$ bar,

CV = 27300 kJ/kg, $m_f = 200$ kg /hr,

$m_g = 17.73$ kg/kg of fuel

To find : i) Boiler efficiency (η_b)

ii) Heat balance sheet

i) Boiler efficiency (η_b) :

$$\eta_b = \frac{m_s (h - h_{f1})}{m_f \times CV}$$

$$h = h_f + x h_{fg} = 729.9 + 0.95\ (2039.6)$$

$$= 2667.52 \text{ kJ/kg}$$

$$h_{f1} = 30 \times 4.187 = 125.61 \text{ kJ/kg}$$

$$\eta_b = \frac{m_s (h - h_{f1})}{CV \times m_f}$$

$$= \frac{1520 (2667.52 - 125.61)}{200 \times 27300} = \mathbf{70.7\ \%}$$

ii) Heat balance sheet :

Step - I : Heat supplied (Q_A)

$$Q_A = m_f \times CV = 1 \times 27300 = 27300 \text{ kJ/kg.}$$

Step II : Heat converted into steam (Q_B)

$$Q_B = \dot{m}_s (h - h_{f1})$$

$$\therefore \quad \dot{m}_s = \frac{m_s}{m_f} = \frac{1520}{200} = 7.6 \text{ kg/kg fuel}$$

$$h = 2667.52 \text{ kJ/kg}, \ h_{f1} = 125.61 \text{ kJ/kg}$$

$$Q_B = 7.6\ (2667.52 - 125.61)$$

$$= 19318.51 \text{ kJ/kg}$$

Step III : Heat carried away by flue gases (Q_C)

$$Q_C = m_g \times C_{pg} \times (T_f - T_b) = 17.73 \times 1 \times [330 - 17]$$

$$Q_C = \mathbf{5549.49\ kJ/kg}$$

Step IV : ' Heat carried away by ash (Q_D)

$$Q_D = m_{ash} \times C \cdot V = \frac{16}{200} \times 2000 = 160 \text{ kJ/kg}$$

$$Q_D = \mathbf{160\ kJ/kg}$$

Step V : Heat unaccounted losses (Q_E)'

$$Q_E = Q_A - [Q_B + Q_C + Q_D]$$

$$= 27300 - [19318.5 + 5549.49 + 160]$$

$$Q_E = \mathbf{2272\ kJ/kg}$$

Heat supplied (Q_A)	kJ/kg	Heat used	kJ/kg	%
$Q_A = m_f \times CV$ for 1 kg of fuel.	27300	Q_B	19318.51	70.7
		Q_C	5549.49	20.3
		Q_D	160	0.58
		Q_E	2272	8.3
	27300	**Total**	**27300**	**99.88**

Ex. 2.12 : *The following results obtained from boiler trial :*

a. Feed water per hour = 700 kg at 27 °C

b. Steam pressure = 8 bar of dryness 0.97.

c. Coal consumption = 100 kg/hr.

d. C.V. of coal = 25000 kJ/kg.

e. Unburnt coal collected = 0.6 kg/hr.

f. Flue gas formed per kg of fuel = 17.3 kg at 327 °C (C_p of flue gas = 1.025 kJ/kg K.)

g. Room temperature = 16 °C

Draw the heat balance sheet on kJ/min basis and boiler efficiency.

Sol.: Given data : $m_s = 700$ kg/hr, $T_w = 27$ ºC, $P_b = 8$ bar, $x = 0.97$, $m_f = 100$ kg/hr, $CV = 25000$ kJ/kg, $m_g = 17.3$ kg/hr, $T_f = 325$ ºC, $T_i = 16$ ºC, $C_{pg} = 1.025$ kJ/Kg K.

To find : i) Draw heat balance sheet of kJ/min basis

ii) Boiler efficiency (η_b) **:**

i) Heat balance sheet on kJ/min basis :

Step I : Heat supplied (Q_A)

$$Q_A = m_f \times CV$$

$$= \frac{100}{60} \times 25000 = \mathbf{41666.7\ kJ/min.}$$

Step II : Heat converted into steam (Q_B)

$$Q_B = m_s (h - h_{f1}) = \frac{700}{60} (h_f + x\, h_{fg} - 4.18 \times 27)$$

$$Q_B = \frac{700}{60} (720.9 + 0.97 \times 2046.5 - 4.18 \times 27)$$

$$= \mathbf{30253.35\ kJ/min.}$$

Step III : Heat carried away by flue gases (Q_C)

$$Q_C = m_g \times C_{pg} \times (T_f - T_i)$$

$$= 17.3 \times \frac{100}{60} \times 1.025 (327 - 16)$$

$$Q_C = \mathbf{9191.3\ kJ/kg}$$

Step IV : Heat carried away by ash (Q_D)'

$$Q_D = \frac{7.5}{60} \times 2000 = \mathbf{250.0\ kJ/min}$$

Step V : 'Heat Unaccounted (Q_E)

$$Q_E = Q_A - [Q_B + Q_C + Q_D + Q_E]$$

$$Q_E = 41666.7 - [30253.35 + 9191.3 + 250]$$

$$Q_E = \mathbf{1972.05\ kJ/min}$$

Heat balance sheet on minute basis :

Heat supplied (Q_A)	kJ/min	Heat used	kJ/min	%
$Q_A = m_f \times CV$ for total fuel	41666.7	Q_B	30253.35	72.6
		Q_C	9191.3	22
		Q_D	250.0	0.59
		Q_E	1972.05	4.74
	41666.7		**41666.7**	**99.93**

ii) Boiler efficiency (η_b)

$$\eta_b = \frac{m_s(h - h_{f1})}{m_f \times CV}$$

$$= \frac{700\,(720.9 + 0.97 \times 2046.5 - 4.18 \times 27)}{100 \times 25000}$$

$$\eta_b = \mathbf{0.7260\ or\ 72.6\ \%}$$

Ex. 2.13 : *The following particulars were recorded during boiler working of pressure 11 bar mass of feed water is 4600 kg/hr. The temperature of feed water is 75 °C. The dryness fraction of the steam is 0.96. Coal used is 490 kg/hr. Calorific value of the coal is 35700 kJ/kg moisture in coal is 4 % by mass of dry fuel gases is 18.57 kg/kg of fuel. Temperature of fuel gases is 300 °C. Boiler house temperature is 16 °C. Specific heat of gases is 0.97 kJ/kg K. Draw a heat balance sheet of a boiler on per kg basis also determine efficiency of the boiler.*

Sol. : Given data : $P_b = 11$ bar, $m_w = 4600$ kg/hr, $T_w = 75$ °C, $x = 0.96$, $CV = 35{,}700$ kJ/kg, $m_m = 4\ \% = 0.04$, $C_{pg} = 0.97$ kJ/kg K, $m_f = 490$ kg/hr, $m_g = 18.57$ kg/kg, $T_f = 300$ °C, $T_b = 16$ °C, $\eta_b = ?$,

$$\eta_b = \frac{m_s(h - h_{f1})}{m_f \times CV},$$

$$h = h_f + x\,h_{fg}$$

$$= 781.1 + (0.96 \times 1998.6)$$

$$= 2699.75\ \text{kJ/kg}$$

$$h_{f1} = 4.187 \times 75$$

$$= 314.025\ \text{kJ/kg}$$

$$\eta_b = \frac{4600\,(2699.75 - 314.025)}{35700 \times 490}$$

$$= \mathbf{62.73\ \%}$$

Step - I : Heat supplied (Q_A)

$$Q_A = m_f \times CV = (1 - 0.04) \times 35700$$

$$= 0.96 \times 35700 = \mathbf{34272\ kJ/kg.}$$

Step II : Heat converted into steam (Q_B)

$$Q_B = \dot{m}_s (h - h_{f1})$$

$$\therefore \quad \dot{m}_s = \frac{m_s}{m_f} = \frac{4600}{490} = 9.38\ \text{kg/kg of fuel}$$

$$\therefore \quad Q_B = 9.38\ (2699.75 - 314.025)$$

$$\therefore \quad Q_B = \mathbf{22378.1005\ kJ/kg.}$$

Step III : Heat carried hot flue gases (Q_C)

$$Q_C = m_g \times C_{pg} \times (T_f - T_b)$$

$$= 18.57 \times 0.97 \times [300 - 16]$$

$$= 284 \times 18.57 \times 0.97 = \mathbf{5115.66\ kJ/kg}$$

Step IV : Heat by moisture (Q_D)

$$Q_D = M_m\,[h_f + h_f g + C_{ps}\,(T_{sup} - t_s) - h_{f2}]$$

$$h_{f2} = 4.187 \times 16 = \mathbf{66.992\ kJ/kg}$$

$$Q_D = 0.04\ [417.5 + 2257.9 + 2.1\ [300 - 99.63] - 66.99]$$

$$= 0.04\ [2675.4 + 353.785]$$

$$Q_D = 121.167\ \text{kJ/kg.}$$

Step V : Heat unaccounted (H_E)

$$Q_E = Q_A - [Q_B + Q_C + Q_D]$$

$$= 34272 - [22378.1005 + 5115.66 + 121.167]$$

$$Q_E = \mathbf{6657.07\ kJ/kg}$$

Heat supplied (Q_A) for 1 kg	kJ	Heat used	kJ	%
$Q_A = m_f \times CV$ Heat supplied for 1 kg of fuel.	Q_A = 34272 kJ/kg	Q_B = Heat convert into steam	22378.1005	65.29 %
		Q_C = Heat by hot fuel	5115.66	14.92 %
		Q_D = Heat by moisture	121.167	0.353 %
		Q_E = Heat unaccounted	6657.07	19.43 %
	100 %			99.98 %

2.15 Introduction to Fuels

- Fuels can be defined as the source of heat energy which is released in a reactive system by chemical or nuclear reaction.
- The mixture of fuel and air is called reactant's(R).
- The products of combustion formed are called products(P).
- Heat is released during the combustion process called as Exothermic reaction.
- Heat is absorbed during combustion reaction is called as Endothermic reaction.
- A chemical fuel is a substance which releases heat energy on combustion.

2.16 Classification of Fuel

Fuels can be classified according to whether :

1. They occur in nature called **primary fuels** or are prepared called **secondary fuels.**
2. They are in solid, liquid or gaseous state.

2.16.1 Solid Fuels

Coal. Its main constituents are carbon, hydrogen, oxygen, nitrogen, sulphur, moisture and ash.

- Coal passes through different stages during its formation from vegetation.
- These stages are enumerated and discussed below :
  Plant debris - Peat - Lignite - Brown coal - Sub-bituminous coal - Bituminous coal - Semi bituminous.
- Coal - Semi anthracite coal - Anthracite coal - Graphite.
- **Peat.** It is the first stage in the formation of coal from wood. It contains huge amount of moisture and

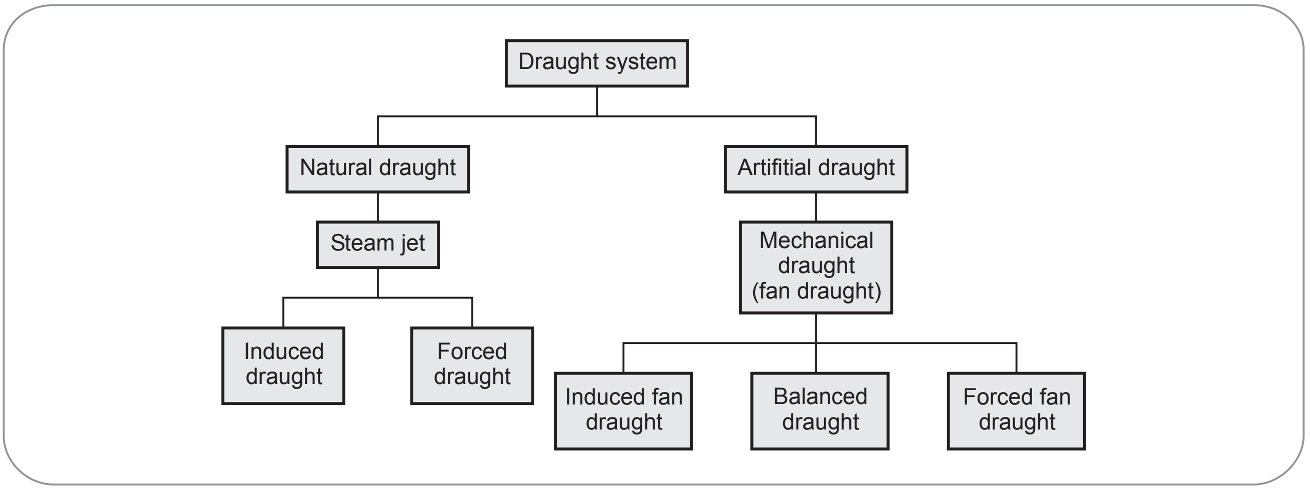


Fig. 2.16

therefore it is dried for about 1 to 2 months before it is put to use. It is used as a domestic fuel in Europe and for power generation in Russia. In India it does not come in the categories of good fuels.

- **Lignites and brown coals.** These are intermediate stages between peat and coal. They have a woody or often a clay like appearance associated with high moisture, high ash and low heat contents. Lignites are usually amorphous in character and impose transport difficulties as they break easily. They burn with a smoky flame. Some of this type are suitable for local use only.
- **Bituminous coal.** It burns with long yellow and smoky flames and has high percentages of volatile matter. The average calorific value of bituminous coal is about 31320 kJ/kg. It may be of two types namely caking or noncaking.
- **Semi-bituminous coal.** It is softer than the anthracite. It burns with a very small amount of smoke. It contains 15 to 20 percent volatile matter and has a tendency to break into small sizes during storage or transportation.
- **Semi-anthracite.** It has less fixed carbon and less lustre as compared to true anthracite and gives out longer and more luminous flames when burnt.
- **Anthracite.** It is very hard coal and has a shining black lustre. It ignites slowly unless the furnace temperature is high. It is non-caking and has high percentage of fixed carbon. It burns either with very short blue flames or without flames. The calorific value of this fuel is high to the tune of 35500 kJ/kg and as such is very suitable for steam generation.
- **Wood charcoal.** It is obtained by destructive distillation of wood. During the process the volatile matter and water are expelled. The physical properties of the residue (charcoal), however depends upon the rate of heating and temperature.
- **Coke.** It consists of carbon, mineral matter with about 2 % sulphur and small quantities of hydrogen, nitrogen and phosphorus. It is solid residue left after the destructive distillation of certain kinds of coals. It is smokeless and clear fuel and can be produced by several processes. It is mainly used in blast furnace to produce heat and at the same time to reduce the iron ore.

2.16.2 Liquid Fuels

- The chief source of liquid fuels is petroleum which is obtained from wells under the earth's crust.
- These fuels have proved more advantageous in comparison to solid fuels in the following respects.
- **Advantages :**
  - Require less space for storage.
  - Higher calorific value.
  - Easy control of consumption.
  - Staff economy.
  - Absence of danger from spontaneous combustion.
  - Easy handling and transportation.
  - Cleanliness.
  - No ash problem.
  - Non-deterioration of the oil in storage.
- **Petroleum.** There are different opinions regarding the origin of petroleum. However, now it is accepted that petroleum has originated probably form organic matter like fish and plant life etc., by bacterial action or by their distillation under pressure and heat. It consists of a mixture of gases, liquids and solid hydrocarbons with small amounts of nitrogen and sulphur compounds. In India, the main sources of petroleum are Assam and Gujarat.
- Heavy fuel oil or crude oil is imported and then refined at different refineries. The refining of crude oil supplies the most product called petrol. Petrol can also be made by polymerization of refinery gases.

  Other liquid fuels are kerosene fuels oils, colloidal fuels and alcohol.

2.16.3 Gaseous Fuels

- **Natural gas.** The main constituents of natural gas are methane (CH_4) and ethane (C_2H_6). It has calorific value nearly 21000 kJ/m^3. Natural gas is used alternately or simultaneously with oil for internal combustion engines.
- **Coal gas.** Mainly consists of hydrogen, carbon monoxide and hydrocarbons. It is prepared by carbonisation of coal. If finds its use in boilers and sometimes used for commercial purposes.
- **Coke-oven gas.** It is obtained during the production of coke by heating the bituminous coal. The volatile content of coal is driven off by heating and major portion of this gas is utilised in heating the ovens. This gas must be throughly filtered before using in gas engines.
- **Blast furnance gas.** It is obtained from smelting operation in which air is forced through layers of coke and iron ore, the example being that of pig iron manufacture where this gas is produced as by product and contains about 20 % carbon monoxide (CO). After filtering it may be blended with richer gas or used in gas engines directly. The geating value of this gas is very low.
- **Producer gas.** It results from the partial oxidation of coal, coke or peat when they are burnt with an insufficient quantity of air. It is produced in specially designed retorts. It has low heating value and in general is suitable for large installations. It is also used in steel industry for firing open hearth furnaces.
- **Water or illuminating gas.** It is produced by blowing steam into white hot coke or coal. THe decomposition of steam takes place linerating free hydrogen and oxygen in the steam combines with carbon to form carbon monoxide according to the reaction :

  $C + H_2O \rightarrow CO + H_2$

  The gas composition varies as the hydrogen content if the coal is used.
- **Sewer gas.** It is obtained form sewage disposal vats in which fermentation and decay occur. It consists of mainly marsh gas (CH_4) and is collected at large disposal plants. It works as a fuel for gas engines which in turn drive the plant pumps and agitators.

Gaseous fueld are becoming popular beause of following advantages they possess.

- **Advantages :**
  1. Better control of combustion.
  2. Much less excess air is needed for complete combustion.
  3. Economy in fuel and more efficiency of furnace operation.
  4. Easy maintenance of oxidizing or reducing atomosphere.
  5. Cleanlines.
  6. No problem of storage if the supply is available from public supply line.
  7. The distribution of gaseous fuels even over a wide area is easy through the pipe lines and as such handling of the fuel is altogether eliminated.
  8. Gaseous fuels give economy of heat and produce higher temperatures (as they can be preheated in regenerative furnaces and thus heat from hot flue gases can be recovered).

2.17 Two Marks Questions with Answers

Q.1 What is the function of a boiler ?

Ans. : • Boiler is a closed vessel that produces saturated steam at the required pressure.

Q.2 Briefly explain the boiler mounting, and name its classifications ?

Ans. : • Boiler mountings are the devise used for safety of the boiler and following are the commonly used mountings.

a. Pressure gauge b. Water level indicator

c. Fusible plug d. Steam stop valve

e. Safety valve

Q.3 What is a valve ?

Ans. : A valve is used to control the flow of fluid inside the pipeline.

Q.4 Write the function of Safety, Stop, and Feed check valves ?

Ans. : • Safety valve : This type of valve is used to keep the boiler safe by controlling the working pressure in the boiler and resist the blasting due to the high pressure the valve is mounted with boiler.

- Stop valve : This type of valve is used to control the flow of steam from the boiler to the engine.
- Feed Check valve: This type of valve is used to control the supply of feed water to the boiler. The water level always remains constant when it works.

Q.5 Why draught is produced in boiler ?

Ans. : • To provide an adequate supply of air for the fuel combustion

- To exhaust the gases of combustion from the combustion chamber
- To discharge the gases of combustion to the atmosphere through the chimney

Q.6 What is the function of a safety valve ?

Ans. : • To blow off steam when the pressure of steam inside the boiler exceeds the working pressure

Q.7 What is the function of economiser ?

Ans. : • To Increase thermal efficiency of boiler.

Q.8 State the difference between cornish boiler and lancashire boiler.

Ans. : • Former contain one fire tube type and latter contains two water tube type boiler

Q.9. What is the function of air preheater ?

Ans. : Air preheater is used to heat the air before blowing it into the furnace.

Q.10 What is the necessity of preheating the air ?

Ans. : • The air is heated by the flue gas so as to improve the performance of boiler.

Q.11 Name the various boiler mountings.

Ans. : • Boiler mountings are used to run a boiler in a safe way ex: safety valve, Water level indicators etc.

Q.12 What is draught in boiler ?

Ans. : • Boiler draught is the pressure difference between the atmosphere and the pressure inside the boiler

Q.13 What is negative draft ?

Ans. : • Negative draft is nothing but a negative pressure maintained in the boiler using induced draft fans

Q.14 What is boiler flue ?

Ans. : • A flue is a duct, pipe, or opening in a chimney for conveying exhaust gases from a fireplace, furnace, water heater, boiler, or generator to the outdoors.

Q.15 What is the function of feed check valve ?

Ans. : The high pressure feed water is supplied to a boiler through this valve against the boiler pressure.

Q.16 What are the advantages of Cochran boiler ?

Ans. : • Low initial installation cost.

- It requires less floor area.
- Easy to operate and handle.
- Transportation of Cochran boiler is easy.
- It can use all types of fuel.

Q.17 what are the dis-advantages of Cochran boiler ?

Ans. : • Low rate of steam generation.

- Inspection and maintenance is difficult.
- High room head is required for its installation due to the vertical design.
- It has limited pressure range.

Q18 State the advantages of water tube boilers over fire tube boilers.

Ans. : • Steam can be raised more quickly

- Steam at higher pressures can be produced
- Higher rate of evaporation
- Failure of water tubes will not affect the working of boiler

Q.19 State the dis-advantages of water tube boilers over fire tube boilers.

Ans. : • Not suitable for ordinary water

- Not suitable for mobile application
- High initial cost and hence not economical

Q.20 Which are the methods used to find out the boiler efficiency ?

Ans. : • Direct method

- Indirect method

Q.21 What is the difference between subcritical and supercritical boiler ?

Ans. : • Large supercritical boilers, when operated at partial load, lower their pressure and usually become subcritical.

- Below the critical pressure, there is a great difference between densities of liquid water and steam

Q.22 What do you mean by high pressure boiler ?

Ans. : A boiler is called a high-pressure boiler when it operates with a steam pressure above 80 bars.

Q.23 What is water level indicator in boiler ?

Ans. : • Water gauge indicates the water level inside the boiler and is hence called as water level indicator.

Q.24 What is benson boiler ?

Ans. : • The benson boiler is a water tube boiler, works on the basic principle of critical pressure of water.

Q.25 How does fusible plug work in a boiler ?

Ans. : • Fusible plug is a small device installed in small horizontal fire tube boilers between furnace and boiler water drum for protection of boiler while lower water level in drum.

Q.26 What is the function of spring loaded safety valve ?

Ans. : • Spring loaded safety valve is a safely mounting fitted on the boiler shell and is essentially required on the boiler shell to safeguard the boiler against high pressure.

Q.27 What is the function of blow-off-cock ?

Ans. : • It is a controllable valve opening at the bottom of water space in the boiler and is used to blow off some water from the bottom which carries mud or other sediments settled during the operation of boiler.

Q.28 What is the function of steam stop valve ?

Ans. : • It is fitted over the boiler in between the steam space and steam supply line. Its function is to regulate the steam supply from boiler to the steam line.

Q.29 What is preventive maintenance of boilers ?

Ans. : • Blow down and test low water cut off

- Blow down gauge glasses
- Blow down boiler
- Check boiler and system for leaks
- Check burner flame

Q.30 What are the advantages of high pressure boiler ?

Ans. : • They require less heat of vaporization.

- They are compact and thus require less floor space.
- Due to the high velocity of water, the tendency of scale formation is minimized.
- All parts are uniformly heated and the danger of overheating is minimized.

Review Questions

1. *Explain with neat sketch Cochran boiler.*
2. *What is the selection criteria of boiler ?*
3. *Distinguish between fire tube and water tube boiler.*
4. *Write a note on high pressure boiler.*
5. *Explain classification of fuel.*

❑❑❑

Notes

UNIT - III

3

Steam Turbines

Syllabus

Types, Impulse and reaction principles, Velocity diagrams, Work done and efficiency - optimal operating conditions. Multi-staging, compounding and governing.

Contents

3.1 Introduction

- A steam turbine is a mechanical device that extracts thermal energy from pressurized steam and converts it into rotary motion.
- Steam turbine depends directly upon dynamic action of the steam.
- As per the Newton's second law of motion, the rate of change of momentum is caused in the steam by allowing high velocity jet of steam to pass over curved plate (blade) and the steam will impart a force to the blade.
- If the blade is free then it will rotate in the direction of applied force.
- The steam from the boiler is expanded in the nozzle where due to fall in pressure of steam, thermal energy of steam is converted into kinetic energy of steam.
- This high velocity jet of steam is impinged on the blades mounted on a shaft. Refer Fig. 3.1.

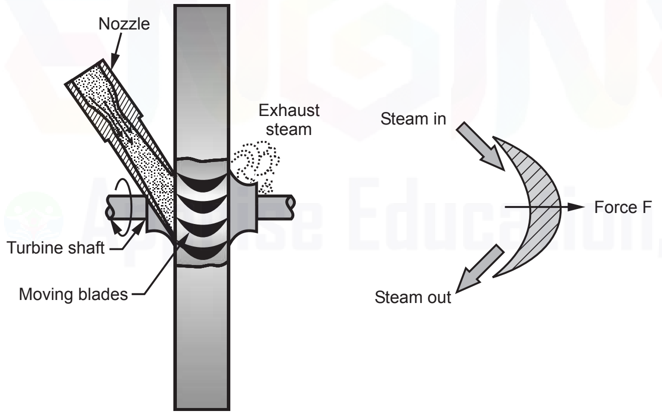


Fig. 3.1 : Working principle of steam turbine

- The change in flow direction of steam causes a force to be exerted on the blades and due to rotation of these blades power is developed.
- A pair of ring of nozzles (fixed blade) are fixed to the casing and a ring of moving blades are fixed to the turbine rotor. This is called as **a single stage** or **a pair of turbine.**
- The fixed and moving blades are designed such that the jet of steam should not strike the blades but the steam should glide over the blades.

3.2 Classification of Steam Turbine

- The most important and common types of steam turbine with respect to action of steam are **impulse** and **reaction turbines.**
- The other kind of classification of steam turbines are as follows :

(i) According to the number of pressure stages :

(a) Single stage turbines with one or more velocity stages : These turbines are used for driving blowers, centrifugal compressors, etc.

(b) Multistage impulse and reaction turbines : They are made for small as well as large power capacities.

(ii) According to the direction of flow of steam :

(a) Axial turbines : In these turbines steam flows parallel to the axis of turbine.

(b) Radial turbines : In these turbines steam flows perpendicular to the axis of turbine.

(iii) According to the number of cylinders :

(a) Single cylinder turbines

(b) Two cylinder turbines

(c) Three cylinder turbines

(d) Four cylinder turbines

(iv) According to the governing method :

a) Turbines with throttle governing

b) Turbines with nozzle governing

c) Turbines with by-pass governing

d) Turbines with combined throttle -bypass or nozzle - bypass governing.

(v) According to the heat drop process :

a) Condensing turbines with generators

b) Non-condensing turbines

c) Back pressure turbines

d) Low pressure turbines

e) Mixed pressure turbines

f) Topping turbines

(vi) According to the condition of steam at inlet to turbine :

a) Low pressure turbines : Steam pressure is upto 2 bar.

b) Medium pressure turbines : Steam pressure is upto 50 bar.

c) High pressure turbines : Steam pressure is above 50 bar

d) Very high pressure turbines : Steam pressure is upto 170 bar.

e) Turbines with super critical pressures : Steam pressure is 225 bar and above.

(vii) According to use in industry :

a) Stationary turbines with uniform speed : Used for driving alternators.

b) Stationary turbines with variable speed : Used for driving air circulators, pumps, blowers, etc.

c) Non stationary turbines with uniform speed : Used in ships, railway locomotives, etc.

3.3 Common Types of Steam Turbine

Steam turbines are mainly divided into two groups :

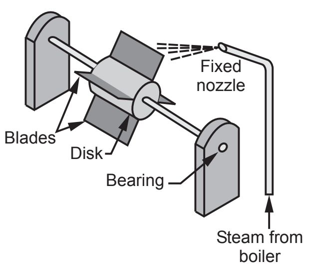
1. Impulse turbine 2. Reaction turbine

1. Impulse Turbine

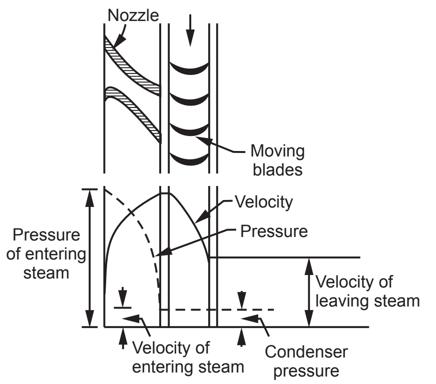
- In these turbines, the steam comes out at a very high velocity through the fixed nozzle and impinges on the blades fixed on the periphery of a rotor. Refer Fig. 3.2 (a).
- The blades change the direction of the steam flow without changing its pressure.
- The resulting force due to the change in momentum causes the rotation of the turbine shaft.
- Fig. 3.2 (b) shows the blade arrangement for impulse turbine and the variation of pressure and velocity of steam passing through the turbine.
- Examples of impulse turbine are De-Laval turbine, Rateau turbine, Curties turbine.
- It is important to note that, in case of impulse turbine the shape of blades in *profile type.*

2. Reaction Turbine

- In these turbines, the high pressure steam from the boiler is passed through the nozzles as shown in Fig. 3.3 (a).
- When the steam comes out through these nozzles, the velocity of the steam increases relative to the rotating disc.
- This results in reacting force of the steam on nozzle which gives rotating motion to the disc and shaft.
- In these turbines steam expands both in fixed and moving blades continuously when the steam passes over them.
- Hence, pressure drop occurs gradually and continuously over both moving and fixed blades.
  For example, Parson's reaction turbine.



(a) Working principle



(b) Arrangement of blades

Fig. 3.2 : Impulse turbine

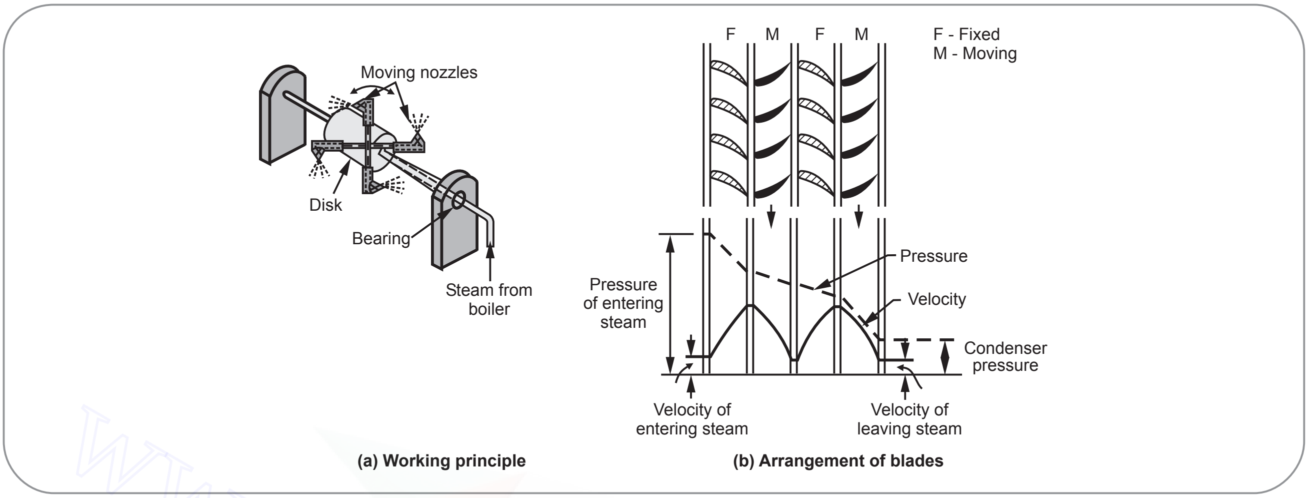


Fig. 3.3 : Reaction turbine

- Fig. 3.3 (b) shows the blade arrangement for reaction turbine and the variation of pressure and velocity of steam passing through the turbine.
- It is important to note that, in case of reaction turbine the shape of blades is *aerofoil.*

3.4 Advantages and Applications of Steam Turbine

Advantages :

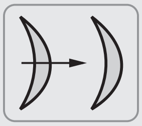
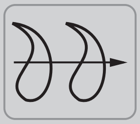
Steam turbines offer following advantages :

- Steam turbine is highly simplified in operation and construction.
- The thermal efficiency of steam turbines is mush higher.
- It is compact and it has low weight to power ratio.
- It can operate at high speeds and greater range of speeds is possible.
- Due to absence of reciprocating parts, the vibrations and noise are greatly minimized.
- Steam turbine can take considerable over load.
- It can be designed in sizes ranging from a few kW to 1000 MW in a single unit.
- In steam turbines there is no condensation loss.
- Life of steam turbine is high.
- Initial cost, maintenance cost and installation cost are low.

Applications :

- Steam turbines are commonly used to operate electric generators in thermal and nuclear power plants to produce electricity.
- In addition to power generation, steam turbines are also used for the following purposes :
  - To propell large ships and submarines.
  - To drive power absorbing machines like compressors, fans, blowers, pumps, etc.
  - Also used in steamers of railway locomotives.

3.5 Comparison between Impulse and Reaction Turbine

Sr. No.	Impulse turbine	Reaction turbine
1.	In these turbines, pressure drops only in nozzle and not in moving blades channel.	In these turbines, pressure drops in nozzle as well as in moving blades channel.
2.	The blades are of profile shape. 	The blades are of aerofoil shape. 
3.	Blade channel area is constant.	Blade channel area is varying.
4.	By using these turbines much power cannot be developed.	Much power can be developed by using these turbines.
5.	These turbines occupy less space for same power.	These tubines occupy more space for same power.
6.	Velocity of steam is slightly higher.	Velocity of steam is lower.
7.	Blade manufacturing is simple and less costly.	Blade manufacturing is difficult hence costly.
8.	Efficiency of these turbines is low.	Efficiency of these turbines is high.

3.6 Compounding of Steam Turbine

AU : May-18

- When the steam is expanded from boiler pressure to condenser pressure in a single stage, the absolute velocity of steam leaving the turbine is very high.
- Also this results in the tremendously high blade velocity and consequently the high rotor speed upto 25000 to 30000 rpm.
- Hence the various methods should be adopted to reduce this rotor speed to a lower value.
- These methods includes mounting the number of rotors on a common shaft. Due to this the jet velocity is absorbed in stages as the steam flows over blades. This is known as **compounding of steam turbine**.
- The following are the methods used for compounding of steam turbine :

(i) Velocity compounding (ii) Pressure compounding (iii) Pressure-velocity compounding

3.6.1 Velocity Compounding

- Fig. 3.4 shows the velocity compounding arrangement in which velocity is decreased gradually over the set of moving and fixed blades.
- The steam is expanded from boiler pressure to condenser inlet pressure through a stationary nozzle. During this the pressure drop occurs and velocity and kinetic energy of steam increases.
- Now the portion of the increased kinetic energy is absorbed by a row of moving blades. This results in the decrease in the velocity as the work is done on the moving blades.
- The steam is then flows through the row of fixed blades. These fixed blades are used to direct the steam over next row of moving blade without altering the velocity.

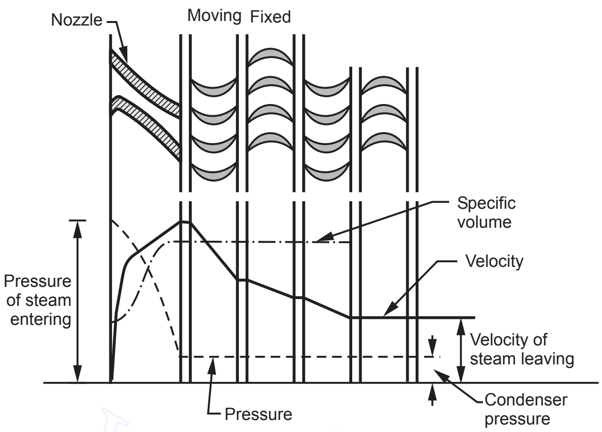


Fig. 3.4 : Velocity compounded impulse turbine

- During the flow of steam over next rows of moving blades where again the work is done and steam leaves the turbine with lower value of velocity.
- As the velocity is gradually decreased over the compounded stages, the method is known as the velocity compounding.

Advantages of velocity compounding

- Less number of stages are employed to reduce the velocity.
- Low initial cost.

Disadvantages of velocity compounding

- High steam consumption.
- Low efficiency.

3.6.2 Pressure Compounding

- Fig. 3.5 shows the pressure compounding arrangement in which pressure of steam decreases gradually over the set of nozzles and blades.
- In first stage, the steam expands partially from boiler pressure in the first set of nozzles. The kinetic energy increases during the flow of steam through nozzle and again get absorbed at the moving blades where pressure remains constant.
- During second stage again the steam expands in the nozzle and pressure falls to a certain value and velocity of steam again increases.

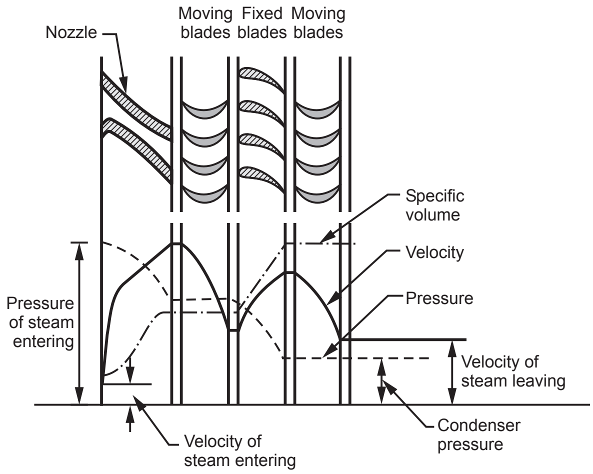


Fig. 3.5 : Pressure compunded steam turbine

- The kinetic energy obtained is then again absorbed by the row of moving blades where pressure remains constant.
- This process repeats in the next stages untill the condenser pressure and lower value of velocity is achieved.
- As the pressure is gradually decreased over the compounded stages of nozzles and blades, the method is known as pressure compounding.

Advantage :

- The speed ratio remains constant throughout the turbine.

Disadvantages :

- Larg number of stages are used in pressure compounding.
- Expensive method of compounding.

3.6.3 Pressure Velocity Compounding

- Fig. 3.6 shows the pressure - velocity compounding arrangement which is the combination of both the methods discussed earlier.
- It consist of number of stages of nozzles, fixed and moving blades. The pressure drop from boiler pressure to condenser pressure is divided into the stages and lower value of velocity also.

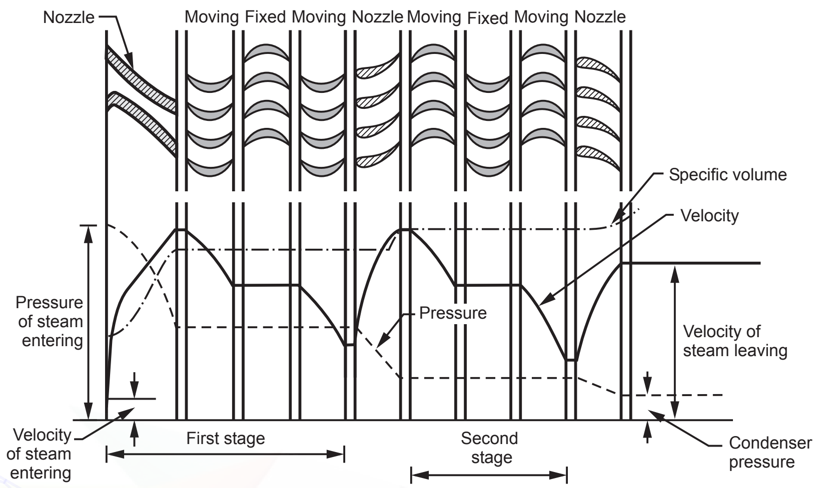


Fig. 3.6 : Pressure and velocity compounded steam turbine

- The nozzles are fitted at the begining of each stage and pressure drop occurs only during the flow through nozzle. The pressure remains constant during remaining stage.
- Also during the flow of steam through nozzle the velocity of steam as well as kinetic energy increases. This increased kinetic energy is absorbed during the flow of steam over moving blades.
- As both the pressure as well as velocity is gradually decreased over the compounded stages it is known as pressure velocity compounding.

3.7 Velocity Diagram for Moving Blade Impulse Turbine

- To find the force on the blades and power developed by the turbine, it is necessary to determine the rate of change of momentum of steam across the moving blades.
- For this purpose, the velocity diagram at inlet and outlet is drawn for the moving blades.
- Let, u = Circumferential or tangential velocity of the blade $= \frac{\pi d_m N}{60}$

d_m = Mean diameter of blade drum in m

N = Speed of turbine in rpm

V_1 = Absolute velocity of steam at inlet to moving blades (exit velocity of steam from nozzle or fixed blades)

V_2 = Absolute velocity of steam at exit to moving blade (Inlet velocity of steam to second ring of nozzles or fixed blades)

V_{w1} = Tangential component of V_1 (velocity of whirl at inlet to moving blades)

V_{w2} = Velocity of whirl at outlet of moving blades

V_{r1} and V_{r2} = Relative velocity of steam at inlet and outlet of moving blades respectively

V_{f1} and V_{f2} = Axial component of V_1 and V_2 respectively (flow velocity at inlet and outlet)

α = Exit angle of nozzle or the angle with which steam enters the moving blades

θ = Inlet angle of moving blades

ϕ = Outlet angle of moving blades

β = Angle with which steam at V_2 leaves the moving blades

(Inlet angle of fixed blades)

$\dot{m}$ = Mass flow rate of steam in kg/sec.

K = Blade velocity coefficient or friction factor $= \dfrac{V_{r2}}{V_{r1}}$

H = Height of blade in m

- Fig. 3.7 shows the velocity diagram of the moving blade at inlet and outlet for an impulse turbine.
- The steam jet from nozzle is impinged on the moving blade at an angle 'α' and the blade starts rotating with velocity 'u'.
- The tangential component of V_1 produces work and the axial component of V_1 is responsible for the flow of steam in axial direction.
- If there is no friction between the steam and the moving blade suface then $V_{r2} = V_{r1}$ $(K = 1)$.
- In this chapter, for convenience the velocity diagram at inlet and outlet of moving blade is combined.
- While drawing the combined velocity diagram, the blade velocity 'u' is taken as common for both the triangles. Refer Fig. 3.8.
- In case of impulse turbine, the relative velocity of steam remains constant or it is reduced slightly due to friction.
- But in case of reaction turbine, the steam expands as it flows over the moving blades. This results in increase in relative velocity of steam at outlet. It means,

For impulse turbine $V_{r2} \le V_{r1}$

For reaction turbine $V_{r2} > V_{r1}$

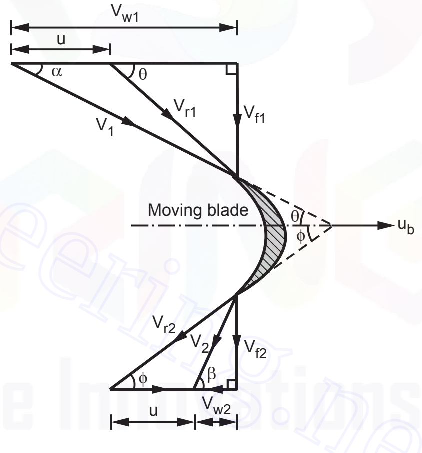


Fig. 3.7 : Inlet and outlet velocity diagram for impluse turbine

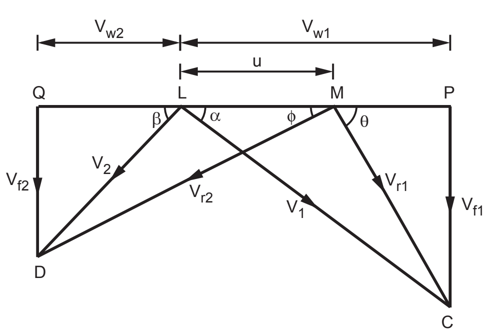


Fig. 3.8 : Combined velocity diagram for impulse turbine

3.7.1 Work Done, Power and Efficiency of Impulse Steam Turbine

i) Work done on the blade

- The work done on the blade is due to the force along the direction of motion of blade.
- The change in velocity of whirl causes the force to act along the direction of motion of blade.
- Now, work done per kg of steam is given by,

$$\text{WD/kg} = \begin{matrix}\text{Force in the direction}\\ \text{of motion of blade}\end{matrix} \times \begin{matrix}\text{Distance travelled in}\\ \text{the direction of force}\end{matrix}$$

$$= [V_{w1} - (-V_{w2})]u = (V_{w1} + V_{w2})\,u,\ \frac{\text{N - m}}{\text{kg}} \qquad \ldots (3.1)$$

- The value of V_{w2} is negative because the steam is discharged opposite to the direction of motion of blade. Hence V_{w1} and V_{w2} are added.
- The work done per second or power developed by the turbine for flow rate of $\dot{m}_s$ kg of steam per second is,

$$\text{WD/sec} = \dot{m}_s (V_{w1} + V_{w2})\,u,\ \frac{\text{N - m}}{\text{s}} \text{ or Watt} \qquad \ldots (3.2)$$

ii) Diagram or blade efficiency

- The diagram or blade efficiency for a single blade stage of an impulse turbine is given by,

$$\eta_b = \frac{\text{Work done on the blade}}{\text{K.E. supplied to the blade}} = \frac{\dot{m}_s (V_{w1} + V_{w2})\,u}{\frac{1}{2}\dot{m}_s\ V_1^2} = \frac{2(V_{w1} + V_{w2})\,u}{V_1^2} \qquad \ldots (3.3)$$

iii) Stage or gross efficiency

- It is defined as the ratio of work done per kg of steam to the theoretical enthalpy drop in the nozzle per kg of steam. It is given by,

$$\eta_{stage} = \frac{\text{Work done on the blade per kg}}{\text{Theoretical enthalpy drop}}$$

$$\therefore \quad \eta_{stage} = \frac{(V_{w1} + V_{w2})\,u}{h_1 - h_2} \qquad \ldots (3.4)$$

- Now multiply and divide above equation by $2\ V_1^2$

$$\eta_{stage} = \frac{(V_{w1} + V_{w2})\,u}{h_1 - h_2} \times \frac{2\,V_1^2}{2\,V_1^2}$$

$$\eta_{stage} = \frac{2(V_{w1} + V_{w2})\,u}{V_1^2} \times \frac{V_1^2}{2(h_1 - h_2)}$$

$$\eta_{stage} = \eta_b \times \eta_n$$

iv) Axial thrust on the wheel (F_a) :

- It is defined as the difference between the flow velocities at the inlet and outlet.

- The change in axial component of absolute velocity (i.e. flow velocity) of steam produces axial thrust on the wheel.
- This is undesirable and thus thrust bearings are provided to take this load. It is given by,

$$F_a = \dot{m}_s (V_{f1} - V_{f2}) \qquad \dots (3.5)$$

v) Loss of kinetic energy in blade friction

- The kinetic energy of steam is lost during the flow over blades due to friction. It is given by,

$$\Delta E = \text{Loss in K.E. during flow}$$

$$= \dot{m}_s (V_{r1}^2 - V_{r2}^2) \qquad \dots (3.6)$$

vi) Blade velcocity coefficient (K)

- If blade friction is neglected, the relative velocity of steam over blade is

$$V_{r1} = V_{r2}$$

- In actual practice, the flow of steam is resisted by friction. This reduces the relative velocity at outlet by 10 to 15 percent i.e.

$$V_{r2} = K\, V_{r1}$$

where K = Blade velocity coefficient

- This coefficient of blade velocity gives the relation between the relative velocity of blade at outlet and inlet.

vii) Blade speed ratio (s)

- It is the ratio of blade speed (u) to the steam velocity at inlet (V_1). It is given as,

$$s = \frac{\text{Blade velocity}}{\text{Steam velocity at inlet}} = \frac{u}{V_1} \qquad \dots (3.7)$$

viii) Blade height (H)

- Volume flow rate or discharge of steam is given as,

$$Q = \text{Area of flow} \times \text{Flow velocity}$$

$$= \pi\left(D + \frac{H}{2} + \frac{H}{2}\right) \times H \times V_f$$

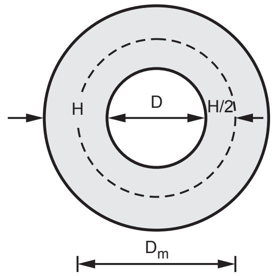


Fig. 3.9

$$\therefore \quad Q = \pi(D+H) \times H \times V_f$$

- Mass flow rate of steam is given by,

$$\dot{m} = \rho Q = \frac{Q}{v_s}, \text{ kg/s} \qquad \dots \left(\because \rho = \frac{1}{v_s}\right)$$

$$\therefore \quad \dot{m} = \frac{\pi(D+H) \times H \times V_f}{v_s}$$

where, v_s = Specific volume of steam in m^3/kg at given stage

H = Blade height

3.7.2 Condition for Maximum Discharge Efficiency for Impulse Turbine

- In impulse steam turbine for maximum blade or diagram efficiency, there is a definite value of speed ratio (u/V_1).
- The blade or diagram efficiency is given by,

$$\eta_b = \frac{2(V_{w1} + V_{w2})u}{V_1^2} \qquad \text{... [From equation (3.3)]}$$

- Now from Fig. 3.8,

$$V_w = MP + MQ = V_{r1}\cos\theta + V_{r2}\cos\phi$$

$$\therefore \quad V_w = V_{r1}\cos\theta\left[1 + \frac{V_{r2}\cos\phi}{V_{r1}\cos\theta}\right]$$

$$\therefore \quad V_w = V_{r1}\cos\theta\,[1 + KC] \qquad \text{... (i)} \left(\text{where} \quad C = \frac{\cos\phi}{\cos\theta}\right)$$

- As angles ϕ and θ are almost similar, the value of C is assumed constant.
- But $MP = V_{r1}\cos\theta = LP - LM = V_1\cos\alpha - u$

Substitute above value in equation (i) we get,

$$V_w = (V_1\cos\alpha - u)(1 + KC) = V_{w1} + V_{w2} \qquad \text{... (ii)}$$

- Now the blade efficiency becomes,

$$\eta_b = \frac{2u\,(V_1\cos\alpha - u)(1 + KC)}{V_1^2}$$

$$\eta_b = 2\,(1 + KC)\left(\frac{uV_1\cos\alpha}{V_1^2} - \frac{u^2}{V_1^2}\right)$$

$$\eta_b = 2\,(1 + KC)(s\cos\alpha - s^2) \qquad \text{...} \left(\because s = \frac{u}{V_1}\right) \text{... (iii)}$$

$$\eta_b = 2s(1 + KC)(\cos\alpha - s) \qquad \text{... (iv)}$$

For maximum efficiency differentiate equation (iii) with respect to 's' and equate it to zero we get,

$$\frac{d}{ds}(\eta_b) = \frac{d}{ds}[2(1 + KC)(s\cos\alpha - s^2)] = 0$$

$$2(1 + KC)(\cos\alpha - 2s) = 0$$

$$\therefore \quad \cos\alpha - 2s = 0$$

$$\therefore \quad s = \frac{\cos\alpha}{2} \qquad \text{... (3.8)}$$

$$\therefore \quad \cos\alpha = 2s = 2\frac{u}{V_1} \qquad \text{... (3.9)}$$

or $$\frac{\cos\alpha}{2} = \frac{u}{V_1}$$

Now substituting equation (3.9) in equation (iii) to get the maximum efficiency

$$\eta_{bmax} = 2\times\frac{\cos\alpha}{2}\times(1+KC)\left(\cos\alpha - \frac{\cos\alpha}{2}\right)$$

$$= 2\times\frac{\cos\alpha}{2}\times(1+KC)\left(\frac{\cos\alpha}{2}\right) = \frac{\cos^2\alpha}{2}(1+KC) \qquad \ldots (3.10)$$

- If we assume that the blades are symmetrical and there is no friction in fluid passage, then $\theta = \phi$ (i.e., C = 1) and K = 1.

$$(\eta_b)_{max} = \cos^2\alpha \qquad \ldots (3.11)$$

- Now the work done per kg of steam is given by,

$$WD/kg = (V_{w1} + V_{w2})\,u = V_w u$$

Substituting equation (ii) in above equation

$$WD/kg = (V_1 \cos\alpha - u)\,(1+KC)\cdot u$$

$$= 2u\,(V_1 \cos\alpha - u) \qquad \ldots (K = 1, C = 1) \ldots (v)$$

The maximum value of work done is obtained by substituting equation (3.9) in above equation (v),

$$WD_{max} = 2u\left(V_1\times\frac{2u}{V_1} - u\right) = 2u^2 \qquad \ldots (3.12)$$

- When $\frac{u}{V_1} = s = 0$, the work done becomes zero as the distance travelled by the blade (u) is zero.
- From the above discussion it is clear that, the maximum efficiency is $\cos^2\alpha$ and maximum work done per kg of steam is $2u^2$ when $\frac{u}{V_1} = \frac{\cos\alpha}{2}$, Refer Fig. 3.10.

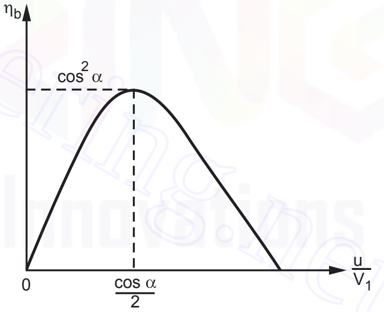


Fig. 3.10

3.8 Solved Examples

Ex. 3.1 : *A stage of a steam turbine is supplied with steam at a pressure of 50 bar and 350°C and exhausts at a pressure of 5 bar. The isentropic efficiency of the stage is 0.80 and the steam consumption is 2500 kg/min. Determine the power output of the stage.*

Sol. : Given data :

$$p_1 = 50 \text{ bar}, T_1 = 350\ °C, p_2 = 5 \text{ bar}, \eta_{isen} = 80\ \% = 0.8,$$

$$\dot{m} = 2500 \text{ kg/min} = \frac{2500}{60} = 41.6667 \text{ kg/sec.}$$

To find : P

Step 1 : Calculate the power output

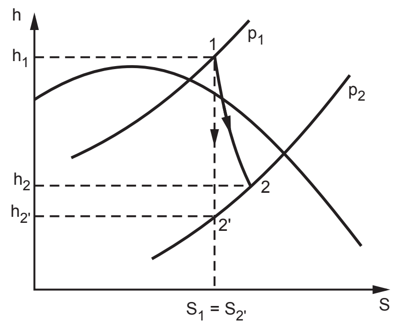


Fig. 3.11

From steam tables, at $p = 50$ bar

For T = 350 °C, specific enthalpy $h = h_1 = 3071.2$ kJ/kg

Specific entropy $S = S_1 = 6.455$ kJ/kg K

From steam tables, at $p = 5$ bar,

Specific enthalpy for water, $h_f = h'_{f2} = 640.1$ kJ/kg

Specific enthalpy for evaporation, $h_{fg} = h'_{fg2} = 2107.4$ kJ/kg

Specific entropy for water, $S_f = S'_{f2} = 1.86$ kJ/kg K

Specific entropy for evaporation, $S_{fg} = S'_{fg2} = 4.959$ kJ/kg K

As isentropic expansion of steam takes place,

$$S_1 = S'_2 \qquad \therefore S_1 = S'_{f2} + x\,S'_{fg2}$$

$$\therefore \quad 6.455 = 1.86 + x \times 4.959 \quad \therefore \; x = 0.9265$$

Thus, enthalpy of steam at $p_2 = 5$ bar

$$\therefore \quad h'_2 = h'_{f2} + x\,h'_{fg2}$$

$$= 640.1 + 0.9265 \times 2107.4$$

$$= 2592.6061 \text{ kJ/kg}$$

Isentropic efficiency,
$$\eta_{isen} = \frac{h_1 - h_2}{h_1 - h'_2} = \frac{(\Delta h)_{actual}}{(\Delta h)_{isen}}$$

$$\therefore \quad 0.8 = \frac{(\Delta h)_{actual}}{3071.2 - 2592.6061}$$

$$\therefore \quad (\Delta h)_{actual} = 382.8751 \text{ kJ/kg}$$

Power developed,
$$P = \dot{m}\,(\Delta h)_{actual}$$

$$= 41.6667 \times 382.8751$$

$$\therefore \quad \mathbf{P = 15.9531 \times 10^3 \; W} \qquad \text{... Ans.}$$

Ex. 3.2 : *In a stage of an impulse turbine provided with a single row wheel, the mean diameter of the blade ring is 750 mm and the speed of rotation is 3200 rpm. The steam issues at a velocity of 300 m/s from the nozzle and the nozzle angle is 20°. The rotor blades are equiangular and the blade friction factor is 0.85. Determine the power developed in the blading if the axial thrust on the blades is 140 N.*

Sol. : Given Data :

$D_m = 750$ mm $= 0.75$ m, $N = 3200$ rpm, $V_1 = 300$ m/sec. $\alpha = 20°$, $\theta = \phi$, $K = 0.85$, $F_a = 140$ N

To find : P

Step 1 : Calculate the power developed in blading

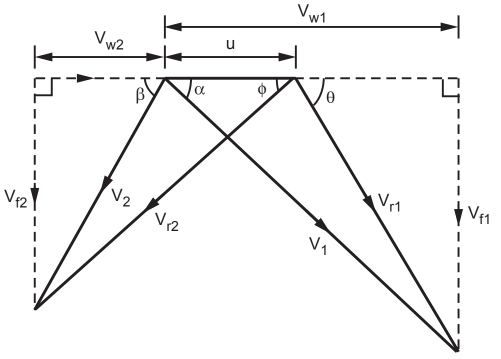


Fig. 3.12

Blade velocity, $u = \frac{\pi D_m N}{60}$

$= \frac{\pi \times 0.75 \times 3200}{60} = 125.6637$ m/sec

From Fig. 3.12, by inlet velocity triangle,

Whirl velocity at inlet, $V_{w1} = V_1 \cos\alpha = 300 \times \cos(20)$

$= 281.9077$ m/sec

Flow velocity at inlet, $V_{f1} = V_1 \sin\alpha = 300 \times \sin(20)$

$= 102.606$ m/sec

And, $\tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{102.606}{281.9077 - 125.6637} = 0.6567$

$\therefore \quad \theta = \tan^{-1}(0.6567) = 33.293°$

For equiangular blades, $\theta = \phi = 33.293°$

Relative velocity at inlet, $V_{r1} = \frac{V_{f1}}{\sin\theta} = \frac{102.606}{\sin(33.293)} = 186.9231$ m/sec

Friction factor, $K = \frac{V_{r2}}{V_{r1}} \quad \therefore \quad 0.85 = \frac{V_{r2}}{186.9231}$

$\therefore \quad V_{r2} = 158.8846$ m/sec

From Fig. 3.12, by outlet velocity triangle,

Whirl velocity at outlet, $V_{w2} = V_{r2} \cos\phi - u$

$V_{w2} = 158.8846 \times \cos(33.293) - 125.6637 = 7.1438$ m/sec

Flow velocity at outlet, $V_{f2} = V_{r2} \sin\phi = 158.8846 \times \sin(33.293) = 87.2150$ m/sec

Axial thrust, $F_a = \dot{m}(V_{f1} - V_{f2}) \quad \therefore 140 = \dot{m}(102.606 - 87.215)$

$\therefore \quad \dot{m} = 9.0962$ kg/sec

Power developed, $P = \dot{m}(V_{w1} + V_{w2})u = 9.0962 \times (281.9077 + 7.1438) \times 125.6637$

$\therefore \quad \mathbf{P = 330.4038 \times 10^3\ W}$ **... Ans.**

Ex. 3.3 : *In a simple impulse turbine, the nozzles are inclined at 20° to the direction of moving blades. The steam leaves the nozzle at 375 m/sec and blade speed is 165 m/sec., find suitable inlet and outlet angles such that axial thrust is zero. The relative velocity of steam as it flows over the blade is reduced by 15 % due to friction. Also find, power developed if mass flow rate is 10 kg/sec.*

Sol. : Given data :

$\alpha = 20°$, $V_1 = 375$ m/sec, $u = 165$ m/sec, $F_a = 0$, $V_{r2} = 0.85\ V_{r1}$, $\dot{m} = 10$ kg/sec.

To find : i) θ, ϕ ii) P

Step 1 : Calculate the inlet, outlet angles for blades

As axial thrust i.e. $F_a = 0,\ V_{f1} = V_{f2}$

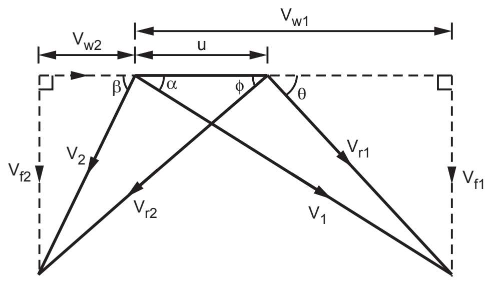


Fig. 3.13

From Fig. 3.13, by inlet velocity triangle,

Whirl velocity at inlet, $V_{w1} = V_1 \cos\alpha$

$= 375 \times \cos(20) = 352.3847$ m/sec

Flow velocity at inlet, $V_{f1} = V_1 \sin\alpha$

$= 375 \times \sin(20) = 128.2575$ m/sec

And, $\tan\theta = \dfrac{V_{f1}}{V_{w1} - u} = \dfrac{128.2575}{352.3847 - 165}$ $\therefore$ $\theta = \mathbf{34.3901°}$ **... Ans.**

Relative velocity at inlet, $V_{r1} = \dfrac{V_{f1}}{\sin\theta} = \dfrac{128.2575}{\sin(34.3901)} = 227.0749$ m/sec

As, $V_{r2} = 0.85\ V_{r1} = 0.85 \times 227.0749 = 193.0137$ m/sec

Also, $V_{f1} = V_{f2} = 128.2575$ m/sec

From Fig. 3.13, by outlet velocity triangle,

$$\sin\phi = \frac{V_{f2}}{V_{r2}} = \frac{128.2575}{193.0137} \quad \therefore \ \phi = \mathbf{41.6439°}$$ **... Ans.**

$$\tan\phi = \frac{V_{f2}}{V_{w2} + u} \quad \therefore \ \tan(41.6439) = \frac{128.2575}{V_{w2} + 165}$$

$\therefore$ $V_{w2} = -20.7628$ m/sec.

Step 2 : Calculate the power developed

Power developed,

$$P = \dot{m}\,(V_{w1} + V_{w2})\,u = 10 \times (352.3847 + (-20.7628)) \times 165$$

$\therefore$ $\mathbf{P = 547.1761 \times 10^3\ W}$ **... Ans.**

Ex. 3.4 : *Steam enters an impulse wheel having a nozzle of 20° at a velocity of 450 m/s. The exit angle of the moving blades is 20° and relative velocity of steam may be assumed to remain constant over the moving blades. If the blade speed is 180 m/s determine :*

(i) blade angle at inlet (ii) work done / kg of steam

(iii) power developed when the turbine is supplied with 2.5 kg/s of steam

(iv) diagram efficiency.

Sol. : Given data :

$\alpha = 20°$, $V_1 = 450$ m/sec, $\phi = 20°$,

$V_{r1} = V_{r2}$, $u = 180$ m/sec, $\dot{m} = 2.5$ kg/sec.

To find : i) θ ii) W iii) P iv) η_b

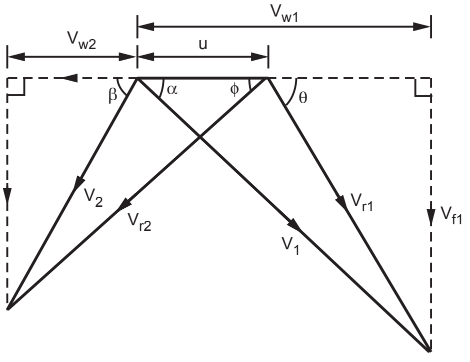


Fig. 3.14

Step 1 : Calculate the blade angle at inlet

From Fig. 3.14, by inlet velocity triangle,

Whirl velocity at inlet, $V_{w1} = V_1 \cos\alpha = 450 \times \cos(20)$

$= 422.8616$ m/sec

Flow velocity at inlet, $V_{f1} = V_1 \sin\alpha = 450 \times \sin(20) = 153.909$ m/sec

And, $$\tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{153.909}{422.8616 - 180} = 0.6337$$

$\therefore$ $\theta = \tan^{-1}(0.6337) = \mathbf{32.3637°}$ **... Ans.**

Relative velocity at inlet, $$V_{r1} = \frac{V_{f1}}{\sin\theta} = \frac{153.909}{\sin(32.3637)} = 287.5235 \text{ m/sec}$$

As, $V_{r1} = V_{r2} = 287.5235$ m/sec

Step 2 : Calculate the work done per kg of steam

From Fig. 3.14, by outlet velocity triangle.

Whirl velocity at outlet, $V_{w2} = V_{r2} \cos\phi - u$

$V_{w2} = 287.5235 \times \cos(20) - 180 = 90.1837$ m/sec

Work done, $W = (V_{w1} + V_{w2})\, u = (422.8616 + 90.1837) \times 180$

$\mathbf{W = 92.3481 \times 10^3 \; J/kg}$ **... Ans.**

Step 3 : Calculate the power developed and diagram efficiency

Power developed, $P = \dot{m}\,(V_{w1} + V_{w2})\, u = 2.5 \times (422.8616 + 90.1837) \times 180$

$\therefore$ $\mathbf{P = 230.8702 \times 10^3 \; W}$ **... Ans.**

Diagram (blade) efficiency,

$$\eta_b = \frac{P}{\frac{1}{2}\dot{m}V_1^2} = \frac{230.8702 \times 10^3}{\frac{1}{2} \times 2.5 \times 450^2}$$

$\therefore$ $\mathbf{\eta_b = 0.9120 = 91.20 \%}$ **... Ans.**

Ex. 3.5 : *Steam issues from the nozzles at angle of 20° at a velocity of 440 m/s. The friction factor is 0.9. For a single stage turbine designed for maximum efficiency, determine :*
(i) the blade velocity, *(ii) the moving blade angles for equi-angular blades*
(iii) the blade efficiency *(iv) the stage efficiency if the nozzle efficiency is 93 %.*
(v) power developed for a mass flow rate of steam of 3 kg/s.

Sol. : Given data :

$\alpha = 20°$, $V_1 = 440$ m/sec, K = 0.9, z = 1, $\eta_n = 93\ \% = 0.93$, $\dot{m} = 3$ kg/sec., $\theta = \phi$.

To find : i) u, η_b ii) θ, ϕ iii) η_{stage} iv) P

Step 1 : Calculate the blade velocity, blade efficiency and stage efficiency

For maximum efficiency condition, $\eta_b = (\eta_b)_{max}$

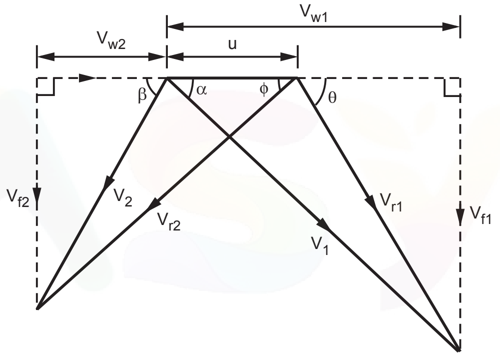


Fig. 3.15

$$\therefore \quad (\eta_b)_{max} = \frac{\cos^2\alpha}{2}\left(1 + K\frac{\cos\phi}{\cos\theta}\right)$$

$$= \frac{(\cos(20))^2}{2}(1 + 0.9 \times 1) \; .... \; [\because \; \theta = \phi]$$

$$\therefore \quad \eta_b = \mathbf{0.8388 = 83.88\ \%} \quad \textbf{... Ans.}$$

$$\text{Stage efficiency, } \eta_{stage} = \eta_b \times \eta_n$$

$$= 0.8388 \times 0.93 = 0.78$$

$$\therefore \quad \eta_{stage} = \mathbf{0.78 = 78\ \%} \quad \textbf{... Ans.}$$

For maximum blade efficiency,

$$\text{Blade speed ratio,} \quad s = \frac{\cos\alpha}{2} = \frac{\cos(20)}{2} = 0.4698$$

$$\text{Also,} \quad s = \frac{u}{V_1} \qquad \therefore \; 0.4698 = \frac{u}{440}$$

$$\therefore \quad \mathbf{u = 206.712\ m/sec} \quad \textbf{... Ans.}$$

Step 2 : Calculate the moving blade angles

From Fig. 3.15, by inlet velocity triangle,

$$\text{Whirl velocity at inlet,} \quad V_{w1} = V_1\cos\alpha = 440 \times \cos(20) = 413.4647 \text{ m/sec}$$

$$\text{Flow velocity at inlet,} \quad V_{f1} = V_1 \sin\alpha = 440 \times \sin(20) = 150.4888 \text{ m/sec}$$

$$\text{Also,} \quad \tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{150.4888}{413.4647 - 206.712} = 0.7278$$

$$\therefore \quad \theta = \tan^{-1}(0.7278) = 36.0496°$$

$$\text{For equiangular blades,} \quad \theta = \phi = \mathbf{36.0496°} \quad \textbf{... Ans.}$$

Also, $V_{f1} = V_{r1} \sin\theta \quad \therefore 150.4888 = V_{r1} \sin(36.0496)$

$\therefore \quad V_{r1} = 255.7222$ m/sec

Step 3 : Calculate the power developed

Friction factor, $K = \dfrac{V_{r2}}{V_{r1}} \quad \therefore \quad 0.9 = \dfrac{V_{r2}}{255.7222}$

$\therefore \quad V_{r2} = 230.1499$ m/sec

From Fig. 3.15, by outlet velocity triangle,

$$\cos\phi = \frac{V_{w2} + u}{V_{r2}} \quad \therefore \quad \cos(36.0496) = \frac{V_{w2} + 206.712}{230.1499}$$

$\therefore \quad V_{w2} = -20.6339$ m/sec

Power developed, $P = \dot{m}(V_{w1} + V_{w2})u$

$= 3 \times (413.4647 + (-20.6339)) \times 206.712$

$\therefore \quad$ $\mathbf{P = 243.6085 \times 10^3\ W}$ **... Ans.**

Ex. 3.6 : *A steam turbine of single row impulse type operating at 3000 rpm has mean diameter of 1.1 m. The nozzle angle is 17°, the ratio of blade velocity to steam velocity is 0.45 and the ratio of relative velocity at outlet to that of inlet is 0.82. The outlet angle of blades is 3° less than the inlet blade angle. Steam flow rate is 10.2 kg/s. Draw the velocity diagram and find resultant thrust on blades, tangential thrust, axial thrust, power developed and blade efficiency of the turbine.*

Sol. : Given data :

$N = 3000$ rpm, $D_m = 1.1$ m, $\alpha = 17°$, $s = \dfrac{u}{V_1} = 0.45$,

$\dfrac{V_{r2}}{V_{r1}} = 0.82$, $\phi = \theta - 3°$, $\dot{m} = 10.2$ kg/sec.

To find : i) F_t ii) F_a iii) F iv) P v) η_b

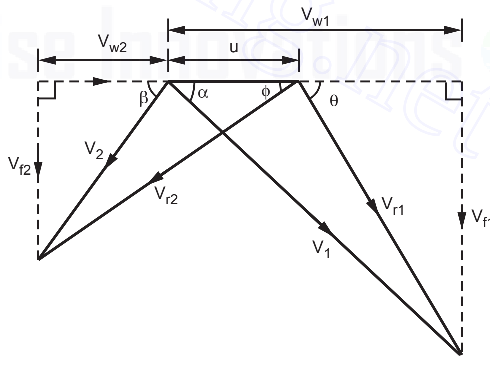


Fig. 3.16

Step 1 : Calculate the tangential thrust

Blade velocity,

$$u = \frac{\pi D_m N}{60} = \frac{\pi \times 1.1 \times 3000}{60} = 172.7875 \text{ m/sec.}$$

As $\dfrac{u}{V_1} = 0.45 \quad \therefore \dfrac{172.7875}{V_1} = 0.45$

$\therefore \quad V_1 = 383.9722$ m/sec

From Fig. 3.16, by inlet velocity triangle,

Whirl velocity at inlet,

$$V_{w1} = V_1 \cos\alpha = 383.9722 \times \cos(17) = 367.1944 \text{ m/sec}$$

Flow velocity at inlet,

$$V_{f1} = V_1 \sin\alpha = 383.9722 \times \sin(17) = 112.2626 \text{ m/sec}$$

Here, $$\tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{112.2626}{367.1944 - 172.7875}$$

$\therefore$ $$\theta = 30.0048°$$

And, $$V_{r1} \cos\theta = V_{w1} - u \quad \therefore \quad V_{r1} \cos(30.0048) = 367.1944 - 172.7875$$

$\therefore$ $$V_{r1} = 224.4926 \text{ m/sec}$$

As, $$\frac{V_{r2}}{V_{r1}} = 0.82 \qquad \therefore \frac{V_{r2}}{224.4926} = 0.82$$

$\therefore$ $$V_{r2} = 184.0839 \text{ m/sec}$$

and $$\phi = \theta - 3° = 30.0048 - 3° = 27.0048°$$

From Fig. 3.16, by outlet velocity triangle,

Whirl velocity at outlet,

$$V_{w2} = V_{r2} \cos\phi - u = 184.0839 \times \cos(27.0048) - 172.7875 = -\ 8.7745 \text{ m/sec}$$

Flow velocity at outlet,

$$V_{f2} = V_{r2} \sin\phi = 184.0839 \times \sin(27.0048) = 83.586 \text{ m/sec}$$

Tangential thrust,

$$F_t = \dot{m}\ (V_{w1} + V_{w2}) = 10.2 \times (367.1944 + (-8.7745))$$

$$\mathbf{F_t = 3655.8829\ N} \qquad \textbf{... Ans.}$$

Step 2 : Calculate the axial thrust and resultant thrust

Axial thrust, $$F_a = \dot{m}\ (V_{f1} - V_{f2}) = 10.2 \times (112.2626 - 83.582)$$

$$\mathbf{F_a = 292.5421\ N} \qquad \textbf{... Ans.}$$

Resultant thrust,

$$F_R = \sqrt{F_t^{\,2} + F_a^{\,2}} = \sqrt{(3655.8829)^2 + (292.5421)^2}$$

$\therefore$ $$\mathbf{F_R = 3667.5687\ N} \qquad \textbf{... Ans.}$$

Step 3 : Calculate the power developed and blade efficiency

Power developed,

$$P = \dot{m}\ (V_{w1} + V_{w2})\,u = 10.2 \times (367.1944 + (-8.7745)) \times 172.7875$$

$\therefore$ $$\mathbf{P = 631.6908 \times 10^3\ W} \qquad \textbf{... Ans.}$$

Blade efficiency,

$$\eta_b = \frac{P}{\frac{1}{2}\dot{m}V_1^2} = \frac{631.6908 \times 10^3}{\frac{1}{2} \times 10.2 \times 383.9722^2}$$

$\therefore$ $$\eta_b = \mathbf{0.8401 = 84.01\ \%}$$ **... Ans.**

Ex. 3.7 : *An impulse turbine has 3 similar stages of the same mean diameter and geometry, each stage develops 500 kW. The peripheral speed of the rotor blades at the mean diameter is 100 m/s; the whirl components of the absolute velocities at entry and exit of the rotor are V_{w1} = 200 m/s and V_{w2} = 0 respectively. The nozzle angles at exit are equal to $\alpha = 65°$. The steam at the exit of the first stage has p = 8.0 bar, T = 200° C. Determine for the first stage :*

i) Mean diameter of the stage for a speed of 3000 r.p.m.

ii) Mass flow rate of steam

iii) Isentropic enthalpy drop for an efficiency of 69 %

iv) Rotor blade angles

v) The blade height of the nozzle and rotor blade at exit.

Sol. : Given data :

$z = 3$, $P_{stage} = 500 \times 10^3$ W, u = 100 m/sec., V_{w1} = 200 m/sec, V_{w2} = 0, $\alpha = 65°$,

At exit of first stage, $p = 8 \times 10^5\ N/mm^2$, $T = 200°$ C, η_n = 69 % = 0.69, N = 3000 rpm.

To find : For first stage i) D_m ii) $\dot{m}$ iii) Δh iv) θ, ϕ v) H

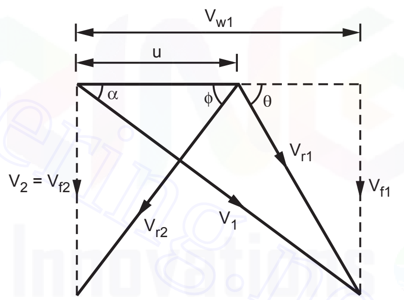


Fig. 3.17

Step 1 : Calculate the mean diameter

Blade velocity,

$$u = \frac{\pi D_m N}{60} \quad \therefore \quad 100 = \frac{\pi \times D_m \times 3000}{60}$$

$\therefore$ $$D_m = \mathbf{0.6366\ m}$$ **... Ans.**

Step 2 : Calculate the mass flow rate of steam

Power developed for a stage,

$$P = \dot{m}(V_{w1} + V_{w2})u \quad \therefore \quad 500 \times 10^3 = \dot{m}\ (200 + 0) \times 100$$

$\therefore$ $$\dot{m} = \mathbf{25\ kg/sec}$$ **... Ans.**

Step 3 : Calculate the isentropic enthalpy drop

From Fig. 3.17, by inlet velocity triangle,

Whirl velocity at inlet,

$$V_{w1} = V_1 \cos\alpha \quad \therefore \quad 200 = V_1 \cos(65)$$

$\therefore$ $$V_1 = 473.2403\ m/sec$$

Nozzle efficiency,

$$\eta_n = \frac{V_1^2}{2\Delta h} \quad \therefore \quad 0.69 = \frac{473.2403^2}{2\Delta h}$$

$\therefore \quad \mathbf{\Delta h = 162.2872 \times 10^3 \ J/kg}$ **... Ans.**

Step 4 : Calculate the blade angles

From Fig. 3.17, by inlet velocity tringle,

Flow velocity at inlet,

$$V_{f1} = V_{w1} \tan\alpha = 200 \times \tan(65) = 428.9013 \text{ m/sec}$$

And, $$\tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{428.9013}{200 - 100}$$

$$\mathbf{\theta = 76.8757^\circ}$$ **... Ans.**

And, $$V_{r1} \cos\theta = V_{w1} - u \quad \therefore \quad V_{r1} \times \cos(76.8757) = 200 - 100$$

$\therefore \quad V_{r1} = V_{r2} = 440.4037$ m/sec ... [$\because$ Blades are frictionless]

From Fig. 3.17, by outlet velocity triangle,

$$\cos\phi = \frac{u}{V_{r2}} = \frac{100}{440.4037} \quad \therefore \ \mathbf{\phi = 76.8756^\circ}$$ **... Ans.**

Step 5 : Calculate the blade height

Specific volume of steam, considering dryness fraction,

$$v_s = x \, v_g$$

As only steam is obtained at exit,

Dryness fraction x = 1

From steam table, at p = 8 bar, T = 200 °C.

Specific volume of steam, $v_g = 0.2608 \ m^3/kg$

$$\therefore \quad v_s = 1 \times 0.2608 = 0.2608 \ m^3/kg$$

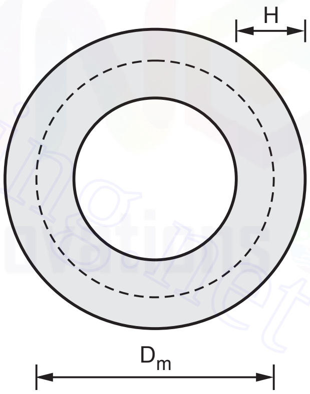


Fig. 3.17 (a)

Discharge of steam,

$$Q = \dot{m} \, v_s = 25 \times 0.2608 = 6.52 \ m^3/sec$$

Also, $$Q = \pi D_m H V_{f1} \quad \therefore 6.52 = \pi \times 0.6366 \times H \times 428.9013$$

$$\therefore \quad \mathbf{H = 7.601 \times 10^{-3} \ m}$$ **... Ans.**

Ex. 3.8 : *Steam issues from the nozzles of an impulse steam turbine with a velocity of 1200 m/s. The nozzle angle is 20°, the mean blade speed is 400 m/s and the inlet and outlet angles of moving blades are equal. The mass of steam flowing through the turbine is 1000 kg/hr. Determine :*

(i) The blade angles *(ii) The tangential force on blades*

(iii) The power developed *(iv) The blade efficiency*

Assume friction factor 0.8

Sol. : Given data :

$$V_1 = 1200 \text{ m/sec. } \alpha = 20^\circ,\ u = 400 \text{ m/sec., } \theta = \phi,\ \dot{m} = 1000 \text{ kg/hr} = \frac{1000}{3600} = 0.2777 \text{ kg/sec. K} = 0.8.$$

To find : i) θ, ϕ ii) F_t iii) P iv) η_b

Step 1 : Calculate the blade angles and relative velocity of steam entering the blades

From Fig. 3.18, by inlet velocity triangle,

Whirl velocity at inlet,

$$V_{w1} = V_1 \cos\alpha = 1200 \times \cos(20) = 1127.6311 \text{ m/sec}$$

Flow velocity at inlet,

$$V_{f1} = V_1 \sin\alpha = 1200 \times \sin(20) = 410.4241 \text{ m/sec}$$

And, $$\tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{410.4241}{1127.6311 - 400} \quad \therefore \theta = 29.4253^\circ$$

As $\theta = \phi$ **= 29.4253°** **... Ans.**

And, $$\sin\theta = \frac{V_{f1}}{V_{r1}} \quad \therefore \sin(29.4253) = \frac{410.4241}{V_{r1}}$$

$\therefore$ $V_{r1} = 835.4035$ m/sec

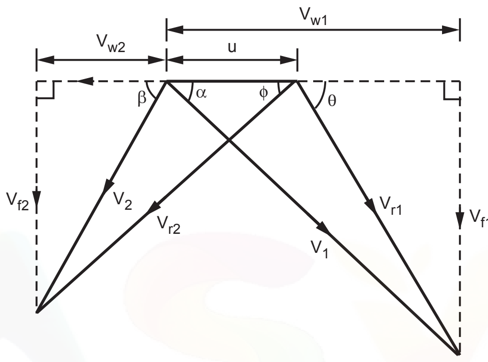


Fig. 3.18

Blade velocity coefficient, $K = \frac{V_{r2}}{V_{r1}}$ $\quad \therefore 0.8 = \frac{V_{r2}}{835.4035}$

$\therefore$ $V_{r2} = 668.3228$ m/sec

Step 2 : Calculate the tangential force on blade

From Fig. 3.18, by outlet velocity triangle,

Whirl velocity at outlet,

$$V_{w2} = V_{r2}\cos\phi - u = 668.3228 \times \cos(29.4253) - 400 = 182.1071 \text{ m/sec}$$

Tangential force, $F_t = \dot{m}(V_{w1} + V_{w2})$

$\therefore$ $\mathbf{F_t} = 0.2777 \times (1127.6311 + 182.1071) = \mathbf{363.7142\ N}$ **... Ans.**

Step 3 : Calculate the power developed and blade efficiency

Power developed,

$$P = \dot{m}(V_{w1} + V_{w2})\, u = 0.2777 \times (1127.6311 + 182.107) \times 400$$

$\therefore$ $\mathbf{P = 145.4857 \times 10^3\ W}$ **... Ans.**

Blade efficiency, $$\eta_b = \frac{P}{\frac{1}{2}\dot{m} V_1^2}$$

$\therefore$ $$\eta_b = \frac{145.48578 \times 10^3}{\frac{1}{2} \times 0.2777 \times (1200)^2} = \mathbf{0.7276 = 72.76\ \%}$$ **... Ans.**

Ex. 3.9 : *In a simple impulse turbine, the nozzles are inclined at 25° to the direction of motion of the moving blades. The steam leaves the nozzles at 370 m/s. The blade speed is 160 m/s. Find suitable inlet and outlet angles for the blades in order that the axial thrust is zero. The relative velocity of steam as it flows over the blade is reduced by 10 % by friction. Determine also the power developed for a flow rate of 10 kg/s.*

Sol. : Given data :

$\alpha = 25°$, $V_1 = 370$ m/sec, u = 160 m/sec., $F_a = 0$,

$V_{r2} = 0.9\ V_{r1}$, $\dot{m} = 10$ kg/sec.

To find : i) θ, ϕ ii) P

Velocity triangle is drawn with help of given data.

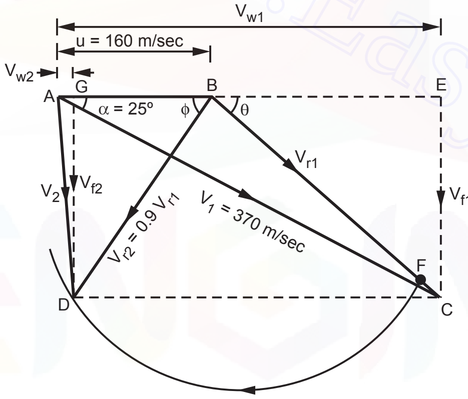


Fig. 3.19

Procedure :

- Consider a suitable scale as **1 cm = 50 m/sec** and draw velocity triangles as shown in Fig. 3.19.
- First, draw horizontal line AB of length $l(AB) = \frac{160}{50} = 3.2$ cm.
- Then, at point A, draw an inclined line AC of length $l(AC) = \frac{370}{50} = 7.4$ cm., at an angle $\alpha = 25°$.
- From point C, draw a perpendicular which intersects horizontal line AB at point E. Join line BC which represents the relative velocity at inlet.

$$V_{r1} = l(BC) \times \text{scale} = 4.7 \times 50 = 235 \text{ m/sec.}$$

∠CBE = blade angle at inlet, θ = **42°** **... Ans.**

- Here the axial thrust being zero i.e. $F_a = 0$, the flow velocities at inlet and outlet are same i.e. $V_{f1} = V_{f2}$. Thus, draw a line from C parallel to horizontal line AB.
- With relative velocity at outlet reduced by 10 % due to friction, mark point on line BC such that $l(BF) = 0.9\ l(BC)$.
- Take point B as centre and BF as the radius, draw an arc. Mark point D as point of intersection of the arc and the horizontal line drawn from point C parallel to horizontal line AB.
- Draw a perpendicular from point D which intersects horizontal line AB at point G. Also, draw the lines AD and BD. Line BD represents the relative velocity at outlet.

$$V_{r2} = l(BD) \times \text{Scale} = 4.2 \times 50 = 210 \text{ m/sec.}$$

∠DBA = The blade angle at outlet, ϕ = **45°** **... Ans.**

- Line GE represents the change in whirl velocity.

$$V_w = l(GE) \times \text{Scale} = 6.5 \times 50 = 325 \text{ m/sec.}$$

Step 1 : Calculate the power developed

Power developed, $P = \dot{m}\, u\, V_w = 10 \times 160 \times 325$

$\therefore$ $\mathbf{P = 520 \times 10^3\ W}$ **... Ans.**

Ex. 3.10 : *A single row steam turbine develops 120 kW at a blade speed of 190 m/sec. when the steam flow is 2 kg/sec. Steam leaves the nozzle at 450 m/sec. The velocity coefficient of the blade is 0.85. Steam leaves the blades axially. Determine nozzle angle and blade angles assuming no shock.*

Sol. : Given data :

$P = 120 \times 10^3$ W, u = 190 m/sec., $\dot{m} = 2$ kg/sec.,

$V_1 = 450$ m/sec, K = 0.85, $\beta = 90°$, $\therefore$ $V_{w2} = 0$

To find : i) α ii) θ, ϕ

Step 1 : Calculate the change in whirl velocity

Power developed, $P = \dot{m}\, u\, V_w$

$$\therefore \quad 120 \times 10^3 = 2 \times 190 \times V_w$$

$$\therefore \quad V_w = 315.7894 \text{ m/sec}$$

Velocity triangle is drawn with help of given data.

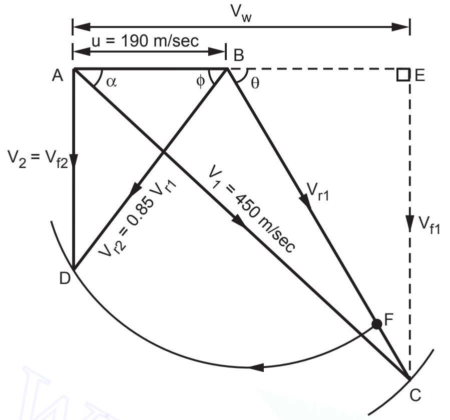


Fig. 3.20

Procedure :

- Consider a suitable scale as 1 cm = 50 m/sec and draw velocity triangles as shown in Fig. 3.20.
- First, draw horizontal line AB of length $l(AB) = \frac{190}{50}$ = 3.8 cm.
- Then, mark point E on horizontal line AB. As AE represents change in whirl velocity.

$$l(AE) = \frac{315.7894}{50} = 6.3 \text{ cm}$$

- Draw perpendicular from point E.
- Take point A as centre, mark an arc with radius $= \frac{450}{50} = 9$ cm. Mark C as point of intersection of the arc and the perpendicular drawn from point E. Join line AC which represents absolute velocity at inlet.

$\angle CBE$ = Blade angle at inlet, $\theta = \mathbf{69^\circ}$ **... Ans.**

$\angle CAE$ = Exit angle for nozzle, $\alpha = \mathbf{47^\circ}$ **... Ans.**

- As steam leaves blades axially, take $\beta = 90^\circ$. Draw perpendicular from point A. With relative velocity at outlet reduced by 15 % due to friction, mark point F on line BC such that $l(BF) = 0.85\ l(BC)$. Take point B as centre and BF as radius, draw an arc. Mark point D as point of intersection of the arc and the perpendicular.
- Join line BD which represents relative velocity at outlet.

$\angle DAB$ = Blade angle at outlet, $\phi = \mathbf{52^\circ}$ **... Ans.**

Ex. 3.11 : *Steam enters the blade row of an impulse turbine with a velocity of 650 m/s at an angle of 20° to the plane of rotation of the blades. The mean blade speed is 270 m/s. The blade angle on the exit side is 35°. The blade friction coefficient is 15 %. Determine :*

(i) The angle of the blade on the entry side,

(ii) The work done per kg of steam,

(iii) The diagram efficiency and

(iv) The axial thrust per kg of steam/second.

Sol. : Given data :

$$V_1 = 650 \text{ m/sec}, \alpha = 20^\circ, u = 270 \text{ m/sec.}$$

$$\phi = 35^\circ, V_{r2} = 0.85\ V_{r1}$$

To find : i) θ ii) W iii) η_b iv) F_a

Procedure :

- Consider a suitable scale as **1 cm = 50 m/sec.** and draw velocity triangles as shown in Fig. 3.21.

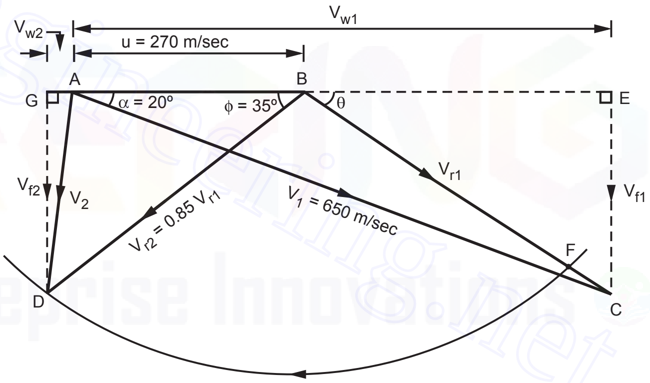


Fig. 3.21

- First draw horizontal line AB of length $l(AB) = \frac{270}{50} = 5.4$ cm
- Then, at point A, draw an inclined line AC of length $l(AC) = \frac{650}{50} = 13$ cm, at an angle $\alpha = 20^\circ$.
- From point C, draw a perpendicular which intersects horizontal line AB at E. Join line BC which represents the relative velocity at inlet.

Velocity triangle is drawn with help of given data.

$$V_{r1} = l(BC) \times \text{Scale} = 8.2 \times 50 = 410 \text{ m/sec.}$$

$$\angle CBE = \text{Blade angle at inlet}, \theta = 33^\circ.$$

Line EC represents the flow velocity at inlet.

$$V_{f1} = l(EC) \times \text{Scale} = 4.5 \times 50 = 225 \text{ m/sec.}$$

- At point B, draw an inclined line at an angle, $\phi = 35°$. With relative velocity at outlet reduced by 15 % due to friction, Mark point F on line BC such that $l(BF) = 0.85\ l(BC)$. Take point B as centre and BF as radius, draw an arc. Mark point D as point of intersection of the arc and the inclined line drawn from point B at an angle, $\phi = 35°$.
- Draw a perpendicular from point D which intersects horizontal line AB at point G. Also, draw the line AD. Line GD represents the flow velocity at outlet.

$$V_{f2} = l(GD) \times \text{Scale} = 4.1 \times 50 = 205 \text{ m/sec.}$$

Line GE represents the change in whirl velocity.

$$V_w = l(GE) \times \text{Scale} = 12.7 \times 50 = 635 \text{ m/sec.}$$

With $\dot{m} = 1$ kg/sec.

Step 1 : Calculate work done per kg, diagram efficiency and axial thrust per kg of steam per sec.

Work done per kg,

$$W = u\, V_w$$

$$\therefore \quad \mathbf{W} = 270 \times 635 = \mathbf{171.45 \times 10^3\ W/kg} \quad \text{... Ans.}$$

Diagram efficiency,

$$\eta_b = \frac{u\, V_w}{\frac{1}{2} V_1^2}$$

$$\therefore \quad \eta_b = \frac{270 \times 635}{\frac{1}{2} \times 650^2} = 0.8115 = \mathbf{81.1597\ \%} \quad \text{... Ans.}$$

Axial thrust per kg of steam/sec.

$$F_a = V_{f1} - V_{f2}$$

$$\therefore \quad \mathbf{F_a} = 225 - 205 = \mathbf{20\ N/kg/sec} \quad \text{... Ans.}$$

Ex. 3.12 : *The velocity of steam exiting the nozzle of the impulse stage of a turbine is 400 m/sec. The blades operate close to the maximum blading efficiency. The nozzle angle is 20°. Considering equiangular blades and neglecting blade friction, calculate for a steam flow of 0.6 kg/sec, the diagram power and the diagram efficiency.*

Sol. : Given data :

$$V_1 = 400 \text{ m/sec}, \alpha = 20°, \theta = \phi, V_{r1} = V_{r2}, \dot{m} = 0.6 \text{ kg/sec.}$$

To find : i) P ii) η_b

Step 1 : Calculate the diagram power

For maximum blade efficiency,

Blade speed ratio, $$s = \frac{\cos\alpha}{2} = \frac{\cos(20)}{2} = 0.4698$$

Also, $s = \frac{u}{V_1}$ $\therefore 0.4698 = \frac{u}{400}$

$\therefore$ $u = 187.92$ m/sec.

From Fig. 3.22, by inlet velocity triangle,

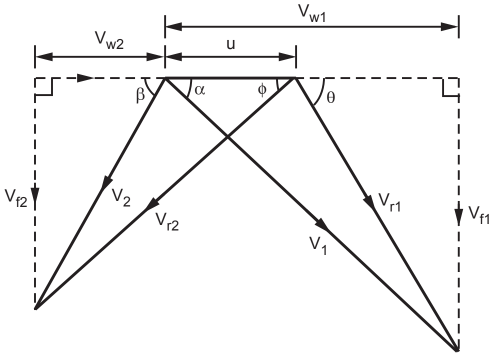


Fig. 3.22

Whirl velocity at inlet, $V_{w1} = V_1 \cos\alpha$

$= 400 \times \cos(20) = 375.877$ m/sec

Flow velocity at inlet, $V_{f1} = V_1 \sin\alpha$

$= 400 \times \sin(20) = 136.808$ m/sec

And, $\tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{136.808}{375.877 - 187.92} = 0.7278$

$\therefore$ $\theta = \tan^{-1}(0.7278) = 36.0496°$

Relative velocity at inlet, $V_{r1} = \frac{V_{f1}}{\sin\theta} = \frac{136.808}{\sin(36.0496)} = 232.4747$ m/sec

Neglecting blade friction, $V_{r1} = V_{r2} = 232.4747$ m/sec.

For equiangular blades, $\theta = \phi = 36.0496°$

From Fig. 3.22, by outlet velocity triangle,

Whirl velocity at outlet, $V_{w2} = V_{r2}\cos\phi - u = 232.4747 \times \cos(36.0496) - 187.92 = 0.03762$ m/sec

Diagram power, $P = \dot{m}\,(V_{w1} + V_{w2})\,u$

$= 0.6 \times (375.877 + 0.03762) \times 187.92$

$\therefore$ $P = \mathbf{42.3851 \times 10^3}$ **W** **... Ans.**

Step 2 : Calculate the diagram efficiency

For maximum blading efficiency, $\eta_b = (\eta_b)_{max}$

For negligible blade friction and equiangular blades,

$$(\eta_b)_{max} = (\cos\alpha)^2 = (\cos(20))^2$$

$\eta_b = 0.88302 =$ **88.302 %** **... Ans.**

Ex. 3.13 : *A single stage impulse turbine is supplied with steam at 4 bar and 160 °C and it is exhausted at a condenser pressure of 0.15 bar at the rate of 60 kg/min. The steam expands in nozzle of nozzle efficiency 90%. The blade speed is 250 m/sec. and nozzles are inclined at 20°. The blade angle at exit of moving blade is 30°. Neglecting friction, find :*

i) Steam jet velocity *ii) Power developed*

iii) Blade efficiency *iv) Stage efficiency.*

Sol. : Given data :

$p_1 = 4$ bar, $T_1 = 160$ ºC, $p_2 = 0.15$ bar, $\dot{m} = 60$ kg/min $= \dfrac{60}{60} = 1$ kg/sec, $\eta_n = 90\ \% = 0.9$, u = 250 m/sec,

$\alpha = 20º$, $\phi = 30º$, $V_{r1} = V_{r2}$, $\Delta h = 705$ kJ/kg

To find : i) V_1 ii) P iii) η_b iv) η_{stage}

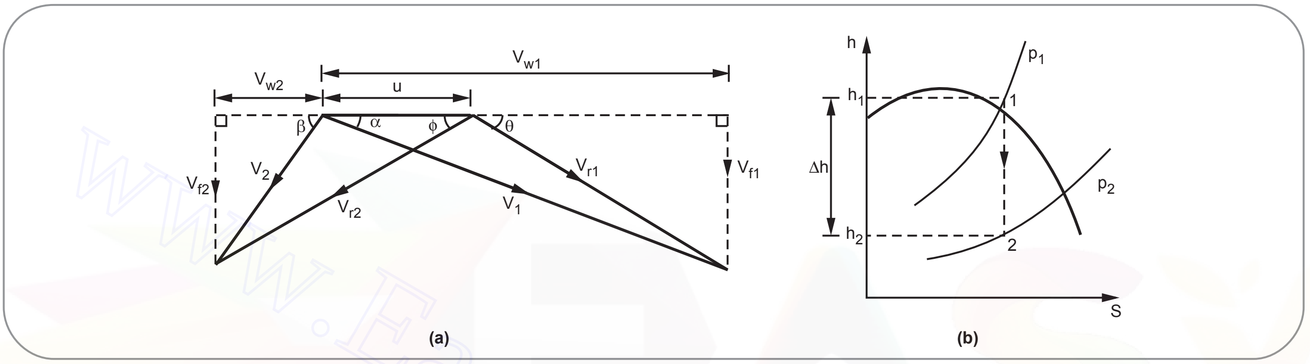


Fig. 3.23

Step 1 : Calculate the steam jet velocity

Nozzle efficiency, $\eta_n = \dfrac{V_1^2}{2\,\Delta h}$

$\therefore \quad 0.90 = \dfrac{V_1^2}{2 \times 705 \times 10^3}$

$\therefore \quad$ **V_1 = 1126.499 m/sec** **... Ans.**

Step 2 : Calculate the power developed

From Fig. 3.23, by inlet velocity triangle,

Whirl velocity at inlet, $V_{w1} = V_1 \cos\alpha = 1126.499 \times \cos(20) = 1058.5627$ m/sec

Flow velocity at inlet, $V_{f1} = V_1 \sin\alpha = 1126.499 \times \sin(20) = 385.2853$ m/sec

And, $\tan\theta = \dfrac{V_{f1}}{V_{w1} - u} = \dfrac{385.2853}{1058.5627 - 250} = 0.4765$

$\therefore \quad \theta = \tan^{-1}(0.4765) = 25.478°$

Relative velocity at inlet, $V_{r1} = \dfrac{V_{f1}}{\sin\theta} = \dfrac{385.2853}{\sin(25.478)} = 895.6696$ m/sec

Neglecting blade friction, $V_{r1} = V_{r2} = 895.6696$ m/sec.

From Fig. 3.23, by outlet velocity triangle,

Whirl velocity at outlet, $V_{w2} = V_{r2} \cos\phi - u$

$$\therefore \qquad V_{w2} = 895.6695 \times \cos(30) - 250 = 525.6726 \text{ m/sec}$$

Power developed, $\quad P = \dot{m}\,(V_{w1} + V_{w2})\,u$

$$\mathbf{P} = 1 \times (1058.5627 + 525.6726) \times 250 = \mathbf{396.0588 \times 10^3\ W} \qquad \textbf{... Ans.}$$

Step 3 : Calculate the blade efficiency and stage efficiency

Blade efficiency, $$\eta_b = \frac{P}{\frac{1}{2}\,\dot{m}\,V_1^2} = \frac{396.0588 \times 10^3}{\frac{1}{2} \times 1 \times 1126.499^2}$$

$$\therefore \qquad \boldsymbol{\eta_b} = \mathbf{0.6242 = 62.4206\ \%} \qquad \textbf{... Ans.}$$

Stage efficiency, $$\eta_{stage} = \eta_b \times \eta_n = 0.6242 \times 0.9 = 0.5617 = 0.5617 \times 100$$

$$\therefore \qquad \boldsymbol{\eta_{stage}} = \mathbf{56.17\ \%} \qquad \textbf{... Ans.}$$

Ex. 3.14 : *The nozzles of De-laval turbine delivers 1.5 kg/sec of steam at a speed of 800 m/sec to the ring of moving blades having speed 200 m/sec. The exit angle of nozzle is 18°. If blade velocity coefficient is 0.75 and exit angle of moving blades is 25°, Find : i) Inlet angle of moving and fixed blades*

ii) Diagram efficiency *iii) Power developed*

iv) Axial thrust on rotor. *v) Energy lost in blades per second.*

Sol. : Given data :

$\dot{m}$ = 1.5 kg/sec, V_1 = 800 m/sec, u = 200 m/sec, α = 18°, K = 0.75, ϕ = 25°.

To find : i) θ, β ii) η_b iii) P iv) F_a v) ΔE

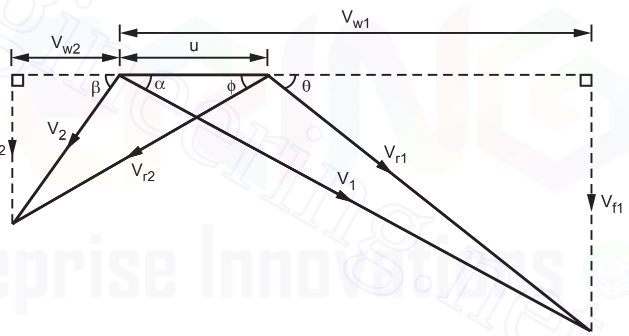


Fig. 3.24

Step 1 : Calculate the inlet angles of moving and fixed blades

From Fig. 3.24, by inlet velocity triangle,

Flow velocity at inlet, $$V_{f1} = V_1 \sin\alpha$$

$$= 800 \times \sin(18) = 247.2135 \text{ m/sec}$$

Whirl velocity at inlet, $$V_{w1} = V_1 \cos\alpha = 800 \times \cos(18) = 760.8452 \text{ m/sec}$$

And, $$\tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{247.2153}{760.8452 - 200} \qquad \therefore\ \theta = \mathbf{23.7872°} \qquad \textbf{... Ans.}$$

Relative velocity at inlet, $$V_{r1} = \frac{V_{f1}}{\sin\theta} = \frac{247.2135}{\sin(23.7872)} = 612.9145 \text{ m/sec}$$

Blade velocity coefficient, $$K = \frac{V_{r2}}{V_{r1}} \qquad \therefore\ 0.75 = \frac{V_{r2}}{612.9145}$$

$$V_{r2} = 459.6858 \text{ m/sec}$$

From Fig. 3.24, by outlet velocity triangle.

Flow velocity at outlet, $V_{f2} = V_{r2} \sin\phi = 459.6858 \times \sin(25) = 194.2716$ m/sec

And, $\tan\phi = \dfrac{V_{f2}}{V_{w2} + u}$ $\therefore \tan(25) = \dfrac{194.2716}{V_{w2} + 200}$

$\therefore$ $V_{w2} = 216.6167$ m/sec

And $\tan\beta = \dfrac{V_{f2}}{V_{w2}} = \dfrac{194.2716}{216.6167}$ $\therefore$ $\boldsymbol{\beta = 41.8871^\circ}$ **... Ans.**

Step 2 : Calculate the power developed and blade efficiency

Power developed,

$$P = \dot{m}(V_{w1} + V_{w2})\,u = 1.5 \times (760.8452 + 216.6167) \times 200$$

$\therefore$ $\mathbf{P = 293.2385 \times 10^3 \ W}$ **... Ans.**

Diagram efficiency (blade efficiency),

$$\eta_b = \frac{P}{\frac{1}{2}\dot{m}V_1^2} = \frac{293.2385 \times 10^3}{\frac{1}{2} \times 1.5 \times 800^2}$$

$\therefore$ $\boldsymbol{\eta_b = 0.6109 = 61.0913\ \%}$ **... Ans.**

Step 3 : Calculate the axial thrust on rotor and energy lost in blades per second

Axial thrust,

$$F_a = \dot{m}(V_{f1} - V_{f2}) = 1.5 \times (247.2135 - 194.2716)$$

$\mathbf{F_a = 79.4128\ N}$ **... Ans.**

Rate of energy loss in blades,

$$\Delta E = \dot{m}(V_{r1}^2 - V_{r2}^2) = 1.5 \times (612.9145^2 - 459.6858^2)$$

$\mathbf{\Delta E = 246.5297 \times 10^3\ W}$ **... Ans.**

Ex. 3.15 : *A simple impulse turbine has one ring of moving blades running at 200 m/s. The absolute velocity of the steam at exit from the stage is 90 m/s at an angle of 75° from the tangential direction. Blade velocity coefficient is 0.82 and flow of steam through the stage is 3 kg/s. If the blades are equiangular, determine :*

(i) Blade angles *(ii) Nozzle angle*

(iii) Absolute velocity of steam issuing from nozzle *(iv) Axial thrust.*

Sol. : Given data :

$u = 200$ m/sec, $V_2 = 90$ m/sec, $\beta = 75^\circ$, $K = 0.82$, $\dot{m} = 3$ kg/sec., $\theta = \phi$

To find : i) θ, ϕ ii) α iii) V_1 iv) F_a

Velocity triangle is drawn with help of given data.

Procedure :

- Consider a suitable scale as **1 cm = 25** m/sec and draw velocity triangles as shown in Fig. 3.25.

- First, draw horizontal line AB of length

$l(AB) = \frac{200}{25} = 8$ cm

- Then, at point A, draw an inclined line AD of length

$l(AD) = \frac{90}{25} = 3.6$ cm, at an angle $\beta = 75°$.

- From point D, draw a perpendicular which intersects horizontal line AB at G. Join line BD which represents the relative velocity at outlet.

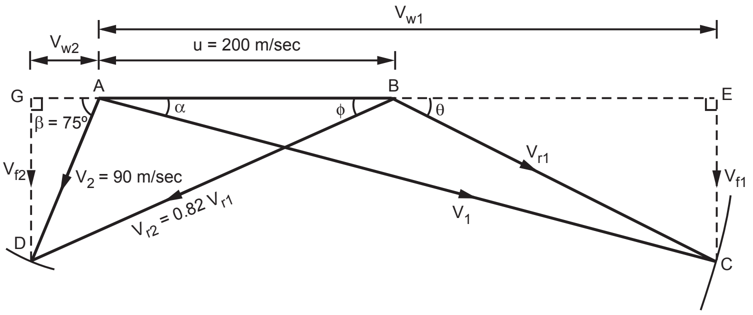


Fig. 3.25

$$V_{r2} = l(BD) \times \text{Scale} = 9.6 \times 25 = 240 \text{ m/sec.}$$

$$\angle DBG = \text{Blade angle at outlet, } \theta = \mathbf{21°} \qquad \textbf{... Ans.}$$

For equiangular blades, $\theta = \phi$

$$\angle CBE = \text{Blade angle at inlet, } \theta = \mathbf{21°} \qquad \textbf{... Ans.}$$

Line GD represents the flow velocity at outlet.

$$V_{f2} = l(GD) \times \text{Scale} = 3.5 \times 25 = 87.5 \text{ m/sec.}$$

- At point B, draw an inclined line at an angle, $\theta = 21°$. With relative velocity at outlet reduced to 82 % due to friction, mark point C on this line such that $l(BC) = \frac{l(BD)}{0.82}$.
- Draw perpendicular from point C, which intersects horizontal line AB at E.

Join line AC which represents the absolute velocity at inlet.

$$\mathbf{V_1} = l(AC) \times \text{Scale} = 19.3 \times 25 = \mathbf{482.5 \text{ m/sec.}} \qquad \textbf{... Ans.}$$

$$\angle CAE = \text{nozzle angle, } \alpha = \mathbf{13°} \qquad \textbf{... Ans.}$$

Line EC represents the flow velocity at inlet.

$$V_{f1} = l(EC) \times \text{Scale} = 4.1 \times 25 = 102.5 \text{ m/sec.}$$

Step 1 : Calculate the axial thrust

Axial thrust, $F_a = \dot{m}\,(V_{f1} - V_{f2})$

$$\mathbf{F_a} = 3 \times (102.5 - 87.5) = \mathbf{45\ N} \qquad \textbf{... Ans.}$$

Ex. 3.16 : *In a De Laval turbine, steam is issued from the nozzle with a velocity of 1500 m/s whereas the mean blade velocity is 500 m/s. The nozzle angle is 20° and the inlet and outlet angles of blades are equal. The mass of the steam flowing through the turbine is at the rate of 1200 kg/hr. Assuming blade velocity coefficient k = 0.8, draw the velocity diagram and determine :*

i) The blade angles. ii) The power developed by turbine. iii) The blade efficiency.

Sol. : Given :

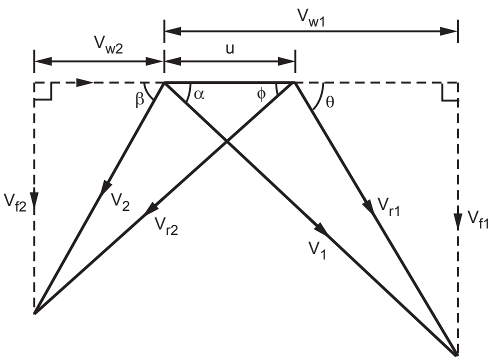


Fig. 3.26

V_1 = 1500 m/s, u = 500 m/s, α = 20°, θ = φ,

m = 1200 kg/hr = 0.33 kg/s, $\frac{V_{r2}}{V_{r1}} = 0.8$

To find : i) θ and φ ii) P iii) η_b

Step 1 : Calculate blade angles

From velocity triangle,

$$V_{w1} = V_1 \cos\alpha$$

$$= 1500 \times \cos 20 = 1409.5 \text{ m/s}$$

$$V_{f1} = V_1 \sin\alpha = 1500 \times \sin 20 = 512 \text{ m/s}$$

$$\tan\theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{512}{1409.5 - 500}$$

∴ θ = **29.37°** and $\theta = \phi$ = **29.37°** **...Ans.**

Step 2 : Calculate power developed by turbine

From velocity triangle,

$$V_{r1} = \sqrt{V_{f1}^2 + (V_{w1} - u_1)^2} = \sqrt{512^2 + (1409.5 - 500)^2}$$

∴ $$V_{r1} = 1043.711 \text{ m/s}$$

$$V_{r2} = 0.8\, V_{r1} = 834.96 \text{ m/s}$$

$$\cos\phi = \frac{V_{w2} + u}{V_{r2}} \qquad \therefore \cos 29.4 = \frac{V_{w2} + 500}{834.96}$$

∴ $$V_{w2} = 227.9 \text{ m/s}$$

Power is given by,

$$P = \frac{\dot{m}\,(V_{w1} + V_{w2}) \times u}{1000}$$

$$= \frac{0.33\,(1409.5 + 227.9) \times 500}{1000}$$

∴ **P = 270.17 kW** **...Ans.**

Step 3 : Calculate blade efficiency

We know that,

$$\eta_b = \frac{2(V_{w1} + V_{w2})}{V_1^2} \times u = \frac{2(1409.5 + 227.9) \times 500}{1500^2}$$

∴ η_b = **0.728 = 72.8 %** **...Ans.**

3.9 Velocity Diagram for Velocity Compounded Impulse Turbine (Multistaging)

- In the impulse turbine, if the steam at high pressure is expanded from the boiler pressure to condenser pressure in one stage, the rotor speed i.e. blade velocity becomes very high.
- Due to this, the absolute velocity of steam becomes maximum.
- To avoid this, the velocity compounded stage is employed to give lower blade speed ratio and better utilization of kinetic energy of steam.
- The velocity compounding is done by absorbing the kinetic energy of steam raised in the nozzles by two or more successive rings of moving blades each separated by ring of fixed blades.
- The function of fixed blades is to change the direction of motion of the steam received from a ring of moving blades and to redirect the steam on to the next row of moving blades. Refer Fig. 3.25.
- Fig. 3.28 shows the velocity triangles for the first and second row of moving blades of velocity compounded arrangement.
- The velocity compounded arrangement facilitates the minimum loss of kinetic energy therefore the efficiency becomes maximum.
- The velocity of blades (u) for both the rows is same as they are mounted on the same shaft and at equal height.
- *Let, the dash terms in the prefix (') indicates the velocities and angles for the second row of moving blades.*

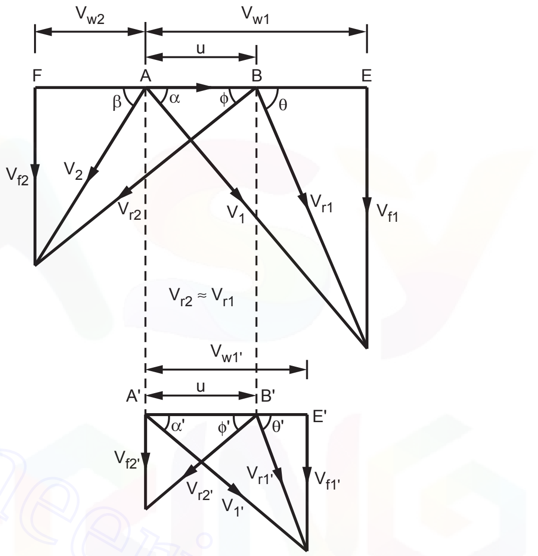


Fig. 3.28 : Velocity triangles for multistage impulse turbine

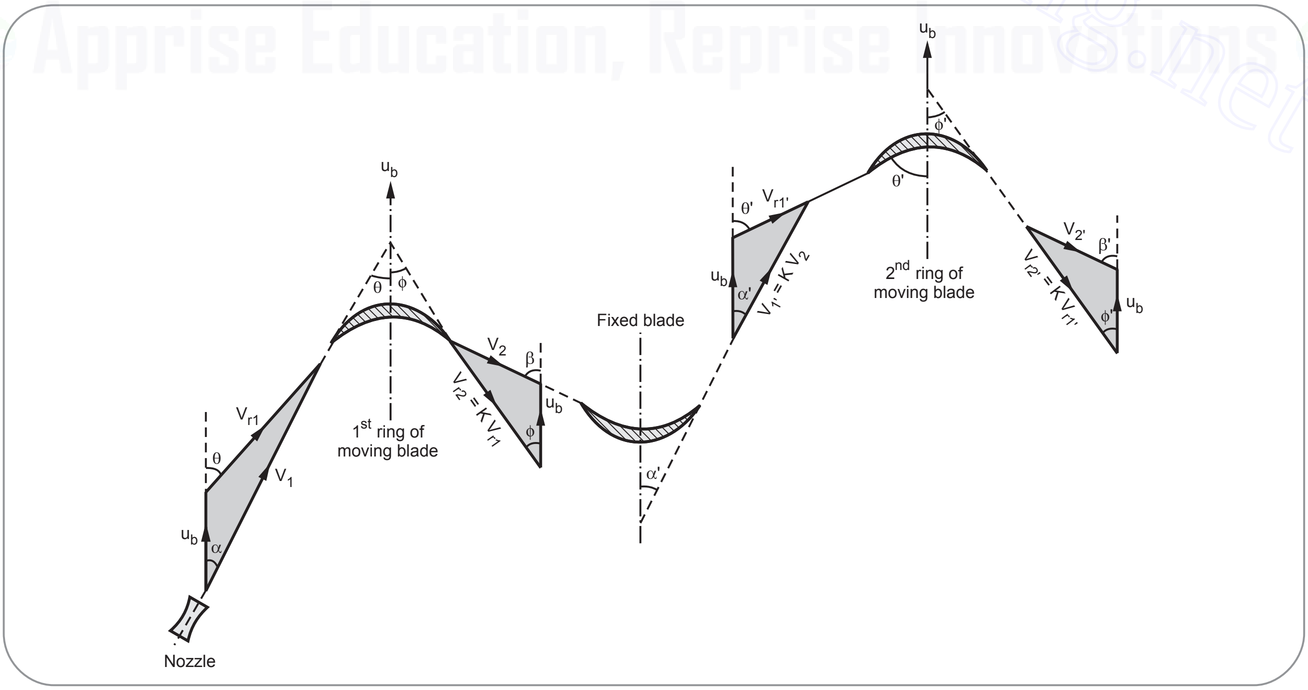


Fig. 3.27 : Steam flow in multistage

For first row of moving blades :

- The work done per kg of steam for the first row is given by,

$$WD_1 = (V_{w1} + V_{w2})\,u = u\,(BE + BF) \quad \ldots \text{(i)}$$

From Fig. 3.28, $\quad BE = V_{r1}\cos\theta$ and $BF = V_{r2}\cos\phi$

$$\therefore \quad WD_1 = (V_{r1}\cos\theta + V_{r2}\cos\phi)\,u \quad \ldots\text{(ii)}$$

- If there is no friction loss $(V_{r1} = V_{r2})$ and symmetrical blading is used $(\theta = \phi)$, then the work done in this stage becomes,

$$\therefore \quad WD_1 = (V_{r1}\cos\theta + V_{r1}\cos\theta)\;u = 2\;V_{r1}\;\cos\theta \times u$$

$$\therefore \quad WD_1 = 2u\,(V_1\cos\alpha - u) \quad \ldots (\because\; V_{r1}\cos\theta = BE = AE - AB)$$

$$\ldots (3.13)$$

- The magnitude of absolute velocity leaving the first row (V_1) is same as that of magnitude of absolute velocity entering the second row.

$$\therefore \quad V_1' = V_2$$

For second row of moving blades

- The work done per kg of steam for second row of moving blades is given by,

$$WD_2 = V_{w1}' \times u \quad \ldots \text{[For axial discharge } \beta' = 90^\circ \text{ and } V_{w2}' = 0]$$

Similar to equation (i), we can write,

$$WD_2 = [A'B' + B'E']\;u = [V_{r2}'\cos\phi' + V_{r1}'\cos\theta']\;\;u$$

But for symmetrical blades, $\theta' = \phi'$ and for negligible friction $V_{r2}' = V_{r1}'$

$$\therefore \quad WD_2 = [V_{r2}'\cos\theta' + V_{r1}'\cos\theta']\;\;u \;\; = 2u\;\;V_{r1}'\;\cos\theta'$$

$$\therefore \quad WD_2 = 2\,u\,(V_1'\cos\alpha' - u) \quad \ldots \text{(iii)}$$

- But, the leaving angle of absolute velocity of first row of moving blades (β) may be equal to the entering angle of absolute velocity of second row of moving blades (α') i.e. $\alpha' = \beta$.

$$\therefore \quad V_1'\cos\alpha' = V_2\cos\beta = V_{r2}\cos\phi - u \quad \ldots (FA = FB - AB)$$

$$= V_{r1}\cos\theta - u = (V_1\cos\alpha - u) - u \quad \ldots (\theta = \phi \text{ and } V_{r2} = V_{r1})$$

$$= V_1\cos\alpha - 2u$$

Substituting the above value in equation (iii),

$$\therefore \quad WD_2 = 2\,u\,[(V_1\cos\alpha - 2u) - u] = 2\,u\,(V_1\cos\alpha - 3u) \quad \ldots (3.14)$$

- Now, the total work done per kg of steam passing through the both stages is given as,

$$WD_T = WD_1 + WD_2 = 2\,u\,(V_1\cos\alpha - u) + 2\,u\,(V_1\cos\alpha - 3u)$$

$$= 2\,u\,(2V_1\cos\alpha - 4u) = 4u\,(V_1\cos\alpha - 2u) \quad \ldots (3.15)$$

- Blade efficiency for two stage impulse turbine is,

$$\eta_b = \frac{WD_T}{\frac{V_1^2}{2}} = \frac{4u\,(V_1 \cos\alpha - 2u)}{\frac{V_1^2}{2}}$$

$$\therefore \quad \eta_b = \frac{8u}{V_1}\left(\cos\alpha - \frac{2u}{V_1}\right) = 8s\,(\cos\alpha - 2s) \qquad \ldots \left(s = \frac{u}{V_1}\right) \ldots \text{(iv)}$$

- For maximum blade efficiency, differentiate the above equation (iv) and equate to zero i.e.

$$\frac{d(\eta_b)}{ds} = 0$$

$$\therefore \quad \frac{d}{ds}[8s\,(\cos\alpha - 2s)] = 0$$

$$\therefore \quad \cos\alpha - 4s = 0 \quad \text{or} \quad s = \frac{\cos\alpha}{4} \qquad \ldots \text{(v)}$$

Substituting this value in equation (iv),

$$\therefore \quad \eta_{bmax} = 8 \times \frac{\cos\alpha}{4}\left[\cos\alpha - 2 \times \frac{\cos\alpha}{4}\right]$$

$$\therefore \quad \eta_{bmax} = 2\cos\alpha\left(\cos\alpha - 0.5\cos\alpha\right) = \cos^2\alpha \qquad \ldots (3.16)$$

- From equation (v) we can write

$$s = \frac{u}{V_1} = \frac{\cos\alpha}{4} \quad \text{or} \quad V_1 = \frac{4u}{\cos\alpha}$$

Substituting this value in the equation (3.16)

$$\therefore \quad (WD_T)_{max} = 4u\,(V_1\cos\alpha - 2u) = 4u\left(\frac{4u}{\cos\alpha} \times \cos\alpha - 2u\right)$$

$$\therefore \quad (WD_T)_{max} = 8u^2$$

- The above analysis is applicable for two stage impulse turbine. The same procedure is used for analysis of more stages.
- If there are 'n' stages, the optimum blade speed ratio is,

$$s = \frac{\cos\alpha}{2n} \qquad \ldots (3.17)$$

$$\text{Work done in the last row} = \frac{WD_T}{2^n} \qquad \ldots (3.18)$$

- From the above equation (3.18) it is clear that, as the number of rows increases, the utility of the last row decreases. *Therefore, practically more than two rows are not preferred for velocity compounded impulse turbine.*

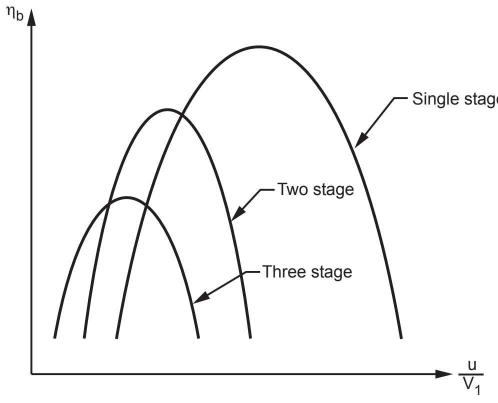


Fig. 3.28 (a)

Advantages of velocity compounded impulse turbine

- The initial cost is less as it requires less number of stages (only 2 or 3).
- The floor space required is less.
- As the total pressure drop occurs in the nozzle itself the turbine housing need not be made of stronger material. This saves the material cost.

Limitation of velocity compounded impulse turbine

- Friction losses are more due to high velocity of steam.
- It has high steam consumption and lower blade efficiency.
- The maximum blade efficiency and optimum value of (u / V_1) decreases with the increase in number rows of moving blades.
- Also the power developed in each successive row decreases with the increase in number of rows.
- All the stages of this type of impulse turbine are not used with same economy.

Ex. 3.17 : *The following data relates to a compounded impulse turbine having two rows of moving blades and one row of fixed blades in between them.*
The velocity of steam leaving the nozzle = 600 m/sec
Blade speed = 125 m/sec, Nozzle angle = 20°
First moving blade discharge angle = 20°
First fixed blade discharge angle = 25°
Second moving blade discharge angle = 30°
Friction loss in each ring = 10 % of relative velocity
Find : i) Diagram efficiency ii) Power developed for a steam flow rate 10 kg/sec.

Sol. : Given data :

$V_1 = 600$ m/sec, $u = 125$ m/sec, $\alpha_1 = 20°$, $\phi_1 = 20°$, $\alpha_2 = 25°$, $\phi_2 = 30°$, $K = 0.9$, $\dot{m} = 10$ kg/sec

To find : i) η_b ii) P

Procedure :

1) For first stage :

- Consider a suitable scale as 1cm = 50 m/sec and draw velocity traingles as shown in Fig 3.29 (a).
- Draw horizontal line A_1B_1 of length $l(A_1B_1) = \frac{125}{50}$ = 2.5 cm and at point A_1, draw an inclined A_1C_1 of length $l(A_1C_1) = \frac{600}{50} = 12$ cm, at angle $\alpha_1 = 20°$.

  (See Fig. 3.29 on next page)

  Join line B_1C_1 which represents relative velocity at inlet for first stage.
- At point B_1, draw an inclined at an angle $\phi_1 = 20°$. Take D_1 as point of intersection of inclined line and an are drawn with center as point B_1 and radius B_1F_1 = $0.9\, l(B_1C_1)$ due to the friction losses in blade.

  Joint line A_1D_1 representing absolute velocity at outlet for first stage.
- Line G_1E_1 represents the change in whirl velocity for first stage

  $V_{w1} + V_{w2} = l(G_1E_1) \times \text{Scale} = 16.8 \times 50 = 840$ m/sec

2) For second stage :

- Consider a suitable scale as 1cm = 50 m/sec and draw velocity triangles as shown in Fig 3.29 (b).
- Take projection of A_1B_1 as A_2B_2 for the second stage. Blade velocity remains same as both the stages are mounted on the same turbine shaft.
- At point A_2 draw an inclined line A_2C_2 of length $l(A_2C_2) = 0.9\, l(A_1D_1)$, due to friction losses in blade = Absolute velocity at inlet for the second stage, at an angle $\alpha_2 = 25°$.
- Join line B_2C_2 which represents relative velocity at inlet for second stage.

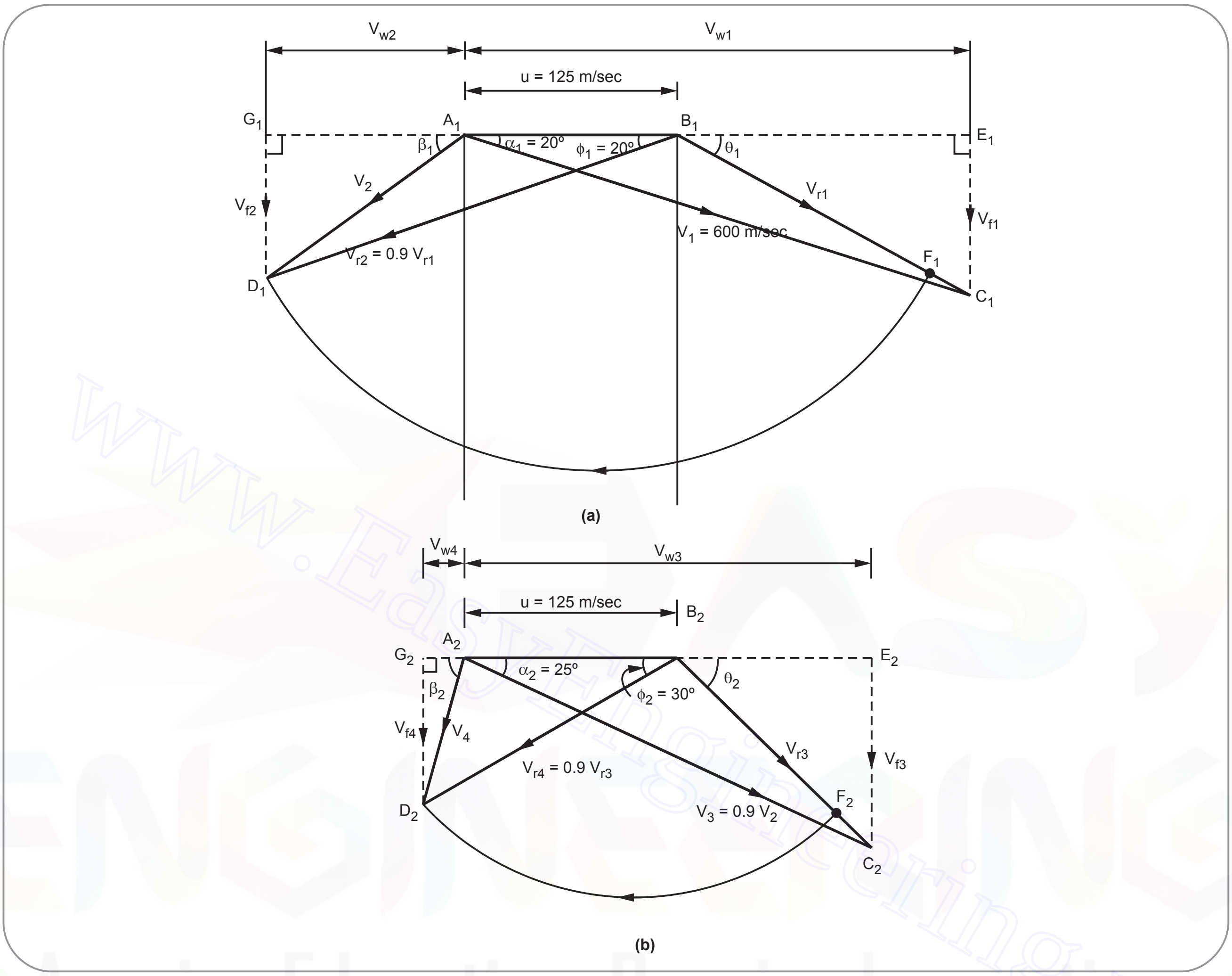


Fig. 3.29

- At point B_2 draw an inclined line at angle $\phi_2 = 30°$. Take D_2 as point of intersection of inclined line and an are drawn with center as point B_2 and radius $B_2F_2 = 0.9\ l(B_2C_2)$ due to friction losses in blade. Join line (A_2D_2) representing absolute velocity at oulet for second stage. Line G_2E_2 represents the change in whirl velocity for second stage i.e.

$$V_{w3} + V_{w4} = (G_2E_2) \times \text{Scale} = 5.7 \times 50 = 285 \text{ m/sec}$$

Step 1 : Calculate the diagram efficiency and power developed

Power developed, $P = \dot{m}\,u\,[(V_{w1} + V_{w2}) + (V_{w3} + V_{w4})] = 10 \times 125 \times [840 + 285]$

$\therefore$ $\mathbf{P = 1406.25 \times 10^3\ W}$ **... Ans.**

Diagram efficiency, $\eta_b = \dfrac{P}{\frac{1}{2}\dot{m}V_1^2}$

$$\therefore \qquad \eta_b = \frac{1406.25 \times 10^3}{\frac{1}{2} \times 10 \times 600^2} = \mathbf{0.7812 = 78.125\ \%} \qquad \textbf{... Ans.}$$

Ex. 3.18 : *The first stage of a turbine is two-row velocity compounded impulse wheel. The steam velocity at inlet is 600 m/sec the mean blade velocity is 120 m/sec. The nozzle angle is 16° and the exit angles first row of moving blades, fixed blades and the second row of moving blades are 18°, 21° and 35° respectively. Calculate :*

i) Blade inlet angles for each row

ii) Calculate for each row of moving blades, the driving force and the axial thrust on the wheel for a mass flow rate of 4 kg/sec.

ii) What whould be the maximum possible diagram efficiency for the given steam inlet and nozzle angle ? Take the blade velocity coefficient as 0.9 for all blades.

Sol. : Given data :

V_1 = 600 m/sec, u = 120 m/sec, α = 16°, ϕ_1 = 18°, α_2 = 21°, ϕ_2 = 35°, $\dot{m}$ = 4 kg/sec, K = 0.9

To find : i) θ_1, θ_2 ii) $F_{t1}\ F_{t2}, F_{a1}, F_{a2}$ iii) $(\eta_b)_{max}$

Procedure :

1) For first stage *:*

- Consider a suitable scale as 1cm = 50 m/sec and draw velocity triangles as shown in Fig 3.30 (a).
- Draw horizontal line A_1B_1 of length $l(A_1B_1) = \frac{120}{50}$ = 2.4 cm and at point A_1, draw an inclined A_1C_1 of length $l(A_1C_1) = \frac{600}{50}$ = 12 cm, at angle α_1 = 16°

  Join line B_1C_1 which represents relative velocity at inlet for first stage.
- At point B_1, draw an inclined line at an angle ϕ_1 = 18°.

  Take D_1 as point of intersection of inclined line and an are drawn with center as point B_1 and radius $B_1F_1 = 0.9\ l(B_1C_1)$ due to the friction losses in blade.

  Joint line A_1D_1 representing absolute velocity at outlet for first stage.
- Line G_1E_1 represents the change in whirl velocity for first stage

  $V_{w1} + V_{w2} = l(G_1E_1) \times \text{Scale} = 17.5 \times 50 = 875$ m/sec

  Line E_1C_1 represents flow velocity at inlet for first stage.

$$V_{f1} = l(E_1C_1) \times \text{Scale} = 3.3 \times 50 = 165 \text{ m/sec}$$

Line E_2C_2 represents flow velocity at inlet for first sgate.

$$V_{f2} = l(G_1D_1) \times \text{Scale} = 2.8 \times 50 = 140 \text{ m/sec}$$

$$\angle C_1 B_1 E_1 = \text{Blade angle at inlet for first stage, } \theta_1 = \mathbf{20^\circ} \qquad \textbf{... Ans.}$$

2) For second stage :

- Consider a suitable scale as 1cm = 50 m/sec and draw velocity triangles as shown in Fig 3.30 (b).
- Take projection of A_1B_1 as A_2B_2 for the second stage. Blade velocity remains same as both the stages are mounted on the same turbine shaft.

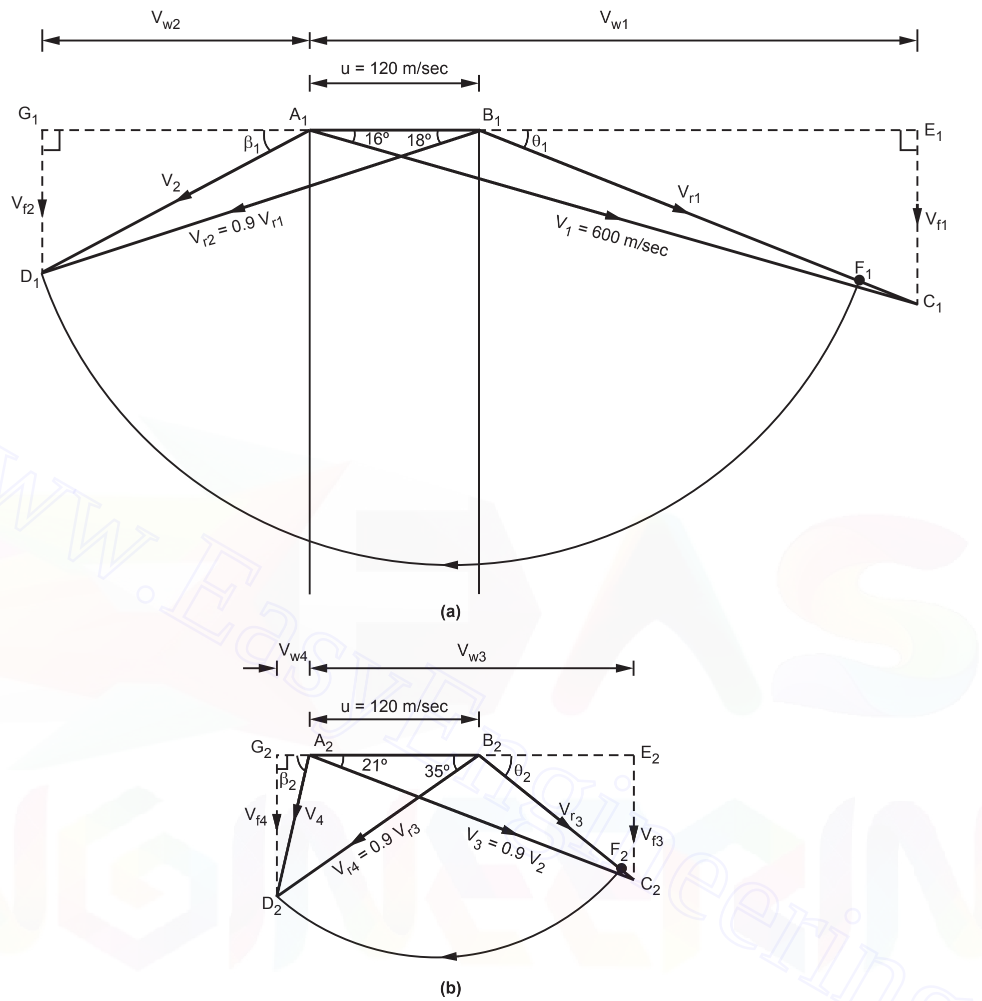


Fig. 3.30

- At point A_2 draw an inclined line A_2C_2 of length $l(A_2C_2) = 0.9\ l(A_1D_1)$, due to friction losses in blade = Absolute velocity at inlet for the second stage, at an angle $\alpha_2 = 21°$.

  Join line B_2C_2 which represents relative velocity at inlet for second stage.
- At point B_2 draw an inclined line at angle $\phi_2 = 35°$. Take D_2 as point of intersection of inclined line and an are drawn with center as point B_2 and radius $B_2F_2 = 0.9\ l(B_2C_2)$ due to friction losses in blade.
- Join line (A_2D_2) representing velocity absolute for second stage.

Line G_2E_2 represents the change in whirl velocity for second stage

$$V_{w3} + V_{w4} = l(G_2E_2) \times \text{Scale} = 5.8 \times 50 = 290 \text{ m/sec}$$

Line E_2C_2 represents flow velocity at inlet for first sgate.

$$V_{f3} = l(E_1C_1) \times \text{Scale} = 2.1 \times 50 = 105 \text{ m/sec}$$

Line G_2D_2 represents flow velocity at inlet for first sgate.

$$V_{f4} = l(G_2D_2) \times \text{Scale} = 1.9 \times 50 = 95 \text{ m/sec}$$

$\angle C_2B_2E_2$ = Blade angle at inlet for first stage, θ_2 = **34°** ... **Ans.**

Step 1 : Calculate the driving force for each stage

Driving force (Tangential force),

For first stage $F_{t1} = \dot{m}(V_{w1} + V_{w2})$

$\therefore$ $\mathbf{F_{t1}} = 4 \times 875 = \mathbf{3500\ N}$ **... Ans.**

For second stage $F_{t2} = \dot{m}(V_{w3} + V_{w4})$

$\therefore$ $\mathbf{F_{t2}} = 4 \times 290 = \mathbf{1160\ N}$ **... Ans.**

Step 2 : Calculate the axial thrust

Axial thrust (Axial force)

For first stage $F_{a1} = \dot{m}(V_{f1} - V_{f2})$

$\therefore$ $\mathbf{F_{a1}} = 4 \times (165 - 140) = \mathbf{100\ N}$ **... Ans.**

For second stage $F_{a2} = \dot{m}(V_{f3} - V_{f4})$

$\therefore$ $\mathbf{F_{a2}} = 4 \times (105 - 95) = \mathbf{40\ N}$ **... Ans.**

Step 3 : Calculate the maximum possible diagram efficiency

Maximum diagram efficiency

$$(\eta_b)_{max} = \cos^2 \alpha_1 = \cos^2 16$$

$$(\eta_b)_{max} = 0.9240 = \mathbf{92.40\ \%}$$ **... Ans.**

3.10 Reheat Factor

- We know that, the expansion of steam in a turbine is adiabatic. But if the friction between the blades and steam is neglected then the process becomes isentropic.
- Due to steam and blade friction leakages, shocks etc the enthalpy drop in the blades is reduced.
- For understanding the concept of friction in steam turbines, consider the expansion of steam in a four stage turbine. Refer Fig, 3.31.
- Let p_1 and T_{sup1} = Inlet pressure and superheated temperature of steam to first stage of turbine.

  p_b = Back pressure or exit pressure from the last stage

  $p_{2,}$ p_4 and p_4 = Intermediate stage pressures
- Consider point A, which represents the state of steam at inlet to first stage turbine at p_1 and T_{sup1}. Line A_1D represents the ideal Rankine enthalpy drop from inlet to exit pressure.
- Line A_1B_1 represents isentropic enthalpy drop in the first stage but due to friction the actual drop in enthalpy is A_1C_1.
- Now, draw horizontal line through point C_1 upto the pressure line of point B_1. This point is marked as A_2.
- As $h_{A2} = h_{C1}$, point A_2 also represents the actual condition of steam at exit of first stage or inlet to the second of turbine.

- Similar to A_1B_1 and A_1C_1 the isentropic and actual enthalpy drops in successive stages are represented by A_2B_2, A_2C_2, A_3B_3 and A_3C_3 and so on.
- The condition of steam at the exit is represented by points A_2, A_3, A_4 and A_5. After joining these points a curve is obtained which is called as condition curve. It represents the approximate process path of steam.
- The sum of the isentropic drops in all the turbine stages i.e. $(A_1B_1 + A_2B_2 + ... = \Sigma AB)$ is called as **commulative enthalpy drop.**
- The sum of actual heat drops in all the stages i.e. $(A_1C_1 + A_2C_2 + ... \Sigma AC)$ is called as total **useful enthalpy drop.**

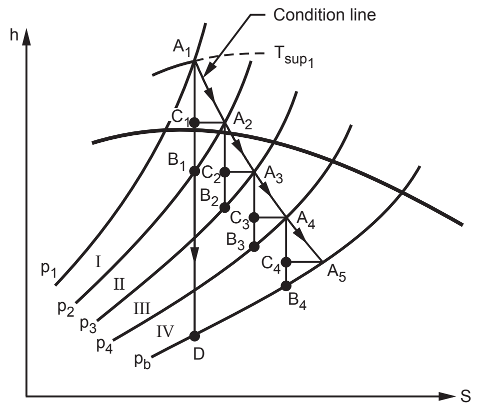


Fig. 3.31 : Reheat factor

- The commulative enthaply drop is always greater than the Rankine enthalpy drop (line A_1D) because the constant pressure lines diverge from left to right on h-S diagram.
- **The Reheat Factor (R.F.)** is defined as the ratio of cummulative enthalpy drop to the Rankine enthalpy drop.
- It is given as,

$$R_f = \frac{\text{Cummulative enthalpy drop}}{\text{Rankine enthalpy drop}} = \frac{\Sigma(AB)}{A_1D} \quad \ldots (i)$$

- **Internal efficiency of turbine (η_i)** is defined as the ratio of total useful enthalpy drop (total actual turbine work) to the Rankine enthalpy drop (Rankine work).

$$\therefore \quad \eta_i = \frac{\text{Total useful enthalpy drop}}{\text{Rankine enthalpy drop}} = \frac{\Sigma(AC)}{A_1D} \quad \ldots (ii)$$

- **Stage efficiency (η_s)** for the given stage is defined as the ratio of actual enthalpy drop to isentropic enthalpy drop.

$$\text{For } 1^{st} \text{ stage, } \eta_{s1} = \frac{A_1C_1}{A_1B_1}$$

$$\text{For } 2^{nd} \text{ stage, } \eta_{s2} = \frac{A_2C_2}{A_2B_2} \text{ and so on}$$

$$\text{For all stage, } \eta_s = \frac{\Sigma(AC)}{\Sigma(AB)} \quad \ldots (iii)$$

- Multiplying and dividing equation (ii) by $\Sigma(AB)$

$$\eta_i = \frac{\Sigma(AC)}{A_1D} \times \frac{\Sigma(AB)}{\Sigma(AB)} = \frac{\Sigma(AB)}{A_1D} \times \frac{\Sigma(AC)}{\Sigma(AB)}$$

$$\therefore \quad \eta_i = R_f \times \eta_s$$

- Reheat factor depends on the friction due to steam flow and not related with the reheating of steam between the stages by using external source.
- The value of R_f varies between 1.02 to 1.06.
- **Overall efficiency** (η_o) of the plant is defined as the ratio of useful work to the heat supplied.

$$\therefore \quad \eta_o = \frac{\text{Useful work}}{\text{Heat supplied}} = \frac{h_{A1} - h_{A5}}{h_{A1} - h_{fD}}$$

- Also, the point on h-S diagram which represents the condition of steam at that point is called as **state point.**

3.11 Reaction Turbine

- The reaction turbines in use are actually impulse reaction turbines. Pure reaction turbines are practically not used.
- The expansion of steam and enthalpy drop occurs in fixed as well as moving blades. Due to this, the blade passages between the consecutive blades are converging type (act as convergent nozzle).
- Steam enters the ring of fixed blades at pressure p_1 and expands upto pressure p_2 before entering the moving blades. Hence, the velocity of steam increases.
- Also, this expansion produces reactive force and the change in direction of velocity vector while passing over the moving blades is accompanied by change in momentum. Hence, impulsive force is also produced.

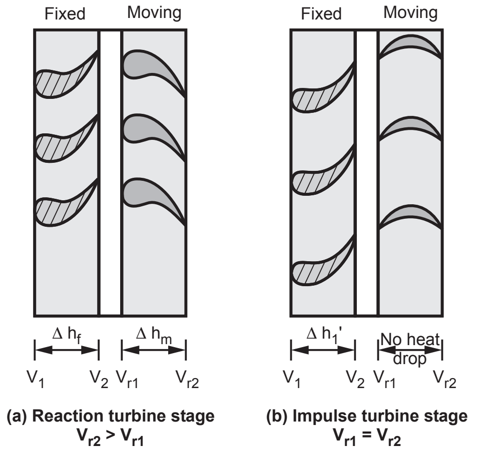


Fig. 3.32

- Due to expansion of steam over the moving blades, the relative velocity at exit of moving blade (V_{r2}) is greater than the relative velocity of steam at inlet to the moving blades (V_{r1}). Refer Fig. 3.32.
- The expansion of steam on h-S diagram is shown in Fig. 3.33 (a).

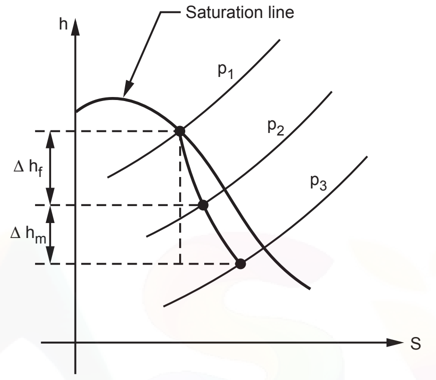


Fig. 3.33 (a) : Expansion of steam

- Fig. 3.33 (b) show the velocity diagram of reaction turbine.

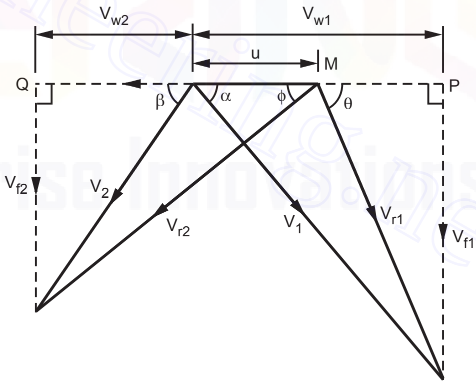


Fig. 3.33 (b) : Velocity diagram of reaction turbine

3.12 Degree of Reaction

- Degree of reaction is a term used in steam turbine which gives relation between the blade angles at inlet and outlet.
- The degree of reaction turbine stage is defined as the ratio of enthalpy drop over moving blades to the enthalpy drop in the stage.

Mathematically,

$$R_D = \frac{\text{Enthalpy drop over moving blade}}{\text{Total enthalpy drop in the stage}}$$

$$\therefore \quad R_D = \frac{\Delta h_m}{\Delta h_f + \Delta h_m} \qquad \text{... (From Fig. 3.33 (a))}$$

- The enthalpy drop in the moving blade is equal to the increase in relative velocity of steam passing through the blades. Mathematically,

$$\therefore \quad \Delta h_m = \frac{V_{r2}^2 - V_{r1}^2}{2} \qquad \text{... (3.19)}$$

- Now the total enthalpy drop in the stage is equal to the work done by the steam in the stage. Mathematically,

$$\Delta h_f + \Delta h_m = u(V_{w1} + V_{w2}) \qquad \text{... (3.20)}$$

From equation (3.19) and (3.20) , the degree of reaction is,

$$R_D = \frac{V_{r2}^2 - V_{r1}^2}{2u(V_{w1} + V_{w2})} \qquad \text{... (3.21)}$$

From Fig. 3.33 (b),

$$\sin\theta = \frac{V_{f1}}{V_{r1}} \qquad \therefore V_{r1} = \frac{V_{f1}}{\sin\theta} = V_{f1}\,\text{cosec}\,\theta$$

and

$$\sin\phi = \frac{V_{f2}}{V_{r2}} \qquad \therefore V_{r2} = \frac{V_{f2}}{\sin\phi} = V_{f2}\,\text{cosec}\,\phi$$

$$V_{w1} + V_{w2} = MP + MQ$$

But,

$$MP = \frac{V_{f1}}{\tan\theta} = V_{f1}\cot\theta$$

$$MQ = \frac{V_{f2}}{\tan\phi} = V_{f2}\cot\phi$$

$$\therefore \quad V_{w1} + V_{w2} = V_{f2}\cot\theta + V_{f_2}\cot\phi = V_f(\cot\theta + \cot\phi)$$

- Assuming the velocity of flow is constant throughout the blades i.e. $V_{f1} = V_{f2} = V_f$ and substituting the value of V_{r1}, V_{r2} and $(V_{w1} + V_{w2})$ in equation (3.21).

$$R_D = \frac{(V_f \cdot \text{cosec}\,\phi)^2 - (V_f \cdot \text{cosec}\,\theta)^2}{2uV_f\,(\cot\theta + \cot\phi)} = \frac{V_f^2\left[(\text{cosec}^2\,\phi) - (\text{cosec}^2\,\theta)\right]}{2uV_f\,(\cot\theta + \cot\phi)}$$

$$\therefore \quad R_D = \frac{V_f}{2u}\left[\frac{(\cot^2\phi + 1) - (\cot^2\theta + 1)}{\cot\theta + \cot\phi}\right] = \frac{V_f}{2u}\left[\frac{\cot^2\phi - \cot^2\theta}{\cot^2\phi + \cot^2\theta}\right]$$

$$\therefore \quad R_D = \frac{V_f\,(\cot\phi - \cot\theta)\,(\cot\phi + \cot\theta)}{2u\,(\cot\phi + \cot\theta)} = \frac{V_f}{2u}\left[\cot\phi - \cot\theta\right] \qquad \text{... (3.22)}$$

For a 50 % degree of reaction $(\Delta h_f = \Delta h_m)$

$$\therefore \quad R_D = \frac{1}{2} = \frac{V_f}{2u}(\cot\phi - \cot\theta)$$

$$\therefore \quad u = V_f \; (\cot\phi - \cot\theta) \qquad \text{... (3.23)}$$

From Fig. 3.33 (b) the value of u can also be written as,

$$\therefore \quad u = V_f (\cot\alpha - \cot\beta) \qquad \text{... (3.24)}$$

$$\text{and,} \quad u = V_f \; (\cot\alpha - \cot\theta) \qquad \text{... (3.25)}$$

By comparing equations (3.23), (3.24), (3.25) we get,

$$\theta = \beta \;\text{ and }\; \phi = \alpha$$

- This means that, if the degree of reaction is 50 % then the moving blades and fixed blades have the same shape.
- This give the symmetrical velocity diagram for inlet and outlet. Such turbine is known as **Parson's reaction turbine.**

3.12.1 Work Power and Efficiency of Reaction Turbine

- For reaction turbine the work done per kg of steam is,

$$WD = (V_{w1} + V_{w2})\, u, \; \frac{N\text{-}m}{kg}$$

- Power developed per stage is,

$$P = \dot{m}\,(V_{w1} + V_{w2})\; u, \; \text{Watts}$$

- Enthalpy drop in a stage is,

$$\Delta h = \Delta h_f + \Delta h_m = \left(\frac{V_1^2}{2}\right) + \frac{(V_{r2}^2 - V_{r1}^2)}{2}, \; \text{J/kg}$$

- Stage efficiency is given by,

$$\eta_{stage} = \frac{\text{Work done on the blade}}{\text{Enthalpy in a stage}} = \frac{u\,(V_{w1} + V_{w2})}{\Delta h}$$

- Volume flow rate of steam is,

$$Q = \text{Area of flow} \times \text{Average velocity of flow}$$

$$Q = \pi D_m H \times V_{f\,avg}, \; m^3/\text{sec}$$

$$\text{But} \quad V_{f\,avg} = \text{Average flow velocity} = \frac{V_{f1} + V_{f2}}{2}$$

$$H = \text{Height of blade}$$

- Mass flow rate of steam

$$\dot{m} = \rho Q = \frac{Q}{v_s} = \frac{\pi D_m H \times V_{favg}}{v_s}$$

where, v_s = Specific volume of steam = x v_g

x = Dryness fraction of steam

v_g = Volume of steam at saturated point in $\frac{m^3}{kg}$

- The value of v_g is found from steam table at given pressure.

3.12.2 Condition for Maximum Efficiency of Reaction Turbine

- The maximum efficiency for the reaction turbine can be determined by using following assumptions :
  - Degree of reaction is 50 % i.e. $R_D = 1/2$
  - The moving and fixed blades are symmetrical i.e. $\theta = \beta$ and $\phi = \alpha$
- The velocity of steam at exit from the previous stage is same as the velocity of steam at the entrance for next stage i.e. $V_1 = V_{r2}$
- From the velocity triangle for reaction turbine as shown in Fig. 3.34.

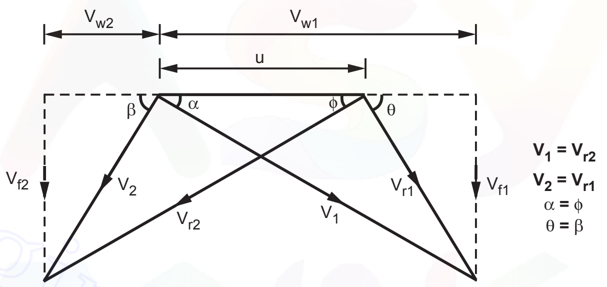


Fig. 3.34

$$\cos\alpha = \frac{V_{w1}}{V_1} \qquad \therefore V_{w1} = V_1 \cos\alpha$$

and $$\cos\phi = \frac{V_{w2} + u_2}{V_{r2}} \qquad \therefore V_{w2} = V_{r2}\cos\phi - u$$

- Now, work done per kg of steam is given by,

$$W = u\,(V_{w1} + V_{w2}) = u\,(V_1\cos\alpha + V_{r2}\cos\phi - u)$$

$$\therefore \quad W = u\,(2V_1\cos\alpha - u) \qquad \ldots (\because \phi = \alpha \text{ and } V_{r2} = V_1)$$

Now multiplying and dividing by V_1^2 we get,

$$\therefore \quad W = u\,(2V_1\cos\alpha - u)\times\frac{V_1^2}{V_1^2}$$

$$\therefore \quad W = V_1^2\left[\frac{2uV_1\cos\alpha}{V_1^2} - \frac{u^2}{V_1^2}\right]$$

$$W = V_1^2\,[2s\cos\alpha - s^2] \qquad \ldots\left(\because s = \frac{u}{V_1}\right)\ldots (3.26)$$

Total energy supplied to the stage is given by,

$$\text{Total Energy supplied to the stage} = \begin{bmatrix}\text{K.E. supplied to} \\ \text{fixed blade}\end{bmatrix} + \begin{bmatrix}\text{K.E supplied to} \\ \text{moving blade}\end{bmatrix}$$

$$\Delta h = \frac{V_1^2}{2} + \frac{V_{r2}^2 - V_{r1}^2}{2} = \frac{V_1^2}{2} + \frac{V_1^2 - V_{r1}^2}{2} \quad \ldots (\therefore V_{r2} = V_1)$$

$$\Delta h = V_1^2 - \frac{V_{r1}^2}{2} \quad \ldots (3.27)$$

But, from triangle LMS, by using cosine rule,

$$V_{r1}^2 = V_1^2 + u^2 - 2 V_1 u \cos\alpha$$

Substituting the above value in equation (3.27)

$$\therefore \quad \Delta h = V_1^2 - \frac{1}{2}(V_1^2 + u^2 - 2 V_1 u \cos\alpha)$$

$$\therefore \quad \Delta h = \frac{2 V_1^2 - V_1^2 - u^2 + 2 V_1 u \cos\alpha}{2} = \frac{V_1^2 + 2 V_1 u \cos\alpha - u^2}{2}$$

$$\therefore \quad \Delta h = \frac{V_1^2}{2}\left[1 + \frac{2 u \cos\alpha}{V_1} - \left(\frac{u}{V_1}\right)^2\right]$$

$$\therefore \quad \Delta h = \frac{V_1^2}{2}[1 + 2s \cos\alpha - s^2] \quad \ldots \left(\because s = \frac{u}{V_1}\right)$$

- Now, the blade efficiency of the reaction turbine is,

$$\eta_b = \frac{\text{Work done per kg of steam}}{\text{Total energy supplied to the stage}} = \frac{WD}{\Delta h}$$

$$\therefore \quad \eta_b = \frac{V_1^2 (2s\cos\alpha - s^2)}{\frac{V_1^2}{2}(1 + 2s\cos\alpha - s^2)} = \frac{2\,(2s\cos\alpha - s^2)}{(1 + 2s\cos\alpha - s^2)}$$

$$\therefore \quad \eta_b = \frac{2s\,(2\cos\alpha - s)}{(1 + 2s\cos\alpha - s^2)}$$

Adding and subtracting 2 in the numerator

$$\therefore \quad \eta_b = \frac{2(1 + 2s\cos\alpha - s^2) - 2}{1 + 2s\cos\alpha - s^2} = 2 - \frac{2}{(1 + 2s\cos\alpha - s^2)}$$

- For maximum blade efficiency, the value of $(1 + 2s\cos\alpha - s^2)$ should be maximum. The condition for maximum efficiency obtained by,

$$\frac{d}{ds}(1+2s\cos\alpha - s^2) = 0$$

$$\therefore \quad 2\cos\alpha - 2s = 0$$

$$\therefore \quad s = \cos\alpha \qquad \text{.... (3.28)}$$

Substituting the above value in equation (i) to obtain the maximum blade efficiency

$$\therefore \quad \eta_{bmax} = 2 - \frac{2}{1+2\cos^2\alpha - \cos^2\alpha}$$

$$= 2\left[1 - \frac{1}{1+\cos^2\alpha}\right]$$

$$\therefore \quad \eta_{bmax} = 2\left[\frac{1+\cos^2\alpha - 1}{1+\cos^2\alpha}\right]$$

$$= \left[\frac{2\cos^2\alpha}{1+\cos^2\alpha}\right] \qquad \text{... (3.29)}$$

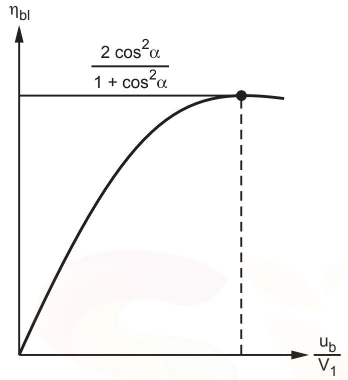


Fig. 3.35

- The variation of blade efficiency with respect to speed ratio $\left(s = \frac{u}{V_1}\right)$ is shown in Fig. 3.35.

3.13 Solved Examples

Ex. 3.19 : *The following data refers to a particular stage of Parson's reaction turbine : Speed of the turbine = 1800 rpm, Mean diameter of the rotor = 1.1 meter, Stage efficiency = 88 %, Blade outlet angle = 16º, Speed ratio = 0.7. Determine the available isentropic enthalpy drop in the stage.*

Sol. : Given data :

R_D = 50 % = 0.5, N = 1800 rpm, D_m = 1.1 m, η_{stage} = 88 % = 0.88, $\alpha = \phi$ = 16º, s = 0.7

To find : Δh

Step 1 : Calculate the isentropic enthalpy drop

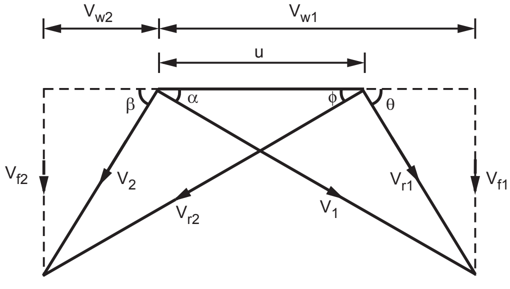


Fig. 3.36

Blade velocity, $u = \frac{\pi D_m N}{60}$

$$= \frac{\pi \times 1.1 \times 1800}{60} = 103.6725 \text{ m/sec}$$

Speed ratio, $s = \frac{u}{V_1} \qquad \therefore \quad 0.7 = \frac{103.6725}{V_1}$

$$\therefore \quad V_1 = 148.1036 \text{ m/sec}$$

For Parson reaction turbine, $R_D = 0.5$

Relative velocity at outlet, $V_{r2} = V_1 = 148.1036$ m/sec

From Fig. 3.36, by inlet velocity triangle,

Whirl velocity at inlet,

$$V_{w1} = V_1 \cdot \cos\alpha = 148.1036 \times \cos(16) = 142.3663 \text{ m/sec}$$

From Fig. 3.36, by outlet velocity triangle,

Whirl velocity at outlet

$$V_{w2} = V_{r2} \cos\phi - u$$

$$= 148.1036 \times \cos(16) - 103.6725$$

$$= 38.6938 \text{ m/sec}$$

Stage efficiency, $$\eta_{stage} = \frac{u(V_{w1} + V_{w2})}{\Delta h}$$

$$\therefore \quad 0.88 = \frac{103.6725 \times (142.3663 + 38.6938)}{\Delta h}$$

$$\therefore \quad \mathbf{\Delta h = 21.3306 \times 10^3 \text{ J/kg}}$$ **... Ans.**

Ex. 3.20 : *In a Parson turbine running at 1800 r.p.m., the available enthalpy drop for an expansion is 70 kJ/kg. If the mean diameter of the rotor is 120 cm, find the number of rows of moving blades required. Assume stage efficiency as 86 %, blade outlet angle is 20 ° and speed ratio is 0.7.*

Sol. : Given data :

$R_D = 50\ \% = 0.5$, $N = 1800$ rpm, $(\Delta h)_{total} = 70$ kJ/kg $= 70 \times 10^3$ J/kg,

$D_m = 120$ cm $= 1.2$ m, $\eta_{stage} = 86\ \% = 0.86$, $\phi = 20°$, $s = 0.7$

To find : Number of rows of moving blades

Step 1 : Calculate the number of rows of moving blades

Blade velocity, $$u = \frac{\pi D_m N}{60}$$

$$= \frac{\pi \times 1.2 \times 1800}{60} = 113.0973 \text{ m/sec}$$

Speed ratio, $$s = \frac{u}{V_1} \qquad \therefore 0.7 = \frac{113.0973}{V_1}$$

$$\therefore \quad V_1 = 161.5676 \text{ m/sec}$$

For Parson steam turbine, $R_D = 0.5$

Relative velocity at outlet, $V_{r2} = V_1 = 161.5676$ m/sec

Exit angle of nozzle, $\alpha = \phi = 20°$

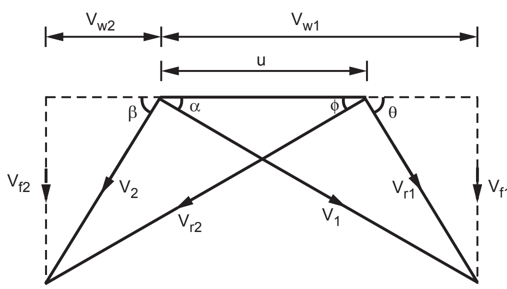


Fig. 3.37

From Fig. 3.37, by inlet velocity triangle,

Whirl velocity at inlet,

$$V_{w1} = V_1 \cos\alpha$$

$$= 161.5676 \times \cos(20) = 151.8238 \text{ m/sec}$$

From Fig. 3.37, by outlet velocity triangle,

Whirl velocity at outlet,

$$V_{w2} = V_{r2} \cos\phi - u$$

$$= 161.5676 \times \cos(20) - 113.0973$$

$$= 38.7265 \text{ m/sec}$$

Isentropic efficiency, $$\eta_{isen} = \frac{u(V_{w1}+V_{w2})}{(\Delta h)_{each}}$$

$$0.86 = \frac{113.0973 \times (161.5676 + 38.7265)}{(\Delta h)_{each}}$$

$$\therefore \quad (\Delta h)_{each} = 26.3403 \times 10^3 \text{ J/kg}$$

Number of rows of moving blades,

$$= \frac{(\Delta h)_{total}}{(\Delta h)_{each}} = \frac{70 \times 10^3}{26.3403 \times 10^3}$$

$$= \mathbf{2.6575 \approx 3}$$ **... Ans.**

Ex. 3.21 : *In a reaction turbine, the blade tips are inclined at 35° and 20° in the direction of motion. The guide blades are of the same shape as the moving blades, but reversed in direction. At a certain place in the turbine, the drum diameter is 1 meter and the blades are 10 cm high. At this place, the steam has a pressure of 1.75 bar and dryness 0.935. If the speed of this turbine is 250 r.p.m. and the steam passes through the blades without shock, find the mass of the steam flow and power developed in the ring of moving blades.*

Sol. : Given data :

$\theta = 35°$, $\phi = 20°$, D = 1 m, H = 10 cm = 0.1 m,

p = 1.75 bar, x = 0.935, N = 250 rpm,

To find : i) $\dot{m}$ ii) P

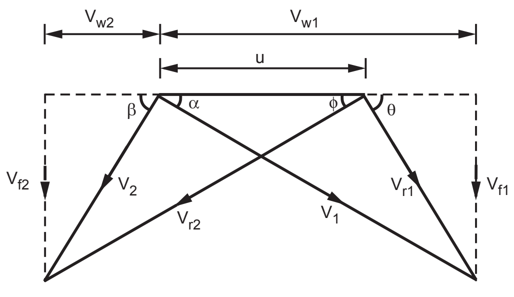


Fig. 3.38 (a)

Step 1 : Calculate mass of steam flow

As guide blades and moving blades are of same shape but reserved in direction, $\theta = \beta = 35°$ and $\phi = \alpha = 20°$

Mean diameter $$D_m = \frac{D+(D+2H)}{2} = \frac{1+(1+2\times 0.1)}{2} = 1.1 \text{ m}$$

Blade velocity, $u = \frac{\pi D_m N}{60} = \frac{\pi \times 1.1 \times 250}{60} = 14.3989$ m/sec

From Fig. 3.38 (a), by inlet velocity triangle,

Flow velocity at inlet $V_{f1} = V_1 \sin\alpha = V_{r1} \sin\theta$

$\therefore \quad V_1 \times \sin(20) = V_{r1} \times \sin(35)$

$\therefore \quad V_1 = 1.677\ V_{r1}$ ... (i)

Then, $V_{w1} - u = V_{r1} \cos\theta$

$\therefore \quad V_1 \cos\ \alpha - u = V_{r1} \cos\theta$

$\therefore \quad V_1 \times \cos(20) - 14.3989 = V_{r1} \times \cos(35)$

$\therefore \quad 0.9396\ V_1 - 0.8191\ V_{r1} = 14.3989$ ... (ii)

Solving Equations (i) and (ii)

$V_{r1} = 19.0308$ m/sec and $V_1 = 31.9146$ m/sec

$\therefore \quad V_{f1} = V_1 \sin\alpha$

$= 31.9146 \times \sin(20) = 10.9154$ m/sec

Whirl velocity at inlet, $V_{w1} = V_1 \cos\alpha$

$= 31.9146 \times \cos(20) = 29.9899$ m/sec

Flow velocity at outlet, $V_{f2} = V_{f1} = 10.9154$ m/sec

Average flow velocity, $(V_f)_{avg} = \frac{V_{f1} + V_{f2}}{2} = \frac{2 V_{f1}}{2} = V_{f1} = 10.9145$ m/sec

From Fig. 3.38 (a), by outlet velocity triangle,

Whirl velocity at outlet,

$$V_{w2} = \frac{V_{f2}}{\tan\beta} = \frac{10.9154}{\tan(35)} = 15.5888 \text{ m/sec}$$

Specific volume of steam considering dryness fraction.

$$v_s = x\, v_g$$

From steam tables,

At $p_x = 1.7$ bar, $v_{gx} = 1.0309\ m^3/kg$

At $p_y = 1.8$ bar, $v_{gy} = 0.97718\ m^3/kg$

$\therefore$ At p = 1.75 bar, specific volume of steam (v_g) is,

$$\frac{p_x - p}{p - p_y} = \frac{v_{gx} - v_g}{v - v_{gy}}$$

$$\frac{1.70 - 1.75}{1.75 - 1.8} = \frac{1.0309 - v_g}{v_g - 0.97718}$$

$$\therefore \quad v_g = 1.004 \ \ m^3/kg$$

$$\therefore \quad v_s = 0.935 \times 1.004 = 0.9387 \ m^3/kg$$

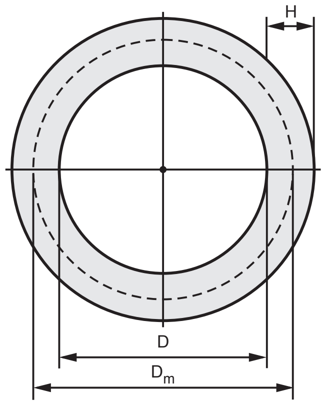


Fig. 3.38 (b)

Discharge of steam $Q = \dot{m} v_s = \dot{m} \times 0.9387$

Also, $\quad Q = \pi D_m H (V_f)_{avg}$

$$\therefore \quad \dot{m} \times 0.9387 = \pi \times 1.1 \times 0.1 \times 10.9154$$

$$\therefore \quad \dot{m} = \mathbf{4.0184 \ kg/sec}$$ **... Ans.**

Step 2 : Calculate the power speed developed

Power developed, $\quad P = \dot{m} u (V_{w1} + V_{w2})$

$$= 4.0184 \times 14.3989 \times (29.9899 + 15.5888)$$

$$\mathbf{P = 2637.2081 \ W}$$ **... Ans.**

Ex. 3.22 : *Show that for Parson's reaction turbine, the degree of reaction is 50 %.*

Sol. : • For Parson's reaction turbine,

$$\alpha = \phi \ \text{ and } \ \theta = \beta$$

Also, $\quad V_1 = V_{r2}$ and $V_2 = V_{r1}$

- We know that, the degree of reaction is given by,

$$R_D = \frac{\text{Enthalpy drop in moving blades}}{\text{Enthalpy drop in a stage}} = \frac{\Delta h_m}{\Delta h_f + \Delta h_m}$$

But, $\quad \Delta h_m = \dfrac{V_{r2}^2 - V_{r1}^2}{2}$

In this case, applying steady flow energy equation to the fixed blades and assuming that the velocity of steam entering the fixed blade is equal to the absolute velocity of steam leaving the previous moving row.

$$\therefore \quad \Delta h_f = \frac{V_1^2 - V_2^2}{2} \qquad \text{...(Here, } V_2 \text{ is also considered)}$$

$$\therefore \quad \Delta h_f = \frac{V_{r2}^2 - V_{r1}^2}{2} \qquad \text{...}(\because V_{r2} = V_1 \text{ and } V_{r1} = V_2)$$

$$\therefore \quad R_D = \frac{\Delta h_m}{\Delta h_m + \Delta h_m} = \frac{\Delta h_m}{2 \Delta h_m} = \frac{1}{2} = 50 \ \% \qquad \text{...Hence proved.}$$

Ex. 3.23 : *A 50 % impulse-reaction turbine runs at 3000 rpm. The angles at exit of fixed bladings and inlet of moving bladings are 20° and 30° respectively. The mean ring diameter is 0.7 m and steam condition is 1.5 bar and 0.96 dry. Calculate*

i) Required height of blades to pass 50 kg/s of steam and

ii) Power developed by the stage. Solve the problem analytically.

Sol. : Given data :

$R_D = 50\ \% = 0.5,\ \ N = 3000$ rpm, $\alpha = \phi = 20°,\ \ \theta = \beta = 30°,\ \ D_m = 0.7$ m,

$p = 1.5$ bar, $x = 0.96,\ \ \dot{m} = 50$ kg/sec

To find : i) H ii) P

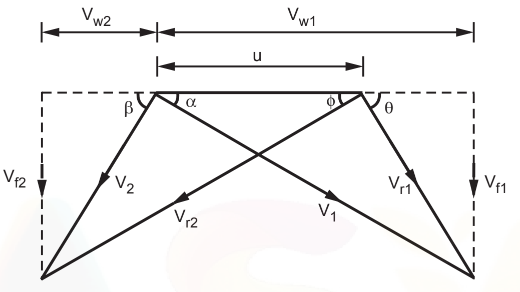


Fig. 3.39

Step 1 : Calculate the height of blades

Blade velocity, $$u = \frac{\pi D_m N}{60}$$

$$= \frac{\pi \times 0.7 \times 3000}{60} = 109.9557 \text{ m/sec}$$

From Fig. 3.39, by inlet velocity triangle,

Flow velocity at inlet, $V_{f1} = V_1 \sin\alpha = V_{r1} \sin\theta$

$\therefore \quad V_1 \sin 20 = V_{r1} \sin (30)$

$\therefore \quad V_1 = 1.4619\ V_{r1}$ … (3.30)

Also, $V_{w1} - u = V_{r1} \cos\theta$

$\therefore \quad V_1 \cos\alpha - u = V_{r1} \cos\theta$

$\therefore \quad V_1 \cos (20) - 109.9557 = V_{r1} \cos (30)$

$\therefore \quad 0.9396\ V_1 - 0.866\ V_{r1} = 109.9557$ … (3.31)

Solving equations (3.30), (3.31)

$\therefore \quad V_1 = 316.6742$ m/sec $\quad V_{r1} = 216.6182$ m/sec

Whirl velocity at inlet, $V_{w1} = V_1 \cos\alpha$

$= 316.6744 \times \cos (20) = 297.5764$ m/sec

Flow velocity at inlet, $V_{f1} = V_1 \sin\alpha$

$= 316.6742 \times \sin (20) = 108.3089$ m/sec

From Fig. 3.39, by outlet velocity triangle,

Flow velocity at outlet, $V_{f2} = V_{f1} = 108.3089$ m/sec

Whirl velocity at outlet, $V_{w2} = \frac{V_{f2}}{\tan\beta} = \frac{108.3089}{\tan(30)} = 187.5965$ m/sec

Specific volume of steam considering dryness fraction,

$$v_s = x\ v_g$$

From steam tables, at $p = 1.5$ bar,

Specific volume of steam, $v_g = 1.159\ m^3/kg$

$\therefore$ $v_s = 0.96 \times 1.159 = 1.1126\ m^3/kg$

Discharge of steam, $Q = \dot{m}\ v_s = 50 \times 1.1126 = 55.63\ m^3/sec$

Average flow velocity, $(v_f)_{avg} = \frac{V_{f1} + V_{f2}}{2}$

$$= \frac{108.3089 + 108.3089}{2} = 108.3089 \text{ m/sec}$$

Also, $Q = \pi D_m H (V_f)_{avg}$

$\therefore$ $55.63 = \pi \times 0.7 \times H \times 108.3089$

$\therefore$ $\mathbf{H = 0.2335\ m}$ **...Ans.**

Step 2 : Calculate the power developed

Power developed, $P = \dot{m}\ u\ (V_{w1} + V_{w2})$

$$= 50 \times 109.9557 \times (297.5764 + 187.5965)$$

$\therefore$ $\mathbf{P = 2667.3762 \times 10^3\ W}$ **... Ans.**

Ex. 3.24 : *5 kg of steam (2 bar, 0.95 dry) flows through a given stage of a reaction turbine per second. The exit angle of fixed blades as well as moving blades is 15° and 3.5 kW of power is developed. If the rotor speed is 360 r.p.m. and tip leakage is 5 percent, calculate the mean drum diameter and the blade height. The axial flow velocity is 0.85 times the blade velocity.*

Sol, : Given data :

$\dot{m} = 5$ kg/sec, $p = 2$ bar, $x = 0.95$, $\alpha = \phi = 15°$,

$P = 3.5 \times 10^3$ W, $N = 360$ rpm, $t_l = 5\ \% = 0.05$, $V_f = 0.85\ u$

To find : i) D_m ii) H

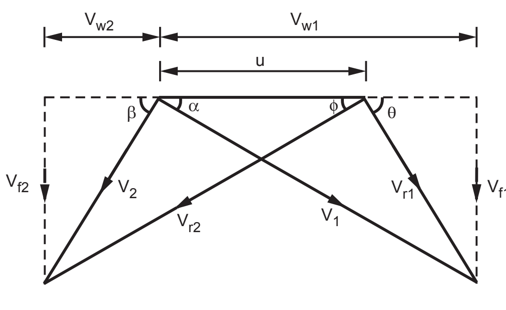


Fig. 3.40

Step 1 : Calculate the mean drum diameter and blade height

Blade velocity, $u = \frac{\pi D_m N}{60}$

$$= \frac{\pi \times D_m \times 360}{60} = 18.8495\ D_m$$

Actual mass flow rate, $\dot{m}_{act} = (1 - t_l)\, \dot{m} = (1 - 0.05) \times 5 = 4.75$ kg/sec

Power developed, $P = \dot{m}_{act}\, u\, (V_{w1} + V_{w2})$

$\therefore \quad 3.5 \times 10^3 = 4.75 \times 18.8495\ D_m \times (V_{w1} + V_{w2})$

$$\therefore \quad V_{w1} + V_{w2} = \frac{39.0908}{D_m} \quad \ldots \text{(i)}$$

As exit angles of fixed and moving blades are same, the reaction steam turbine is considered as Parson's steam turbine.

$\therefore$ Absolute velocity at inlet, $V_1 = V_{r2}$

Flow velocity inlet, $V_{f1} = V_{r2} = V_f$

Refer Fig. 3.40, by inlet velocity triangle,

Flow velocity at inlet $V_{f1} = V_1 \sin \alpha$

$\therefore \quad 0.85\ u = V_1 \sin \alpha \quad \ldots [\because V_{f1} = 0.8\ u]$

$\therefore \quad 0.85 \times 18.8495\ D_m = V_1 \times \sin(15)$

$\therefore \quad V_1 = 61.9045\ D_m$

Refer Fig. 3.40 also, $V_{w1} + V_{w2} = V_1 \cos\alpha + V_{r2} \cos\phi - u$

$= 2\, V_1 \cos\alpha - u \quad \ldots [\because \phi = \alpha,\ V_{r2} = V_1]$

$= 2 \times 61.9045\ D_m \times \cos(15) - 18.8475\ D_m$

$\therefore \quad V_{w1} + V_{w2} = 100.7428\ D_m \quad \ldots \text{(ii)}$

Equating equations (i) and (ii),

$$\frac{39.0908}{D_m} = 100.7428\ D_m$$

$\therefore \quad \mathbf{D_m = 0.6229\ m} \quad$ **... Ans.**

Specific volume of steam considering dryness fraction,

$$v_s = x\ v_g$$

From steam tables, at $p = 2$ bar,

Specific volume of steam, $v_g = 0.8854\ m^3/kg$

$\therefore \quad v_s = 0.95 \times 0.8854 = 0.8411\ m^3/kg$

Blade velocity, $u = 18.8495\ D_m$

$= 18.8495 \times 0.6229 = 11.7413$ m/sec

Flow velocity, $V_{f1} = V_{f2} = 0.85\ u$

$= 0.85 \times 11.7413 = 9.9801$ m/sec

Average flow velocity, $(V_f)_{avg} = \frac{V_{f1} + V_{f2}}{2} = \frac{2V_{f1}}{2} = V_{f1}$

$\therefore \quad (V_f)_{avg} = 9.9801$ m/sec

Discharge of steam, $Q = \dot{m}_{act}\, v_s = 4.75 \times 0.8411 = 3.9952\ m^3/sec$

Also, $Q = \pi\, D_m\, H\, (V_f)_{avg}$

$3.9952 = \pi \times 0.6229 \times H \times 9.9801$

$\therefore \quad$ **H = 0.2045 m** **... Ans.**

Ex. 3.25 : *In a certain stage of a reaction turbine, the steam leaves the fixed blade at a pressure of 3 bar, 0.98 dry and a velocity of 130 m/s. The blades are 20 mm high and the discharge angle for both the blades is 20°. The ratio of the axial velocity of flow to the blade velocity is 0.7 at inlet and 0.76 at exit from moving blades. If the turbine uses 4 kg/s of steam with 5 % tip leakage, determine the mean blade diameter and power developed in the ring.*

Sol. : Given data :

$p = 3$ bar, $x = 0.98$, $V_1 = 130$ m/sec., $H = 20$ mm $= 0.02$ m,

$\alpha = \phi = 20^o$, $\frac{V_{f1}}{u} = 0.7$, $\frac{V_{f2}}{u} = 0.76$, $\dot{m} = 4$ kg/sec, $t_l = 5\ \% = 0.05$

To find : i) D_m ii) P

Step 1 : Calculate the mean blade diameter

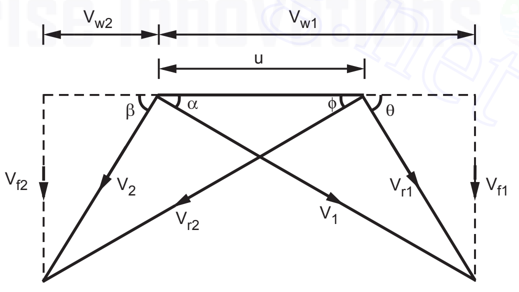


Fig. 3.41

Actual mass flow rate, $\dot{m}_{act} = (1 - t_l)\ \dot{m}$

$= (1 - 0.05) \times 4 = 3.8$ kg/sec

From Fig. 3.41, by inlet velocity triangle,

Flow velocity at inlet, $V_{f_1} = V_1 \sin\alpha$

$= 130 \times \sin(20) = 44.4626$ m/sec

Also, $\frac{V_{f1}}{u} = 0.7 \quad \therefore \frac{44.4626}{u} = 0.7$

$\therefore \quad u = 63.518$ m/sec

Whirl velocity at inlet, $V_{w1} = V_1 \cos\alpha$

$= 130 \times \cos(20) = 122.16$ m/sec

From Fig. 3.41, by outlet velocity triangle,

Flow velocity at outlet, $\frac{V_{f2}}{u} = 0.76 \quad \therefore \frac{V_{f2}}{63.518} = 0.76$

$\therefore \quad V_{f2} = 48.2736$ m/sec

Whirl velocity at outlet, $V_{w2} = \frac{V_{f2}}{\tan\phi} - u$

$= \frac{48.2736}{\tan(20)} - 63.518 = 69.1126$ m/sec

Average flow velocity, $(V_f)_{avg} = \frac{V_{f1} + V_{f2}}{2} = \frac{44.4626 + 48.2736}{2}$

$= 46.3681$ m/sec

Specific volume of steam considering dryness fraction,

$v_s = x\ v_g$

From steam tables, at $p = 3$ bar,

Specific volume of steam, $v_g = 0.60553\ m^3/kg$

$\therefore \quad v_s = 0.98 \times 0.60553$

$= 0.5934\ m^3/kg$

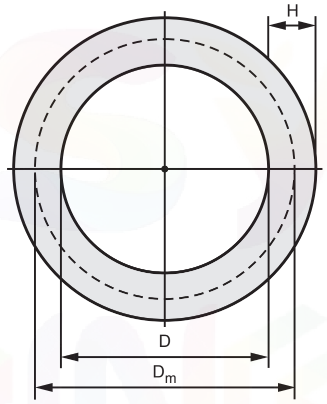


Fig. 3.41 (a)

Discharge of steam, $Q = \dot{m}_{act} \times v_s$

$= 3.8 \times 0.5934$

$= 2.2549\ m^3/sec$

Also, $Q = \pi D_m H (V_f)_{avg}$

$2.2549 = \pi \times D_m \times 0.02 \times 46.3681$

$\therefore \quad D_m = \mathbf{0.7739\ m}$ **... Ans.**

Step 2 : Calculate the power developed

Power developed, $P = \dot{m}_{act}\ u\,(V_{w1} + V_{w2})$

$= 3.8 \times 63.518 \times (122.16 + 69.1126)$

$\therefore \quad \mathbf{P = 46.1671 \times 10^3\ W}$ **... Ans.**

Ex. 3.26 : *A Parsons reaction turbines at 400 rpm develops 5 MW using 6 kg/kWh of steam. The exit angle of the blades are 20° and the velocity of steam is 1.35 times the blade velocity and pressure at exit is 1.2 bar and dryness fractions is 0.95.*

Calculate for this (i) A suitable blade height, assuming $\frac{D_m}{H} = 12$ *and (ii) Input power.*

Sol. : Given data :

$$R_D = 50\ \% = 0.5,\ N = 400 \text{ rpm},\quad P = 5\times 10^6 \text{ W},\ \dot{m} = 6\frac{\text{kg}}{\text{kW-hr}} = \frac{6\times 5\times 10^3}{3600} = 8.3333 \text{ kg/sec},$$

$$\alpha = \phi = 20^\circ,\ V_1 = 1.35 \text{ u},\quad p = 1.2 \text{ bar},\ x = 0.95,\quad \frac{D_m}{H} = 12$$

To find : i) H ii) P_i

Step 1 : Calculate the blade height

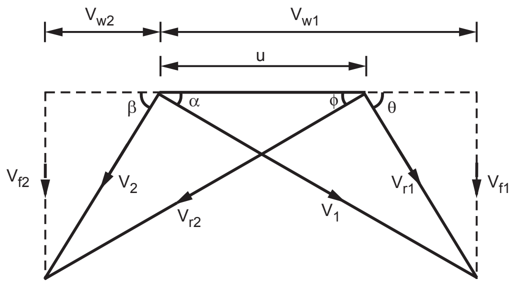


Fig. 3.42 (a)

From Fig. 3.42 (a), by inlet velocity triangle,

Whirl velocity at inlet, $V_{w1} = V_1 \cos\alpha$

$= 1.35\text{u}\times \cos(20) = 1.2685 \text{ u}$

From Fig. 3.42 (a), by outlet velocity triangle,

Whirl velocity at outlet, $V_{w2} = V_{r2} \cos\phi - u$ $\left[\because R_D = 50\ \% = 0.5 \quad \because V_{r2} = V_1\right]$

$= 1.35 \text{ u}\times \cos(20) - \text{u}$

$= 1.2685\text{u} - \text{u} = 0.2685 \text{ u}$

Power developed, $P = \dot{m}\ u\,(V_{w1} + V_{w2})$

$\therefore$ $5\times 10^6 = 8.3333\times u\times(1.2685\ u + 0.2685\ u)$

$\therefore$ $u = 624.7978$ m/sec

$\therefore$ Absolute velocity at inlet, $V_1 = 1.35$ u

$= 1.35\times 624.7978 = 843.477$ m/sec

Flow velocity at inlet, $V_{f1} = V_1 \times \sin\alpha$

$= 843.477\times \sin(20) = 288.4861$ m/sec

For Parson reaction turbine, $R_D = 0.5$

$V_{f1} = V_{f2} = 288.4861$ m/sec

Relative velocity at outlet, $V_{r2} = V_1 = 843.477$ m/sec

Average flow velocity, $(V_f)_{avg} = \dfrac{V_{f1} + V_{f2}}{2} = 288.4861$ m/sec

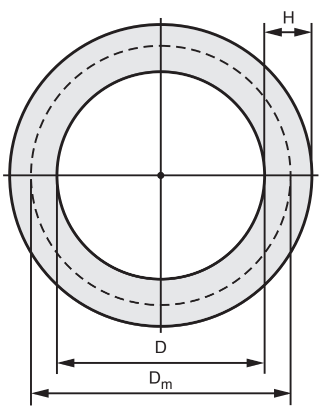


Fig. 3.42 (b)

Specific volume of steam considering dryness fraction,

$v_s = x\ v_g$

From steam tables, at $p = 1.2$ bar,

Specific volume of steam, $v_g = 1.4281 \ m^3/kg$

$\therefore \quad v_s = 0.95 \times 1.4281 = 1.3566 \ m^3/kg$

Discharge of steam, $Q = \dot{m} \ v_s = 8.3333 \times 1.3566$

$= 11.3049 \ m^3/sec$

Also, $Q = \pi D_m \ H (V_f)_{avg}$

$$\therefore \quad 11.3049 = \pi \times 12 \ H \times H \times 288.4861 \qquad \ldots \left[\because \frac{D_m}{H} = 12\right]$$

$$\mathbf{H = 0.03224 \ m = 3.224 \ cm} \qquad \textbf{... Ans.}$$

Step 2 : Calculate the input power

$\therefore \quad V_{w1} = 1.2685 \ u$

$= 1.2685 \times 624.7978$

$= 792.556 \ m/sec$

$\therefore \quad V_{w2} = 0.2685 \ u$

$= 0.2685 \times 624.7978$

$= 167.7582 \ m/sec$

From Fig. 3.42 (a), by inlet velocity triangle,

$$\tan \theta = \frac{V_{f1}}{V_{w1} - u} = \frac{288.4861}{792.556 - 624.7978}$$

$$\therefore \quad \theta = 59.8214^o$$

Relative velocity at inlet, $V_{r1} = \dfrac{V_{f1}}{\sin \theta} = \dfrac{288.4861}{\sin (59.8214)} = 333.7172 \ m/sec$

Input power, $P_i = \dot{m} \left(\dfrac{V_1^2}{2} + \dfrac{V_{r2}^2 - V_{r1}^2}{2} \right)$

$$= 8.3333 \times \left(\frac{843.477^2}{2} + \frac{843.477^2 - 333.7172^2}{2} \right)$$

$$\therefore \quad \mathbf{P_i = 5464.727 \times 10^3 \ W} \qquad \textbf{...Ans.}$$

Ex. 3.27 : *For a certain stage of 50 % reaction turbine mean rotor diameter is 1.35 m and speed ratio is 0.69. The rotor speed is 3000 rpm and outlet blade angle is 55°, find :*

i) Inlet blade angle ii) Blade efficiency and maximum blade efficiency.

Sol. : Given data :

$R_D = 50\ \% = 0.5,\quad D_m = 1.35$ m, $\quad s = 0.69,\quad N = 3000$ rpm, $\phi = 55°$.

To find : i) θ ii) η_b iii) $(\eta_b)_{max}$

Step 1 : Calculate the inlet blade angle

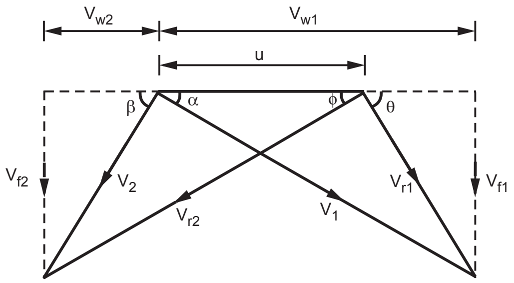


Fig. 3.43

Blade velocity, $$u = \frac{\pi D_m N}{60} = \frac{\pi \times 1.35 \times 3000}{60}$$

$$= 212.0575 \text{ m/sec}$$

Speed ratio, $$s = \frac{u}{V_1}$$

$$\therefore \quad 0.69 = \frac{212.0575}{V_1} \qquad \therefore V_1 = 307.3297 \text{ m/sec}$$

For 50 % reaction turbine, $\alpha = \phi = 55°$

Relative velocity at outlet, $V_{r2} = V_1 = 307.3297$ m/sec

From Fig. 3.43, by inlet velocity triangle,

Whirl velocity at inlet, $$V_{w1} = V_1 \cos\alpha = 307.3297 \times \cos(55)$$

$$= 176.277 \text{ m/sec}$$

Flow velocity at inlet, $$V_{f1} = V_1 \sin\alpha = 307.3297 \times \sin(55)$$

$$= 251.7497 \text{ m/sec}$$

And, $$\tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{251.7497}{176.277 - 212.0575}$$

$$\therefore \quad \theta = -81.9108° = -81.9108 + 180$$

$$\therefore \quad \theta = \mathbf{98.0892°}$$ **... Ans.**

Step 2 : Calculate the blade efficiency

Blade efficiency, $$\eta_b = \frac{2s\,(2\cos\alpha - s)}{1 - s^2 + 2s\cos\alpha}$$

$$= \frac{2 \times 0.69 \times (2 \times \cos(55) - 0.69)}{1 - 0.69^2 + 2 \times 0.69 \times \cos(55)}$$

$$\therefore \quad \eta_b = \mathbf{0.47959 = 47.959\ \%}$$ **... Ans.**

Step 3 : Calculate the maximum efficiency

Maximum efficiency, $$(\eta_b)_{max} = \frac{2\cos^2\alpha}{1+\cos^2\alpha} = \frac{2\times(\cos(55))^2}{1+(\cos(55))^2}$$

$\therefore$ $(\eta_b)_{max}$ = **0.4950 = 49.50 %** **... Ans.**

Ex. 3.28 : *A reaction steam turbine runs at 3000 rpm having outlet blade angle 20° and speed ratio 0.72. The mean diameter of rotor is 1500 mm. Calculate the diagram efficiency and percentage increase in diagram efficiency if the rotor is designed to run at its best theoretical speed. Take exit blade angle as 20°.*

Sol. : Given data :

$N = 3000$ rpm, $\phi = 20°$, $s = 0.72$, $D_m = 1500$ mm $= 1.5$ m, $\alpha = 20°$

To find : i) η_b ii) N_{th} iii) Percentage increase in diagram efficiency

Step 1 : Calculate the diagram efficiency

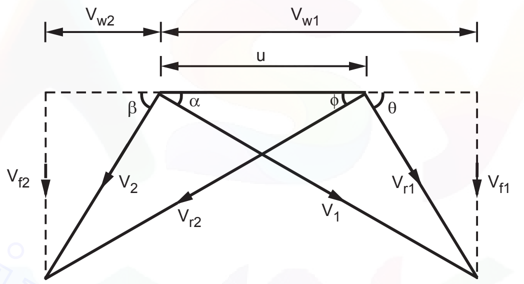


Fig. 3.44

Blade velocity, $$u = \frac{\pi D_m N}{60}$$

$$= \frac{\pi\times 1.5\times 3000}{60} = 235.6194 \text{ m/sec}$$

Speed ratio, $$s = \frac{u}{V_1}$$

$$\therefore \quad 0.72 = \frac{235.6194}{V_1} \quad \therefore \quad V_1 = 327.2491 \text{ m/sec}$$

As exit angles of fixed and moving blades are same, the reaction steam turbine is considered as Parson's steam turbine.

$\therefore$Relative velocity at outlet, $V_{r2} = V_1 = 327.2491$ m/sec

From Fig. 3.44, by inlet velocity triangle,

Whirl velocity at inlet, $$V_{w1} = V_1\cos\alpha$$

$$= 327.2491\times\cos(20) = 307.5135 \text{ m/sec}$$

Flow velocity at inlet, $$V_{f1} = V_1\sin\alpha$$

$$= 327.2491\times\sin(20) = 111.9257 \text{ m/sec}$$

Relative velocity at inlet, $$V_{r1} = \sqrt{V_{f1}^2+(V_{w1}-u)^2}$$

$$= \sqrt{111.9257^2+(307.5135-235.6194)^2}$$

$$= 133.0267 \text{ m/sec}$$

From Fig. 3.44, by outlet velocity triangle,

Whirl velocity at inlet, $V_{w2} = V_{r2} \cos \phi - u$

$= 327.2491 \times \cos (20) - 235.6194$

$= 71.8941$ m/sec

Diagram efficiency, $$\eta_b = \frac{P}{\dot{m}\left(\frac{V_1^2}{2} + \frac{V_{r2}^2 - V_{r1}^2}{2}\right)}$$

$$= \frac{u\,(V_{w1} + V_{w2})}{\frac{V_1^2}{2} + \frac{V_{r2}^2 - V_{r1}^2}{2}}$$

$$= \frac{235.6194 \times (307.5135 + 71.8941)}{\frac{327.2491^2}{2} + \frac{327.2491^2 - 133.0267^2}{2}}$$

$\therefore$ η_b = **0.9099 = 90.9939 %** ...**Ans.**

Step 2 : Calculate the percentage increase in diagram efficiency and rotor speed for best theoretical speed

Here maximum efficiency condition is considered.

Speed ratio, $$s = \cos \alpha = \frac{(u)_{th}}{V_1}$$

$$\therefore \quad \cos (20) = \frac{(u)_{th}}{327.2491} \quad \therefore \quad (u)_{th} = 307.5135 \text{ m/sec}$$

Blade efficiency, $$(\eta_b)_{max} = \frac{2 \cos^2 \alpha}{1 + \cos^2 \alpha}$$

$$= \frac{2 \times \cos^2 (20)}{1 + \cos^2 (20)} = 0.9378 = 93.7877 \%$$

Percentage increase in diagram efficiency,

$$= \frac{(\eta_b)_{max} - \eta_b}{\eta_b}$$

$$= \frac{93.7877 - 90.9939}{90.9939}$$

= **0.0307 = 3.07 %** ...**Ans.**

Theoretical blade velocity, $(u)_{th} = \dfrac{\pi D_m N_{th}}{60}$

$$307.5135 = \frac{\pi \times 1.5 \times N_{th}}{60}$$

$\therefore$ $\mathbf{N_{th} = 3915.3834\ rpm}$ **...Ans.**

Ex. 3.29 : *A stage in a 50 % reaction turbine delivers dry saturated steam at 2.7 bar from the fixed blades at 90 m/s. The mean blade height is 40 mm and the moving blade exit angle is 20°. The axial velocity of steam is 0.75 times the mean blade velocity. Steam flow rate is 9000 kg/h. The effect of blade tip thickness on the annulus area can be neglected. Calculate :*

i) Wheel speed in rpm, ii) Diagram efficiency,

iii) Diagram power iv) Enthalpy drop of steam in this stage.

Sol. : Given data :

$R_D = 50\ \% = 0.5$, $p = 2.7$ bar, $x = 1$, $V_1 = 90$ m/sec, $H = 40$ mm $= 0.04$ m,

$\phi = \alpha = 20°$, $V_f = 0.75\ u$, $\dot{m} = 9000\ \text{kg/hr} = \dfrac{9000}{3600} = 2.5$ kg/sec

To find : i) N ii) η_b iii) P iv) Δh

Step 1 : Calculate the wheel speed

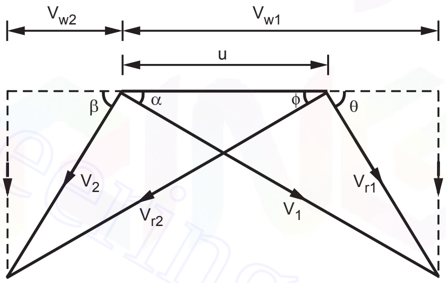


Fig. 3.45

For 50 % reaction design i.e. $R_D = 0.5$,

Flow velocity, $V_{f1} = V_{f2} = V_f = 0.75\ u$

Average flow velocity, $(V_f)_{avg} = \dfrac{V_{f1} + V_{f2}}{2} = V_f$

Relative velocity at exit and absolute at inlet,

$$V_{r2} = V_1 = 90 \text{ m/sec}$$

Blade angles, $\phi = \alpha = 20°$, $\theta = \beta$

From Fig. 3.45, by inlet velocity triangle,

Flow velocity at inlet, $V_{f1} = V_1 \sin \alpha$

$= 90 \times \sin (20) = 30.7818$ m/sec

Whirl velocity at inlet, $V_{w1} = V_1 \cos \alpha$

$= 90 \times \cos (20) = 84.5723$ m/sec

Also, $V_{f1} = 0.75\ u$ $\quad \therefore 30.7818 = 0.75\ u$

$\therefore$ $u = 41.0424$ m/sec

Specific volume of steam considering dryness fraction,

$$v_s = x\ v_g$$

From steam tables, at $p = 2.7$ bar,

Specific volume of steam, $v_g = 0.6684\ m^3/kg$

$\therefore \quad v_s = 1 \times 0.6684 = 0.6684\ m^3/kg$

Discharge of steam, $Q = \dot{m}\, v_s = 2.5 \times 0.6684$

$$= 1.671\ m^3/sec$$

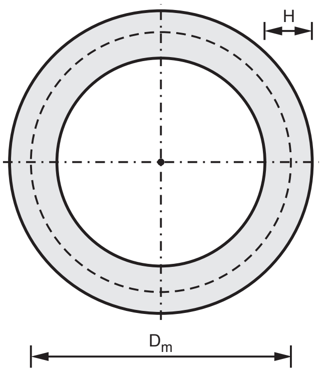


Fig. 3.45 (a)

Also, $Q = \pi D_m\ H\,(V_f)_{avg}$

$\therefore \quad 1.671 = \pi \times D_m \times 0.04 \times 30.7818$

$\therefore \quad D_m = 0.4319\ m$

Blade velocity, $u = \dfrac{\pi D_m N}{60}$

$\therefore \quad 41.0424 = \dfrac{\pi \times 0.4319 \times N}{60}$

$\therefore \quad \mathbf{N = 1814.8925\ rpm}$ **...Ans.**

Step 2 : Calculate the diagram power and diagram efficiency

From Fig. 3.45, by outlet velocity triangle,

Whirl velocity at outlet, $V_{w2} = V_{r2}\cos\phi - u$

$$= 90 \times \cos(20) - 41.0424$$

$$= 43.5299\ m/sec$$

Diagram power, $P = \dot{m}\ u\,(V_{w1} + V_{w2})$

$$= 2.5 \times 41.0424 \times (84.5723 + 43.5299)$$

$\therefore \quad \mathbf{P = 13.144 \times 10^3\ W}$ **...Ans.**

From Fig. 3.45, by inlet velocity triangle,

$$\tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{30.7818}{84.5723 - 41.0424}$$

$\therefore \quad \theta = 35.2657^\circ$

Relative velocity of inlet, $V_{r1} = \dfrac{V_{f1}}{\sin\theta}$

$$= \frac{30.7818}{\sin(35.2657)} = 53.3139 \text{ m/sec}$$

Diagram efficiency, $$\eta_b = \frac{P}{\dot{m}\left(\frac{V_1^2}{2} + \frac{V_{r2}^2 - V_{r1}^2}{2}\right)}$$

$$= \frac{13.144 \times 10^3}{2.5 \times \left(\frac{90^2}{2} + \frac{90^2 - 53.3139^2}{2}\right)}$$

$\therefore$ η_b = **0.7872 = 78.7205 %** ...Ans.

Step 3 : Calculate the enthalpy drop of steam

Enthalpy drop of steam, $\Delta h = (V_{w1} + V_{w2})\, u$

$= (84.5723 + 43.5299) \times 41.0424$

$\therefore$ Δh = **5257.6217 J/kg** ...Ans.

Ex. 3.30 : *The total tangential force on one ring of Parson's turbine is 1200 N. When the blade speed is 100 m/s. The mass flow rate is 8 kg/s. The blade outlet angle is 20°. Determine blade velocity at outlet from the blade. If friction losses which would occur with pure impulse are 25 % of the kinetic energy corresponding to the relative velocity at entry to each ring of blades and expansion losses are 10 % of the heat drop in blade, Determine the heat drop per stage, Stage efficiency, Blade efficiency and Maximum utilization factor.*

Sol. : Given data :

$F_t = 1200$ N, $u = 100$ m/sec, $R_D = 50\ \% = 0.5$, $\dot{m} = 8$ kg/sec, $\phi = \alpha = 20°$,

Friction losses $= \frac{0.25\, V_{r1}^2}{2} = \frac{0.25\, V_2^2}{2}$, Expansion losses = 10 % heat drop in blades = 0.1 heat drop in blades

To find : i) Δh ii) η_{stage} iii) η_b iv) $(\eta_b)_{max}$

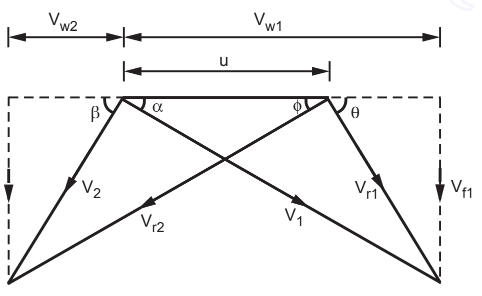


Fig. 3.46

Step 1 : Calculate the heat drop per stage

Power developed, $P = F_t \times u$

$= 1200 \times 100 = 120 \times 10^3$ W

For Parson's reaction turbine i.e. $R_D = 0.5$,

Whirl velocity at outlet, $V_{w2} = V_{w1} - u$

Tangential force, $F_t = \dot{m}(V_{w1} + V_{w2}) = \dot{m}(V_{w1} + V_{w1} - u)$

$\therefore$ $1200 = 8 \times (V_{w1} + V_{w1} - 100)$

$\therefore$ $V_{w1} = 125$ m/sec

$$\therefore \quad V_{w2} = V_{w1} - u$$

$$= 125 - 100 = 25 \text{ m/sec}$$

From Fig. 3.46, by inlet velocity triangle,

Absolute velocity at inlet, $V_1 = \dfrac{V_{w1}}{\cos\alpha}$

$$= \frac{125}{\cos(20)} = 133.0222 \text{ m/sec}$$

Flow velocity at inlet, $V_{f1} = V_1 \sin\alpha$

$$= 133.0222 \times \sin(20) = 45.4962 \text{ m/sec}$$

Relative velocity at inlet, $V_{r1} = \sqrt{V_{f1}^2 + (V_{w1} - u)^2}$

$$= \sqrt{45.4962^2 + (125 - 100)^2}$$

$$= 51.9124 \text{ m/sec}$$

For Parson's reaction turbine i.e. $R_D = 0.5$,

Flow velocity at outlet, $V_{f2} = V_{f1} = 45.4962$ m/sec

Absolute velocity at outlet, $V_2 = V_{r1} = 51.9124$ m/sec

Relative velocity at outlet, $V_{r2} = V_1 = 133.0222$ m/sec

Enthalpy drop for fixed and moving blades remains same.

$$(\Delta h)_{fixed} = (\Delta h)_{moving}$$

With the fixed blade acting as a nozzle,

$$\text{Friction loss} = \frac{0.25\, V_2^2}{2}$$

$$\therefore \quad \text{Available energy at entry} = \frac{V_2^2}{2} - \frac{0.25\, V_2^2}{2} = \frac{0.75\, V_2^2}{2}$$

With expansion losses = 0.1 heat drop in blades

$$\therefore \quad \text{Isentropic efficiency, } \eta_{isen} = 90\ \% = 0.9$$

$$\text{Available energy at exit} = \frac{V_1^2}{2}$$

Also,

$$\eta_{isen} = \frac{\dfrac{V_1^2}{2} - \dfrac{0.75\, V_2^2}{2}}{(\Delta h)_{fixed}}$$

$$\therefore \qquad 0.90 = \frac{\frac{133.0222^2}{2} - \frac{0.75 \times 51.9124^2}{2}}{(\Delta h)_{fixed}}$$

$$\therefore \qquad (\Delta h)_{fixed} = 8707.6292 \text{ J/kg}$$

Heat drop per stage, $\Delta h = (\Delta h)_{fixed} + (\Delta h)_{moving}$

$$= 8707.6292 + 8707.6292 \qquad \ldots[\because (\Delta h)_{fixed} = (\Delta h)_{moving}]$$

$$\therefore \qquad \Delta h = \mathbf{17.4152 \times 10^3 \text{ J/kg}} \qquad \ldots \textbf{Ans.}$$

Step 2 : Calculate blade efficiency, stage efficiency and Maximum utilization factor

Blade efficiency, $\eta_b = \dfrac{P}{\dot{m}\left(\dfrac{V_1^2}{2} + \dfrac{V_{r2}^2 - V_{r1}^2}{2}\right)}$

$$= \frac{120 \times 10^3}{8 \times \left(\frac{133.0222^2}{2} + \frac{133.0222^2 - 51.9124^2}{2}\right)}$$

$$\therefore \qquad \eta_b = \mathbf{0.9175 = 91.7573 \ \%} \qquad \ldots \textbf{Ans.}$$

Stage efficiency, $\eta_{stage} = \dfrac{P}{\dot{m}\ \Delta h}$

$$= \frac{120 \times 10^3}{8 \times 17.4152 \times 10^3}$$

$$\therefore \qquad \eta_{stage} = \mathbf{0.8613 = 86.1316 \ \%} \qquad \ldots \textbf{Ans.}$$

Maximum utilization factor, $(\eta_b)_{max} = \dfrac{2\ \cos^2 \alpha}{1 + \cos^2 \alpha}$

$$= \frac{2 \times (\cos 20)^2}{1 + (\cos 20)^2}$$

$$\therefore \qquad (\eta_b)_{max} = \mathbf{0.9378 = 93.7877 \ \%} \qquad \ldots \textbf{Ans.}$$

Ex. 3.31 : *The initial pressure and temperature of steam entering a reaction turbine of axial type are 100 bar and 550 °C respectively. The steam flows at 120 kg/s and the exit angle of first stage of nozzle blades is 70 degrees. The turbine is a single-stage machine with 50 % degree of reaction at the mean blade height. The stage efficiency is 85 %. Assuming maximum blade efficiency, determine :*

i) Rotor blade angles at inlet and outlet, *ii) Absolute steam velocity at rotor inlet,*

iii) Power developed, *iv) Final state of the steam after expansion.*

Mean diameter of rotor is 105 cm and the speed of rotation is 3200 rpm. All angles are measured with respect to axial direction only.

Sol. : Given data :

$p = 100$ bar, $T_1 = 550$ °C, $\dot{m} = 120$ kg/sec, $\alpha = 70°$, $R_D = 50\ \% = 0.5$, $\eta_{stage} = 85\ \% = 0.85$,

$D_m = 105$ cm $= 1.05$ m, $N = 3200$ rpm

To find : i) θ, ϕ ii) V_1 iii) P iv) p_2, T_2

Step 1 : Calculate the rotor blades angles at inlet, outlet and absolute velocity at rotor inlet

Blade velocity, $u = \dfrac{\pi D_m N}{60}$

$= \dfrac{\pi \times 1.05 \times 3200}{60} = 175.9291$ m/sec

For 50 % reaction design, i.e. $R_D = 0.5$,

Exit angle of nozzle, $\alpha = \phi = 70°$

For maximum blade efficiency,

Blade speed ratio, $s = \dfrac{u}{V_1} = \cos\alpha$

$\therefore \dfrac{175.9291}{V_1} = \cos(70)$

$\therefore$ $\mathbf{V_1 = 514.3822}$ **m/sec** **...Ans.**

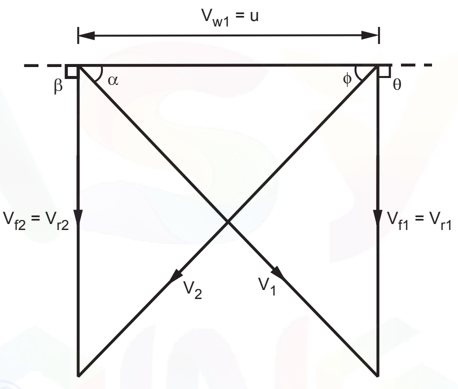


Fig. 3.47

From Fig. 3.47, by inlet velocity triangle,

Flow velocity at inlet, $V_{f1} = V_1 \sin\alpha$

$= 514.3822 \times \sin(70) = 483.3611$ m/sec

Whirl velocity at inlet, $V_{w1} = V_1 \cos\alpha$

$= 514.3822 \times \cos(70) = 175.9291$ m/sec

For 50 % reaction design i.e. $R_D = 0.5$,

Whirl velocity at outlet, $V_{w2} = V_{w1} - u$

$= 175.9291 - 175.9291 = 0$

Rotor blade angle at outlet, $\phi = \alpha = \mathbf{70°}$ **...Ans.**

As whirl velocity at outlet, $V_{w2} = 0$

Rotor blade angle at inlet, $\theta = \beta = \mathbf{90°}$ **...Ans.**

Step 2 : Calculate the power developed

Power developed, $P = \dot{m}\, u\,(V_{w1} + V_{w2})$

$$= 120 \times 175.9291 \times (175.9291 + 0)$$

$$\therefore \quad \mathbf{P = 3714.1257 \times 10^3\ W} \qquad \textbf{... Ans.}$$

Step 3 : Calculate the final state of steam after expansion

Stage efficiency,

$$\eta_{stage} = \frac{P}{\dot{m}\,\Delta h}$$

$$\therefore \quad 0.85 = \frac{3714.1257 \times 10^3}{120 \times \Delta h}$$

$$\therefore \quad \Delta h = 36.4129 \times 10^3 \text{ J/kg}$$

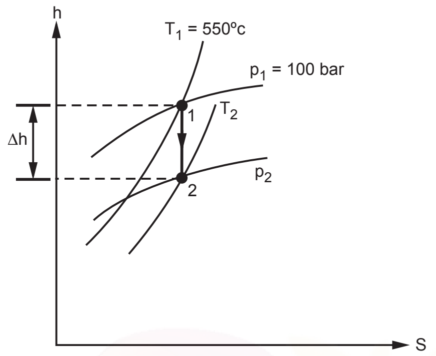


Fig. 3.47 (a)

Refer Fig. 3.47 (a),

- For the final state, first plot point 1 with p_1 = 100 bar, T_1 = 550° on h-S diagram (Mollier Diagram).
- From point 1, draw a straight line vertically downwards having length Δh = 36.4129 kJ/kg.
- Mark the end point of this line as point 2 read the pressure and temperature. Final state of steam is :

$$\mathbf{p_2 = 83\ bar \quad T_2 = 525\ ^\circ C} \qquad \textbf{...Ans.}$$

Ex. 3.32 : *In Parson's reaction turbine running at 1400 rpm with 50 % reaction turbine develops 75 kW per kg per second of steam. The exit angle of the blades is 20° and steam velocity is 1.4 times the blades velocity. Determine: i) Blade velocity ii) Inlet angle of the moving blades.*

Sol. : Given :

$\alpha = \phi = 20°$, $V_1 = 1.4$ u, N = 1400 rpm, P = 75 kW, $\dot{m} = 1$ kg/s

To find : i) u ii) θ

Step 1 : Calculate blade velocity

From velocity triangle,

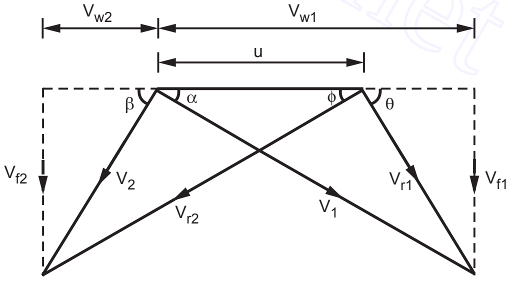


Fig. 3.48

$$V_{w1} = V_1 \cos\alpha = 1.4 \text{ u} \cos 20 = 1.3154 \text{ u}$$

$$V_{r2} = V_1 = 1.4 \text{ u}$$

$$V_{w2} = V_{r2} \cos\phi - u = 1.44 \cos 20 - u$$

$$\therefore \quad V_{w2} = 0.3154 \text{ u}$$

Power is given by,

$$P = \frac{\dot{m}\,(V_{w1} + V_{w2})\,u}{1000}$$

$$\therefore \quad 75 = \frac{1(1.3154\text{ u} + 0.3154\text{ u})\text{ u}}{1000}$$

$$\therefore \quad \mathbf{u = 214.5\ m/s} \qquad \textbf{...Ans.}$$

Step 2 : Calculate inlet angles of blade

We know that,

$$V_1 = 1.4\ u = 1.4 \times 214.5 = 303.3 \text{ m/s}$$

From velocity triangle,

$$V_{f1} = V_1 \sin\alpha = 303.3 \times \sin 20 = 102.7 \text{ m/s}$$

$$V_{w1} = V_1 \cos\alpha = 303.3 \times \cos 20 = 282.2 \text{ m/s}$$

$$\tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{102.7}{282.2 - 214.5}$$

$\therefore$ $\theta = \mathbf{56.6^o}$ **...Ans.**

Ex. 3.33 : *In a De Laval turbine steam issues from the nozzle with a velocity of 1200 m/s. The nozzle angle is 20°, the mean blade velocity is 400 m/s and the inlet and outlet angles of blades are equal. The mass of steam flowing through the turbine per hour is 100 kg. Calculate :*

i) Blade angles.

ii) Relative velocity of steam entering the blades.

iii) Tangential force on the blades.

iv) Power developed

v) Blade efficiency.

Take blade velocity co-efficient are 0.8. **AU : May-15, Marks 16**

Sol. : Given data :

$$\dot{m} = 1000 \text{ kg/hr} = \frac{1000}{3600} \text{ kg/s} = 0.278 \text{ kg/sec}$$

$$V_1 = 1200 \text{ m/s}, \quad u = 400 \text{ m/s}, \quad \alpha = 20\ ℃, \quad K = 0.8, \quad \theta = \phi$$

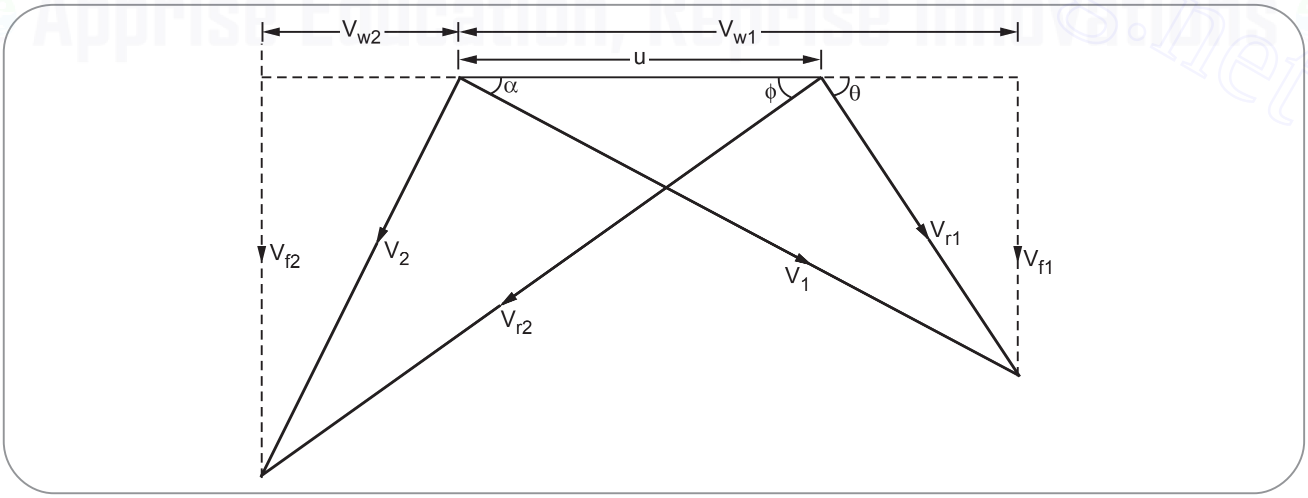


Fig. 3.49

a) Blade angles

$$V_{f_1} = V_1 \sin\alpha = 410.42 \text{ m/s}$$

$$V_{w1} = V_1 \cos\alpha = 1127.63 \text{ m/s}$$

$$\tan\theta = \left(\frac{V_{f_1}}{V_{w1} - u}\right) = \left(\frac{410.42}{1127.63 - 400}\right)$$

$$\theta = \mathbf{29.43°}$$

$$\therefore \quad \theta = \phi = \mathbf{29.43°}$$

b) Relative velocity of steam entering the blades

$$V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{410.42}{\sin(29.43)}$$

$$V_{r_1} = \mathbf{835.3\ m/s}$$

But, $$K = \frac{V_{r2}}{V_{r1}} = 0.8$$

$$V_{r_2} = (0.8 \times 835.3) = 668.24 \text{ m/s}$$

Consider outlet velocity triangle,

$$V_{w_2} = (V_{r_2} \cos\phi - u)$$

$$= (668.24 \cos(29.43 - 400)$$

$$V_{w2} = 182 \text{ m/s}$$

c) Tangential force on the blades

$$F_T = \dot{m}\,(V_{w_1} - V_{w_2})$$

$$= 0.278\,(1127.63 - 182) = \mathbf{262.885\ N}$$

d) Power developed,

$$P = \dot{m}\,(V_{w_1} - V_{w_2}) \times u$$

$$= (262.885) \times 400 = \mathbf{105.154\ kW}$$

e) Power Efficiency (η_b)

$$= \frac{P}{\frac{1}{2}\dot{m}\,V_1^{\,2}}$$

$$= \left[\frac{2 \times 105.154 \times 10^3}{0.287 \times 1200^2}\right] \times 100$$

$$= \mathbf{50.89\ \%}$$

Ex. 3.34 : *A simple impulse turbine has one ring of moving blades running at 150 m/sec. The absolute velocity of steam at exit from the stage is 85 m/sec at an angle of 80° from the tangential direction. Blade velocity co-efficient is 0.82 and the rate of steam flowing through the stage is 2.5 kg/sec. If the blades are equiangular, determine : (i) Blade angles; (ii) Nozzle angle; (iii) Absolute velocity of steam issuing from the nozzle; (iv) Axial thrust.* **AU : May-16, Marks 16**

Sol. : $u = 150$ m/s, $V_2 = 85$ m/s,

$\beta = 80°$, $K = 0.82$, $\dot{m} = 2.5$ kg/sec, $\theta = \phi$

i) Blade angles

Consider outlet velocity triangle

$$V_{w_2} = V_2 \cos\beta = 85 \cos 80° = 14.76 \text{ m/s}$$

$$V_{f_2} = V_2 \sin\beta = 85 \sin 80° = 83.71 \text{ m/s}$$

$$\tan\phi = \left(\frac{V_{f_2}}{V_{w_2} + u}\right)$$

$$\tan\phi = \left(\frac{83.71}{14.76 + 150}\right)$$

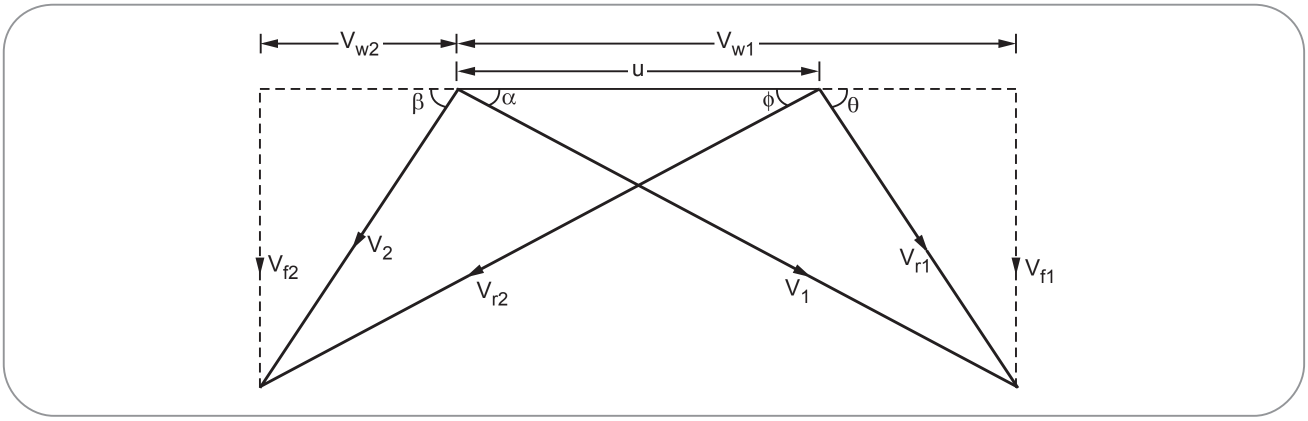


Fig. 3.50

$$\boxed{\phi = 26.93° = \theta} \quad \text{.... Blade angles}$$

$$\sin\phi = \frac{V_{f_2}}{V_{r_2}}$$

$$V_{r_2} = \frac{V_{f_2}}{\sin\phi} = \frac{83.71}{\sin(26.93°)} = \mathbf{184.83\ m/s}$$

$$\therefore \quad K = \frac{V_{r_2}}{V_{r_1}} \Rightarrow V_{r_1} = \frac{V_{r_2}}{K} = \frac{184.83}{0.82} = 225.4\ m/s$$

$$\sin\theta = \frac{V_{f_1}}{V_{r_1}}$$

$$V_{f_1} = V_{r_1}\sin\theta = (225.4)\sin(26.93)$$

$$= \mathbf{102.08\ m/s}$$

$$\text{and} \quad \tan\theta = \frac{V_{f_1}}{(V_{w_1} - u)}$$

$$\tan(26.93) = \frac{102.08}{(V_{w_1} - 150)}$$

$$V_{w_1} = 350.96\ m/s$$

ii) Nozzle angle,

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}}$$

$$= \left(\frac{102.08}{350.96}\right)$$

$$\boxed{\alpha = 16.22°}$$

iii) Absolute velocity and seam issuing from nozzle

$$V_{w_1} = V_1\cos\alpha$$

$$V_1 = \frac{V_{w_1}}{\cos\alpha} = \frac{350.96}{\cos(16.22)}$$

$$V_1 = \mathbf{365.51\ m/s}$$

iv) Axial thrust

$$F_{axial} = \dot{m}(V_{f_1} - V_{f_1})$$

$$= (2.5)(102.08 - 83.71)$$

$$= \mathbf{45.925\ N}$$

3.14 Energy Losses in Steam Turbine

- The energy loss in steam turbines is defined as, the increase in heat energy required for mechanical work actually as compared to the theoretical value in which the process of expansion takes place adiabatically.
- The losses in steam turbine are divided into the main groups :

(1) Internal losses	(2) External

1) **Internal losses :** These losses are related with the steam conditions during the flow in turbine. It includes the following losses :

   i) Losses in regulating valves and nozzles or guide blades.

   ii) Losses in moving blades.
   - Friction losses.
   - Losses due to steam leakage through the annular space.
   - Losses due to turning of steam jet in the blades.
   - Impingement losses etc.

   iii) Exit velocity losses.

   iv) Losses due to exhaust piping.

   v) Losses due to steam wetness.

   vi) Losses due to clearance between the rotor and guide blade discs.

2) **External losses :** These losses are not related with the steam conditions. It includes the following losses :

   i) Losses due to leakage of steam from the seals.

   ii) Mechanical losses.

3.14.1 Methods to Improve Thermal Efficiency

The efficiency of steam turbines can be increased by using any of the following methods :

- Reheating of steam in between the stages.
- Utilisation of low pressure steam for process industry.

- In case of impulse turbine, to improve part load efficiency use nozzle governing.
- Use by-pass governing or both throttle and by-pass governing to improve efficiency at part loads.
- Bleeding of steam for heating the feed water in between the stages.

3.15 Govering of Steam Turbine

- We know that, the purpose of governing is to maintain the speed of turbine fairly constant irrespective of the load.
- The output power of turbine is controlled by varying the steam flow with the help of valves interposed between the boiler and the turbine.
- Depending upon the methods of varying steam flow rate, there are various governing methods used.
- The following are the various governing methods used for the steam turbine :

(i) Throttle governing	(ii) Nozzle governing
(iii) By-pass governing	(iv) Combination of above

3.15.1 Throttle Govering

- Fig. 3.51, shows the simple throttle arrangement. The purpose of throttle governing is to throttle the steam whenever there is reduction in the load as compared to design load before it is supplied to the turbine.
- This helps in maintaining the speed of the turbine.
- To start the turbine for full load running the steam inlet is opened i.e. the throttle valve is opened.
- If the load on the turbine is reduced, the energy supplied to the turbine will be in excess and hence speed of the turbine increases.
- Due to this increased speed, the governor sleeve will lift as well as the pilot piston valve spindle will also get lifted.
- The upper port is then opened to oil pressure and lower port to the oil return.
- The relay piston will thus close the throttle valve partially and amount of steam supplied to the turbine will reduce and hence the speed on turbine comes to normal.
- The lowering of throttle valve will also lower the pilot piston spindle and close the ports to stabilize the relay piston.

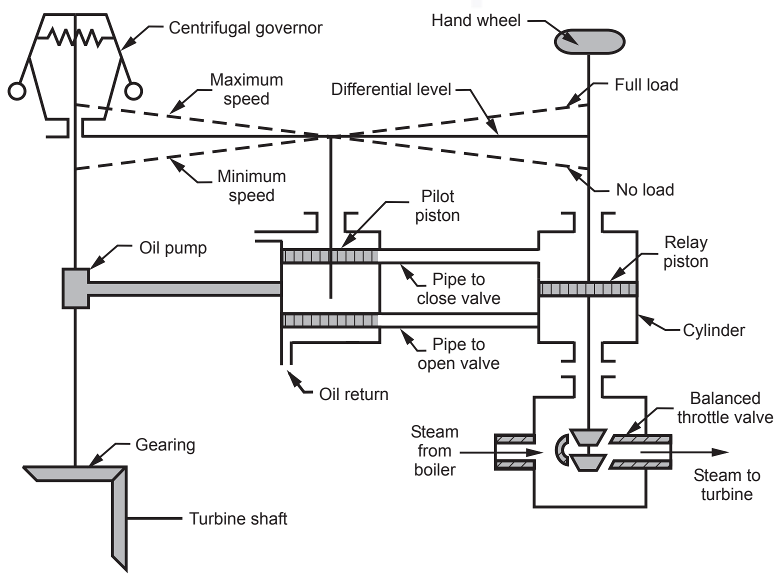


Fig. 3.51 : Throttle governing of steam turbine

- Due to restriction of passage in the valve, the steam is throttled from p_1 to p_2 and specific ideal output of turbine thus reduces from $h_1 - h_2$ to $h_3 - h_4$.
- Though the effort of the governor may not be sufficient to move the throttle valve against the piston, the oil operated relay (servo-mechanism) is used in circuit to amplify the small force produced by the governor.

Advantages :

- Simple in operation.
- Less initial cost.
- Lower admission losses.
- Used on small turbines (impulse as well as reaction turbines).

Disadvantages :

- Severe throttling losses.
- Thermodynamically not efficient as available heat drop is low.
- Reduced efficiency of turbine if throttling is carried out at low loads.

3.15.2 Nozzle Governing

- Nozzle governing is the more efficient type of governing in a steam turbine.
- In this type of governing, the nozzles are grouped together and each group is controlled by a seperate valve. Refer Fig. 3.52.

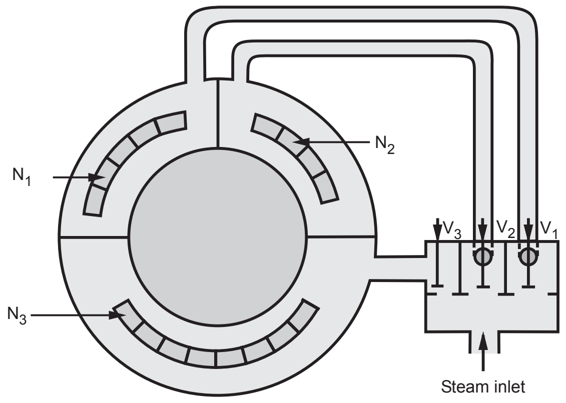


Fig. 3.52

- The groups may contain number of nozzles (3 to 5 or more) as shown in Fig. 3.52 by N_1, N_2 and N_3. These groups are controlled by individual valve V_1, V_2 and V_3 respectively.
- Under the full load condition the valves remains fully open.
- With the variation in load, the supply from the steam nozzle may be varied accordingly by shutting off or opening the nozzle.
- The nozzle control can only be applied to the first stage of a turbine.
- Also it is suitable for impulse turbine and larger turbines having an impulse stage followed by the impulse-reaction turbine.

Advantages :

- No throttling losses.
- More efficient than throttle governing as available heat drop is high.
- Used for medium and large turbines having initial impulse stage.

Disadvantages :

- High admission losses.
- Only applied to the first stage of a turbine.
- Pressure drop at entry to second stage when some of the nozzles cut off.

3.15.3 By-pass Governing

- By-pass governing is used when the turbine is throttle governed.
- The steam turbine working under designed or economic load have full admission of steam in the high pressure stages.
- At the maximum load, the turbine would require the additional steam. As this additional steam could not pass through the first stage since the additional nozzles are absent.

- Thus, the by-pass governing is used to admit the additional steam through by-pass valve to the later stages. Refer Fig. 3.53.
- This by-pass valve opens when throttle valve has opened and steam is by-passed through the valve to lower stages in the turbine.
- The supply of steam in lower stages increase the work output in these stages but the overall efficiency is reduced.

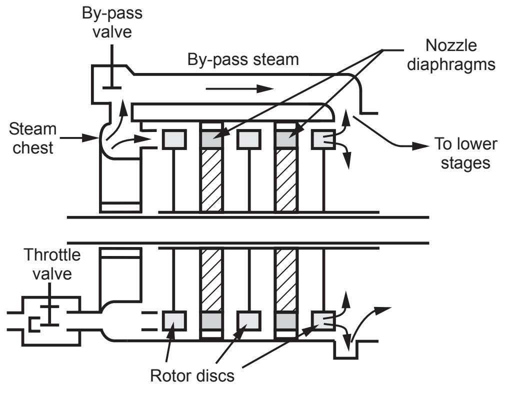


Fig. 3.53 : By-pass governing

3.16 Selection of Steam Turbine

The selection of steam turbine depends on the following factors :

i) Capacity of the plant
ii) Thermal efficiency of turbine
iii) Reliability of turbine
iv) Location of plant with reference to availability of water for condensate
v) Plant load factor and capacity factor
vi) Size of turbine
vii) Cost of turbine
viii) Maintenance of turbine
ix) Type of turbine (impulse or reaction)

3.17 List of Formulae

1) Nozzle

- Velocity at the outlet of nozzle (V_2) :

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

- Work done per kg of steam :

$$WD = \frac{n}{n-1}(p_1 V_1 - p_2 V_2)$$

- Mass flow rate through nozzle :

$$\dot{m} = \sqrt{n\left(\frac{p_1}{V_1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}}$$

- Nozzle efficiency :

$$\eta_n = \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}} = \frac{h_1 - h'_3}{h_1 - h_3} = \frac{h_1 - h'_6}{h_1 - h_6}$$

or
$$\eta_n = \frac{V_{2'}^2 - V_1^2}{V_3^2 - V_1^2}$$

2) Steam turbine

i) Impulse steam turbine :

- $V_{r2} \leq V_{r1}$
- Work done on the blade

$$\text{WD/sec} = \dot{m}_s (V_{w1} + V_{w2})\, u, \ \frac{\text{N - m}}{\text{s}} \text{ or Watt.}$$

- Blade efficiency or Diagram efficiency

$$\eta_b = \frac{\text{WD on the blade}}{\text{K.E. supplied to the blade}} = \frac{2(V_{w1} + V_{w2})\,u}{V_1^2}$$

- Stage efficiency or gross efficiency

$$\eta_{stage} = \frac{\text{WD on the blade}}{\text{Theoretical enthalpy drop}} = \frac{(V_{w1} + V_{w2})\,u}{h_1 - h_2}$$

$$\text{Also, } \eta_{stage} = \eta_b \times \eta_n$$

- Axial thrust on the wheel (F_a) :

$$F_a = \dot{m}_s (V_{f1} - V_{f2})$$

- Loss of kinetic energy due to blade friction :

$$\Delta E = \dot{m}_s (V_{r1} - V_{r2})$$

- Blade velocity coefficient :

$$V_{r2} = K\, V_{r1}$$

- Blade speed ratio :

$$s = \frac{u}{V_1}$$

- For blade height (H)

$$\dot{m} = \frac{\pi (D + H) \times H \times V_f}{V_s}$$

- Maximum diagram or blade efficiency :

Maximum blade efficiency occurs at $\frac{u}{V_1} = \frac{\cos\alpha}{2}$

$$(\eta_b)_{max} = \cos^2\alpha \quad \text{and} \quad (WD)_{max} = 2u^2$$

- Power developed (P) :

$$P = \dot{m}\,(\Delta h)_{actual} = \dot{m}\,(V_{w1} + V_{w2})\,u$$

- Multistage impulse turbine blade efficiency :

$$\eta_b = \frac{8u}{V_1}\left(\cos\alpha - \frac{2u}{V_1}\right) = 8s\,(\cos\alpha - 2s)$$

$$\eta_{b_{max}} = \cos^2\alpha \quad \text{and} \quad WD_{max} = 8u^2$$

ii) Reaction steam turbine

- Degree of reaction :

$$R_D = \frac{\Delta h_m}{\Delta h_f + \Delta h_m} = \frac{V_{r2}^2 - V_{r1}^2}{2\,u\,(V_{w1} + V_{w2})}$$

For 50 % reaction turbine,

$$R_D = \frac{1}{2} = \frac{V_f}{2u}(\cot\phi - \cot\theta)$$

- For reaction turbine the work done per kg of steam :

$$WD = (V_{w1} + V_{w2})\,u, \quad \frac{N\text{ - }m}{kg}$$

- Power developed per stage :

$$P = \dot{m}\,(V_{w1} + V_{w2})\,u, \text{ Watts}$$

- Enthalpy drop in a stage :

$$\Delta h = \Delta h_f + \Delta h_m = \left(\frac{V_1^2}{2}\right) + \frac{(V_{r2}^2 - V_{r1}^2)}{2}, \text{ J/kg}$$

- Stage efficiency :

$$\eta_{stage} = \frac{\text{Work done on the blade}}{\text{Enthalpy in a stage}} = \frac{u\,(V_{w1} + V_{w2})}{\Delta h}$$

- Volume flow rate of steam :

$$Q = \text{Area of flow} \times \text{Average velocity of flow}$$

$$Q = \pi D_m H \times V_{f\,avg}, \text{ m}^3\text{/sec}$$

But $$V_{f\,avg} = \text{Average flow velocity} = \frac{V_{f1} + V_{f2}}{2}$$

$$H = \text{Height of blade}$$

- Mass flow rate of steam

$$\dot{m} = \rho Q = \frac{Q}{v_s} = \frac{\pi D_m H \times V_{favg}}{v_s}$$

where, v_s = Specific volume of steam = $x\ v_g$

x = Dryness fraction of steam

v_g = Volume of steam at saturated point in m^3/kg

- The value of v_g is found from steam table at given pressure.
- Blade efficiency for reaction turbine :

$$\eta_b = \frac{2s(2\cos\alpha - s)}{(1+2s\cos\alpha - s^2)} = 2 - \frac{2}{(1+2s\cos\alpha - s^2)}$$

$$\eta_{bmax} = \frac{2\cos^2\alpha}{1+\cos^2\alpha}$$

3.18 Two Marks Questions with Answers

Q.1 What is a steam turbine ?

Ans. : Steam turbine is a high speed rotating machine which converts the heat energy of steam into work energy.

Q.2 What is an impulse steam turbine ?

Ans. : In this case, velocity of steam from the nozzle is very high. The jet of steam coming out of the nozzle strikes the blades mounted on the rotor. The rotor is connected to the generator and thus produces power. It has been observed that the velocity of steam jet is almost two times the velocity of the moving blades.

Q.3 What is a stage in a steam turbine ?

Ans. : In an impulse turbine, the stage is a set of moving blades behind the nozzle.

In a reaction turbine, each row of blades is called a stage.

Q.4 State the operating principle of an impulse turbine ?

Ans. : The velocity of the steam is about twice as fast as the velocity of the blades. Only turbines utilizing fixed nozzles are classified as impulse turbines.

Q.5 State the operating principle of a reaction turbine ?

Ans. : The steam is directed into the moving blades by fixed blades designed to expand the steam. The result is a small increase in velocity over that of the moving blades. These blades form a wall of moving nozzles that further expand the steam. The steam flow is partially reversed by the moving blades, producing a reaction on the blades. Since the pressure drop is small across each row of nozzles (blades), the speed is comparatively low.

Q.6 What is meant by governing a steam turbine ?

Ans. : Steam turbine governing means controlling the flow rate of steam into a steam turbine to maintain a constant speed of rotation under all loads. The variation in speed can have a significant impact on its performance.

Q.7 What are the different methods of steam governing in a steam turbine ?

Ans. : There are four different methods of governing the turbines.

(i) Throttle governing

(ii) Nozzle governing

(iii) By pass governing

Q.8 Define critical speed ?

Ans. : It is the speed at which the machine vibrates most violently. It is due to many causes, such as imbalance or harmonic vibrations set up by the entire machine.

Q.9 What is bleeding ?

Ans. : Bleeding is the process of draining steam from the turbine at certain point during its expansion, and using this steam for heating the feed water supplied to the boiler.

Q.10 Define heat rate in turbine ?

Ans. : Heat required for unit of power generated in specific conditions and specific fuel burning.

Q.11 Define extraction turbine ?

Ans. : In an extraction turbine, steam is withdrawn from one or more stages, at one or more pressures, for heating, plant process or feed water heater needs. They are often called bleeder turbines.

Q.12 How many governors are needed for safe turbine operation ?

Ans. : Two governors are needed for safe turbine operation.

Q.13 What is a radial-flow turbine ?

Ans. : In a radial-flow turbine, steam flows outward from the shaft to the casing.

Q.14 What are two types of clearance in a turbine ?

Ans. : Radial - clearance at the tips of the rotor and casing.

Axial - the fore-and-aft clearance, at the sides of the rotor and the casing.

Q.15 Define topping and superposed turbines ?

Ans. : Topping and superposed turbines are high-pressure, non-condensing units that can be added to an older, moderate-pressure plant. Topping turbines receive high-pressure steam from new high-pressure boilers. The exhaust steam of the new turbine has the same pressure as the old boilers and is used to supply the old turbines.

Q.16 What is meant by the water rate of a turbine ?

Ans. : Water rate is another term used for the steam rate.

Q.17 State any two disadvantages of velocity compounding ?

Ans. : Steam velocity is too high and that is responsible for appreciable friction losses.

Blade efficiency decreases with the increase of the number of stages.

Q.18 State the advantages of welded rotors ?

Ans. : Welded rotor is a composed body built up by welding the individual segments. So the limitations on forgings capacity do not apply.

Welding discs together results in a lower stress level. Therefore, more ductile materials can be chosen to resist SCC attack.

There are no keyways. So regions of high stress concentrations are eliminated.

Q.19 What is the difference between an HP turbine and an IP turbine ?

Ans. : In a HP turbine in which steam enters the turbine from just the boiler and it is partially expanded to perform work where as in IP turbine after partial expansion in HP turbine it goes to boiler for further heating and then it goes to IP turbine.

Q.20 Where Da-laval turbines are mostly used ?

Ans. : For small power purposes and high speeds.

Q.21 How we can define the degree of reaction ?

Ans. : It is the ratio of heat drop in the moving blades to the total heat drop in the fixed blades.

Q.22 Define rankine efficiency.

Ans. : The ratio of isentropic heat drop to the heat supplied is called rankine efficiency.

Q.23 How the efficiency of steam turbine is improved ?

Ans. : 1) Reheating of steam

2) Regenerative feed heating

3) Binary vapour plants

Q.24 What is the effect of reheating in a turbine ?

Ans. : Increases the workdone through the turbine

Increases the efficiency of the turbine

Reduces wear on the blades

Q.25 What are the various losses in steam turbine ?

Ans. : 1) Profile loss

2) Secondary loss

3) Tip leakage loss

4) Disc windage loss

5) Wetness loss

6) Annulus loss

Q.26 What is profile loss ?

Ans. : Due to formation of boundary layer on blade surfaces. Profile loss is a boundary layer phenomenon and therefore subject to factors that influence boundary layer development. These factors are Reynolds number, surface roughness, exit mach number and trailing edge thickness.

Q.27 What is the cause of turbine deposits ?

Ans. : The turbine deposits are steam-born foreign matters settled on turbine blades.

Q.28 What is the effect of over speed of rotor ?

Ans. : Over speed rotor grows radially causing heavy rub in the casing and the seal system. As a result, considerable amount of shroud-band and rivet head damage occurs.

Q.29 What is the role of Turbine's blades in turbine ?

Ans. : A turbine's blades are designed to control the speed, direction and pressure of the steam as it passes through the turbine.

Q.30 What are the parts of turbine blade ?

Ans. : Root-The root is a constructional feature of turbine blades, which fixes the blade into the turbine rotor.

Profile-The profile converts kinetic energy of steam into mechanical energy of the blade.

Shroud-The shroud reduces the vibration of the blade which can be induced by the flowing of high pressure steam through the blades.

Q.31 What is the need for compounding in steam turbines ? (Refer section 3.6) **AU : Dec.-15**

Q.32 Distinguish between impulse and reaction principles. (Refer section 3.5) **AU : May - 16**

Q.33 Define the term compounding in turbines. (Refer section 3.6) **AU : Dec.-17**

Q.34 Distinguish between impulse and reaction turbine. (Refer section 3.5) **AU : May-18**

Review Questions

1. *State the advantages and applications of steam turbine.*
2. *Explain the compounding of steam turbine.*
3. *What is reheat factor.*
4. *Explain energy losses in steam turbine.*
5. *Write a short note on governing of steam turbine.*

3.19 University Question with Answer

May-2018

Q.1 *Elucidate the working of velocity, pressure and velocity pressure compounding methods with neat sketch.* **(Refer sections 3.6.1, 3.6.2 and 3.6.3) [13]**

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UNIT - IV

4

Cogeneration and Residual Heat Recovery

Syllabus

Cogeneration Principles, Cycle Analysis, Applications, Source and utilisation of residual heat. Heat pipes, Heat pumps, Recuperative and Regenerative heat exchangers. Economic Aspects.

Contents

4.1 Introduction

- Cogeneration power plant is used to generate continuous useful energy.
- The obtained useful energy in the form of mechanical or electrical energy.
- This cycle is also called as combined heat and power (CHP).
- This cycle has about 85 % of overall efficiency.
- The cogeneration power plant produces more electricity compared to combined power plant.
- The cogeneration can be calculated as,

$$\eta_C = \frac{E+H}{Q} = \frac{E+H}{H.S.}$$

where, E = Electrical energy generated

H = Heat energy generated

Q = Total heat supplied

- Effectiveness of co-generation power plant.

$$\in = \frac{W_T + H.S.}{(H.S.)_1} = \frac{W_T + Q_S}{Q_{S1}}$$

4.2 Block Diagram of Cogeneration Power Plant

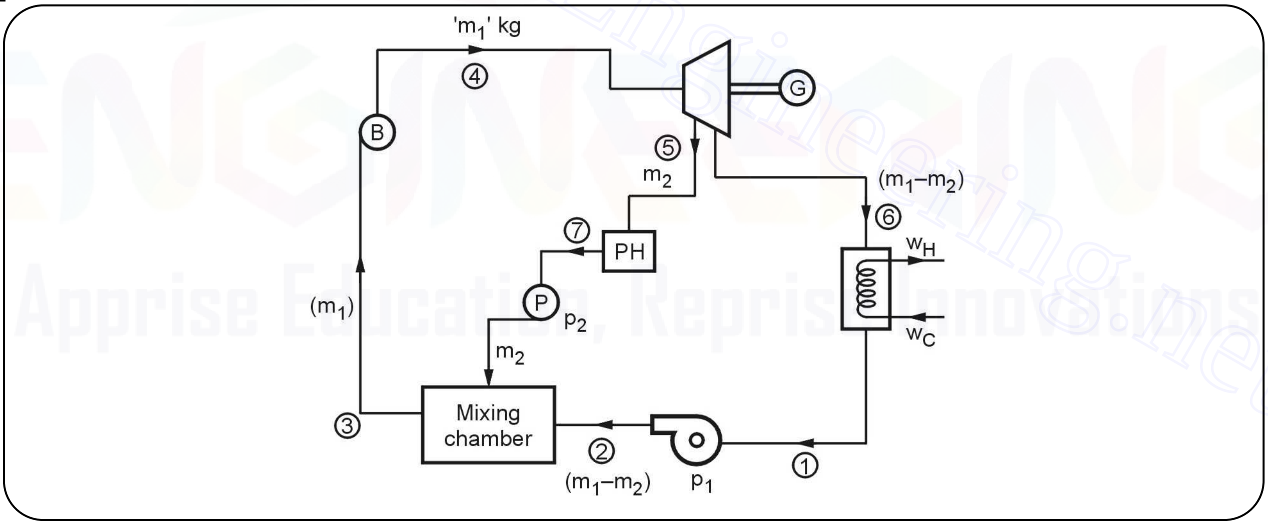

Fig. 4.1

4.3 Working Principle of Cogeneration

- From above figure, the condensed water is transferred to the feed water heater with the help of pump 1.
- The partial steam extracted from the turbine is mixed with liquid entering the feed water heater where heat transfer occurs from steam to liquid.
- Now, homogeneous mixture is entered into the mixing chamber where the mass from the process heating also enters, again heating of homogeneous mixture takes place.
- With the help of pump-1 this available mass is pumped to the boiler, the generation of steam takes place.
- This high pressure and temperature steam now transfer to turbine where adiabatic expansion takes place.
- This cycle will repeat in same way.

4.3.1 T-S Diagram of Cogeneration Power Plant

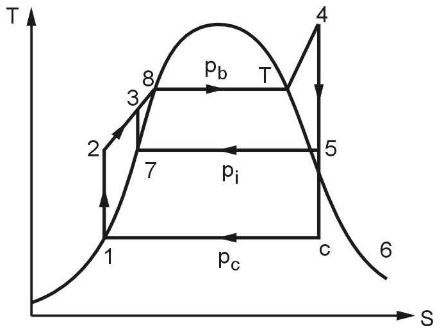

Fig. 4.2

4.4 Solved Numericals

Ex. 4.1 : *In co-generation steam power plant the boiler generates steam at 50 bar and 400 °C, which is supplied to turbine for expansion steam at 5 bar is extracted from a turbine for a process heating and remainder continuous to expand upto a condenser pressure of 0.05 bar. Determine : i) Power output in turbine ii) Process heat utilize in kJ/kg; iii)* η_{Reg}

Sol. : Given data :

$$p_b = 50 \text{ bar}; \quad p_i = 5 \text{ bar} \quad p_c = 0.05 \text{ bar}; \quad T_{sup} = 400\ °C$$

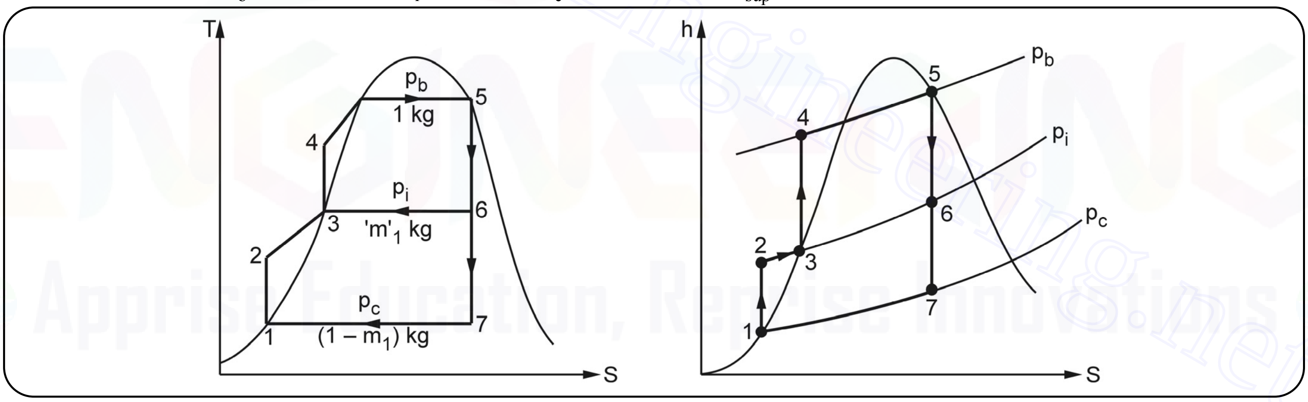

Fig. 4.3

p	t_s	h_f	h_{fg}	h_g	s_f	S_{fg}	S_g
50	263.9	1154.5	1639.7	2794.2	2.921	3.053	5.974
5	151.8	640.1	2107.1	2747.5	1.860	4.959	6.819
0.05	32.90	137.8	2423.8	2561.6	0.476	7.920	8.396

$$h_1 = h_f = 137.8 \text{ kJ/kg}$$

$$h_2 = \left(\frac{p_i - p_c}{10}\right) + h_1 = 138.295 \text{ kJ/kg}$$

$$h_3 = h_f = 640.1 \text{ kJ/kg}$$

$$h_4 = \left(\frac{p_b - p_i}{10}\right) + h_3 = 644.6 \text{ kJ/kg}$$

$$h_5 = h_g + C_{ps}(t_{sup} - t_s) = 2794.2 + 2.1\ (400 - 263.9) = 3080.01 \text{ kJ/kg}$$

$$S_5 = S_6$$

$$S_g + C_{ps}\, ln\left(\frac{t_{sup}}{t_s}\right) = S_f + x_6 \times S_{fg}$$

$$5.974 + 2.1 \times ln\left(\frac{673}{536.9}\right) = 1.860 + x_6 \times 4.959$$

$$x_6 = 0.925$$

$$h_6 = h_f + x_6 \times h_{fg} = 2589.445 \text{ kJ/kg}$$

$$S_6 = S_7$$

$$S_f + x_6\, S_{fg} = S_f + x_7 \times S_{fg}$$

$$1.860 + 0.925 \times 4.959 = 0.476 + x_7 \times 7.920$$

$$x_7 = 0.753$$

$$h_7 = h_f + x_7 \times h_{fg} = 1962.9214 \text{ kJ/kg}$$

$$m_1 = \frac{h_3 - h_2}{h_6 - h_2} = 0.2047 \text{ kg/kg of steam}$$

$$W_T = (h_5 - h_6) + (1 - M_1)(h_6 - h_7)$$

$$= 988.839 \text{ kJ/kg}$$

$$\text{H.S.} = h_5 - h_4 = 2435.41$$

i) $W_{net} = W_T - W_P = \mathbf{983.9454\ kJ/kg}$

ii) Process heat utilize $= m_1 (h_6 - h_3) = \mathbf{398.95\ kJ/kg}$

iii) $$\eta_{Reg} = \frac{W_{net}}{\text{H.S.}}$$

$$= \frac{W_T - W_P}{\text{H.S.}} = \mathbf{40.40\ \%}$$

Ex. 4.2 : *In cogeneration steam power plant boiler generates steam at 60 bar and 450 °C, which supply to turbine for expansion steam at 6 bar is extracted from a turbine for a process heating and remainder continues to expand upto a condenser pressure of 0.5 bar. The mass flow rate of steam is 15 kg/s. If the amount of steam extracted for process heating is 5 kg/s. Which is condense at 6 bar from the process reheater. Find the following :*
i) Power output in turbine ii) Process heat utilize in kW; iii) η_{Reg} *iv) Specific steam consumption, v) Work ratio vi) Heat rejected in kW. Neglect the pump work.*

Sol. : Given data :

$$p_b = 60 \text{ bar}; \quad p_i = 6 \text{ bar}; \quad p_c = 0.5 \text{ bar}; \quad T_{sup} = 450\ °C$$

Total mass of steam = 15 kg (turbine)

M_1 = 5 kg (process heating)

m_2 = $(15 - M_1)$

= 10 kg (condenser)

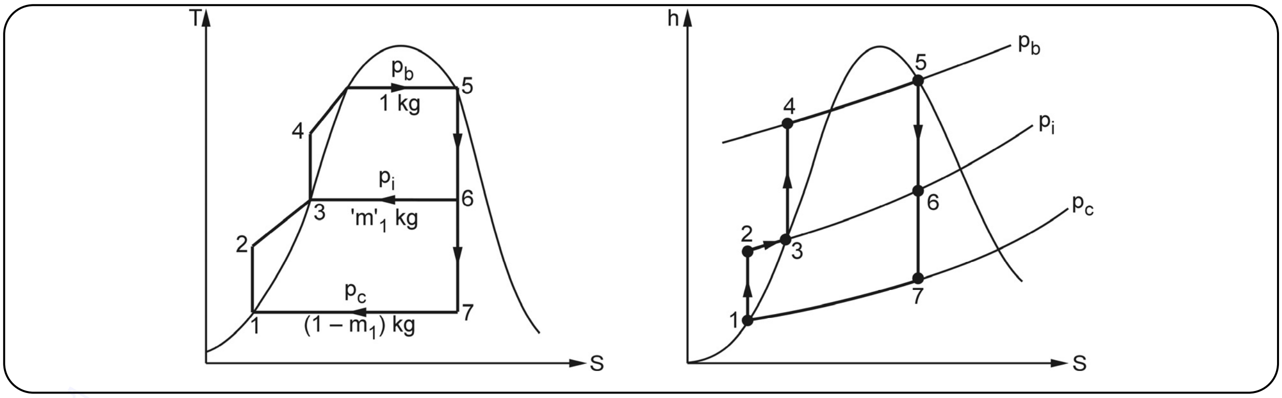

Fig. 4.4

From steam table,

p	t_s	h_f	h_{fg}	h_g	s_f	S_{fg}	S_g
60	275.6	1213.7	1571.3	2785.0	3.027	2.863	5.890
6	158.8	670.4	2085.1	2755.5	1.931	4.827	6.758
0.5	81.35	340.6	2305.4	2646.0	1.091	6.504	7.595

$$h_1 = h_f = 340.6 \text{ kJ/kg}$$

$$h_2 = \left(\frac{p_i - p_c}{10}\right) + h_1 = \left(\frac{6 - 0.5}{10}\right) + 340.6 = 341.15 \text{ kJ/kg}$$

$$h_3 = h_f = 670.4 \text{ kJ/kg}$$

$$h_4 = \left(\frac{p_b - p_i}{10}\right) + h_3 = \left(\frac{60 - 6}{10}\right) + 670.4 = 675.8 \text{ kJ/kg}$$

$$h_5 = h_g + m\,C_{ps}(T_{sup} - t_{sat})$$

$$= 2785 + 1 \times 2.1 \times (450 - 275.6) = 3151.24 \text{ kJ/kg}$$

$$S_5 = S_6$$

$$S_g + m\,C_{ps}\,ln\left(\frac{T_{sup}}{t_{sat}}\right) = S_f + x_6\,S_{fg}$$

$$5.890 + 1 \times 2.1 \times ln\left(\frac{723}{548.6}\right) = 1.931 + x_6 \times 4.827$$

$$x_6 = 0.9402$$

$$h_6 = h_f + x_6\,h_{fg}$$

$$= 670.4 + 0.940 \times 2085.1 = 2630.394 \text{ kJ/kg}$$

$$S_6 = S_7$$

$$S_f + x_6 S_{fg} = S_f + x_7 \times S_{fg}$$

$$1.931 + 0.942 \times 4.827 = 1.091 + x_7 \times 6.504$$

$$x_7 = 0.8282$$

$$h_7 = h_f + x_7 h_{fg}$$

$$= 340.6 + 0.8282 \times 2305.4 = 2249.93 \text{ kJ/kg}$$

$$\text{H.S.} = (h_5 - h_4) \times 15 = 2475.44 \text{ kJ/kg}$$

i) $$W_T = 15 (h_5 - h_6) + (h_6 - h_7) (15 - M_1) = 11617.33 \text{ kW}$$

ii) $$\text{H.S.} = (h_5 - h_4) \times 15 = 37137.6 \text{ kW}$$

v) $$W_{ratio} = \frac{W_{net}}{W_T} = 1$$

iii) $$\eta_{Reg} = \frac{W_T}{\text{H.S.}} = \frac{11617.33}{37131.6} = 31.28 \% \text{ (Neglect the pump work)}$$

vi) $$\text{H.R.} = (h_7 - h_1) \times 10 = 19093.33 \text{ kW}$$

$$\text{H.V.} = (h_6 - h_3) \times M_1 = 9799.97 \text{ kW}$$

iv) $$\text{S.R.} = \frac{3600}{W_T} = 0.8098 \text{ kg/kW.hr}$$

Ex. 4.3 : *In a co-generation plant, 25 kg/sec steam enters turbine at 40 bar and 400 ºC. 20 % of steam is withdrawn for process heating at 3 bar and remaining to expand in turbine up to condenser pressure of 0.08 bar. Neglect pump work. Represent cycle on T-S diagram. Find :*

i) Thermal efficiency of cycle ii) Capacity of power plant in MW iii) Effectiveness of cogeneration. Neglect pump work.

Sol. : Plotting 4, 5 and 6 points on h-S diagram

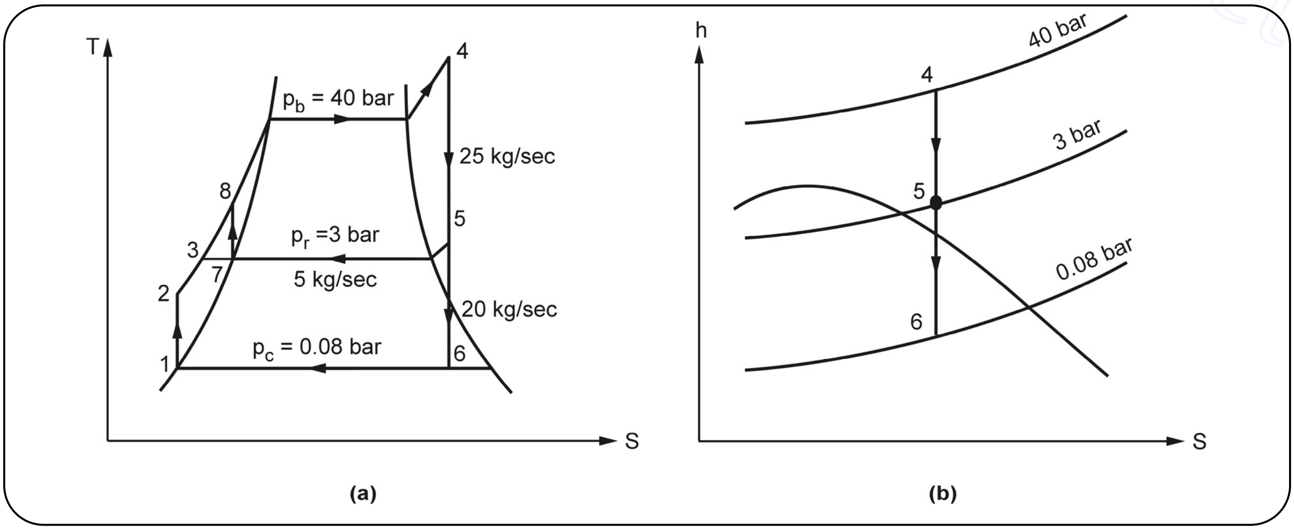

Fig. 4.5

Plotting 4, 5, and 6 points on h-s diagram

$h_4 = 3210$ $h_5 = 2640$

$h_6 = 2130$

Also from steam table :

At 0.08 bar, $h_{f1} = h_1 = 173.9$ kJ/kg

and at 3 bar, $h_{f7} = h_7 = 561.5$ kJ/kg

i) Thermal efficiency of cycle :

$$\text{Turbine work, } W_T = \dot{m}(h_4 - h_5) + (\dot{m} - 5)(h_5 - h_6)$$

$$= 25(3210 - 2640) + (25 - 5)(2640 - 2130)$$

$$\therefore \quad W_T = 20400 \text{ kJ/sec.}$$

Process heat energy utilised :

$$Q_s = \dot{m}_s (h_4 - h_5) = 5(2640 - 561.5)$$

$$Q_s = 10392.5 \text{ kJ/sec.}$$

Let "h_3" is the enthalpy of feed water after mixing the condensate of condenser and process heater.

By heat balance,

$$25 \times h_3 = 20 \times h_1 + 5 h_7$$

$$25 \times h_3 = 20 \times 173.9 + 5 \times 561.5$$

$$h_3 = 251.42 \text{ kJ/kg}$$

$\therefore$ Rate of heat supplied in boiler,

$$Q_{s1} = 25(h_4 - h_3) = 25(3210 - 251.42)$$

$$Q_{s1} = 73964.5 \text{ kJ/sec.}$$

$$\text{Cycle efficiency, } \eta = \frac{W_T}{Q_{s1}} = \frac{20400}{73964.5}$$

$$\eta = 27.58\ \%$$ **… Ans.**

ii) Capacity of power plant :

$$P = W_T = 20400 \text{ kW}$$

$$\therefore \quad \boxed{P = 20.4 \text{ MW}}$$ **… Ans.**

iii) Effectiveness of co-generation :

$$\in = \frac{W_T + Q_s}{Q_{s1}} = \frac{20400 + 10392.5}{73.964.5}$$

$$\therefore \quad \boxed{\in = 41.63\ \%}$$ **… Ans.**

4.5 Introduction to Waste Heat

- Waste heat is heat, which is generated in a process by way of fuel combustion or chemical reaction, and then "dumped" into the environment even though it could still be reused for some useful and economic purpose.
- Industrial waste heat refers to energy that is generated in industrial processes without being put to practical use. Sources of waste heat include hot combustion gases discharged to the atmosphere, heated products exiting industrial processes, and heat transfer from hot equipment surfaces.
- The exact quantity of industrial waste heat is poorly quantified, but various studies have estimated that as much as 20 to 50 % of industrial energy consumption is ultimately discharged as waste heat.
- While some waste heat losses from industrial processes are inevitable, facilities can reduce these losses by improving equipment efficiency or installing waste heat recovery technologies.
- Waste heat recovery entails capturing and reusing the waste heat in industrial processes for heating or for generating mechanical or electrical work.
- Large quantity of hot flue gases is generated from Boilers, Kilns, Ovens and Furnaces. If some of this waste heat could be recovered, a considerable amount of primary fuel could be saved.
- The energy lost in waste gases cannot be fully recovered. Example uses for waste heat include generating electricity, preheating combustion air, preheating furnace loads, absorption cooling and space heating.

Waste Heat Sources	Uses for Waste Heat
○ Combustion exhausts :	• Combustion air preheating
○ Glass melting furnace	• Boiler feed water preheating
○ Cement kiln	• Load preheating
○ Fume incinerator	• Power generation
○ Aluminum reverberatory furnace	• Steam generation for use in : power generation mechanical power process steam
○ Boiler	• Space heating
○ Process off gases :	• Water preheating
○ Steel electric arc furnace	• Transfer to liquid or gaseous process streams
○ Aluminum reverberatory furnace	
• Cooling water from :	
○ Furnaces Air compressors	
○ Internal Combustion engines	
• Conductive, convective, and radiative losses from equipment :	
○ Hall Hèroult cells	

4.6 Commercial Waste Heat Recovery Devices

4.6.1 Recuperators

- In a recuperator, heat exchange takes place between the flue gases and the air through metallic or ceramic walls.
- Duct or tubes carry the air for combustion to be pre-heated, the other side contains the waste heat stream.
- A recuperator for recovering waste heat from flue gases is shown in Fig. 4.6.
- The simplest configuration for a recuperator is the metallic radiation recuperator, which consists of two concentric lengths of metal tubing as shown in Fig. 4.7.
- The inner tube carries the hot exhaust gases while the external annulus carries the combustion air from the atmosphere to the air inlets of the furnace burners.
- The hot gases are cooled by the incoming combustion air which now carries additional energy into the combustion chamber.
- This is energy which does not have to be supplied by the fuel.

- Consequently, less fuel is burned for a given furnace loading.

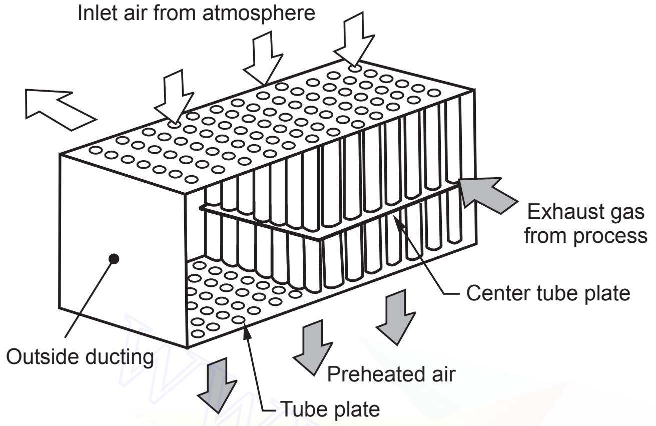

Fig. 4.6 Waste heat recovery using recuperator

- The saving in fuel also means a decrease in combustion air and therefore stack losses are decreased not only by lowering the stack gas temperatures but also by discharging smaller quantities of exhaust gas.
- The radiation recuperator gets its name from the fact that a substantial portion of the heat transfer from the hot gases to the surface of the inner tube takes place by radiative heat transfer.
- The cold air in the annuals, however, is almost transparent to infrared radiation so that only convection heat transfer takes place to the incoming air.
- As shown in the diagram, the two gas flows are usually parallel, although the configuration would be simpler and the heat transfer more efficient if the flows were opposed in direction (or counterflow).
- The reason for the use of parallel flow is that recuperators frequently serve the additional function of cooling the duct carrying away the exhaust gases and consequently extending its service life.
- A second common configuration for recuperators is called the tube type or convective recuperator.
- As seen in the Fig. 4.8, the hot gases are carried through a number of parallel small diameter tubes, while the incoming air to be heated enters a shell surrounding the tubes and passes over the hot tubes one or more times in a direction normal to their axes.
- If the tubes are baffled to allow the gas to pass over them twice, the heat exchanger is termed a two-pass recuperator; if two baffles are used, a three-pass recuperator, etc.
- Although baffling increases both the cost of the exchanger and the pressure drop in the combustion air path, it increases the effectiveness of heat exchange.
- Shell and tube type recuperators are generally more compact and have a higher effectiveness than radiation recuperators, because of the larger heat transfer area made possible through the use of multiple tubes and multiple passes of the gases.

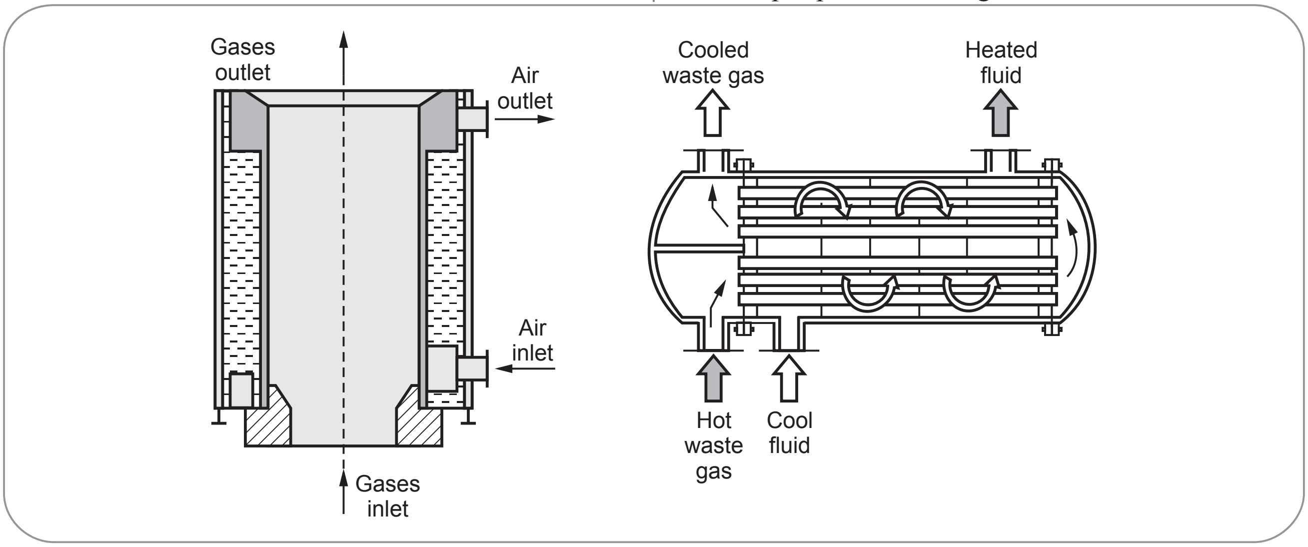

Fig. 4.7 Convective recuperator

4.6.2 Radiation / Convective Hybrid Recuperator

- For maximum effectiveness of heat transfer, combinations of radiation and convective designs are used, with the high-temperature radiation recuperator being first followed by convection type.
- These are more expensive than simple metallic radiation recuperators, but are less bulky.
- A convective/radiative hybrid recuperator is shown in Fig. 4.8.

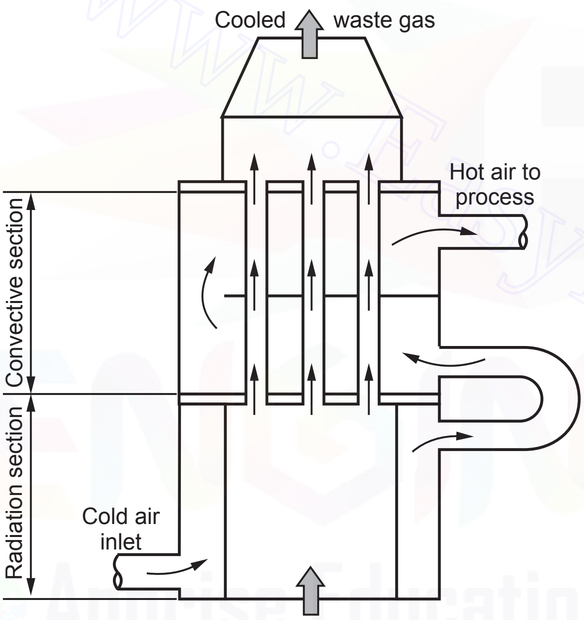

Fig. 4.8 Convective radiative recuperator

4.6.3 Ceramic Recuperator

- The principal limitation on the heat recovery of metal recuperators is the reduced life of the liner at inlet temperatures exceeding 1100 °C.
- In order to overcome the temperature limitations of metal recuperators, ceramic tube recuperators have been developed whose materials allow operation on the gas side to 1550 °C and on the preheated air side to 815 °C on a more or less practical basis.
- Early ceramic recuperators were built of tile and joined with furnace cement, and thermal cycling caused cracking of joints and rapid deterioration of the tubes.
- Later developments introduced various kinds of short silicon carbide tubes which can be joined by flexible seals located in the air headers.
- Earlier designs had experienced leakage rates from 8 to 60 percent. The new designs are reported to last two years with air preheat temperatures as high as 700 °C, with much lower leakage rates.

4.6.4 Regenerator

- The regeneration which is preferable for large capacities has been very widely used in glass and steel melting furnaces.
- Important relations exist between the size of the regenerator, time between reversals, thickness of brick, conductivity of brick and heat storage ratio of the brick.
- In a regenerator, the time between the reversals is an important aspect. Long periods would mean higher thermal storage and hence higher cost.
- Also long periods of reversal result in lower average temperature of preheat and consequently reduce fuel economy. (Refer Fig. 4.9).

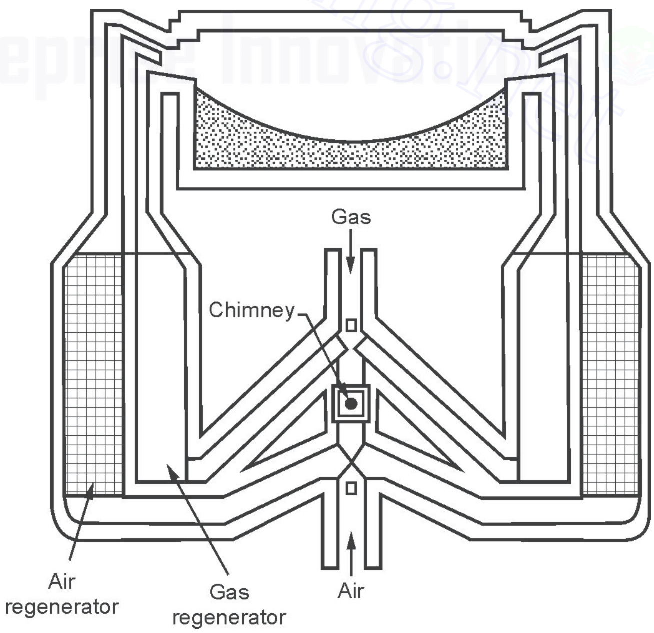

Fig. 4.9 Regenerator

- Accumulation of dust and slagging on the surfaces reduce efficiency of the heat transfer as the furnace becomes old.
- Heat losses from the walls of the regenerator and air in leaks during the gas period and out-leaks during air period also reduces the heat transfer.

4.6.5 Heat Pipe

- A heat pipe can transfer up to 100 times more thermal energy than copper, the best known conductor. In other words, heat pipe is a thermal energy absorbing and transferring system and have no moving parts and hence require minimum maintenance.

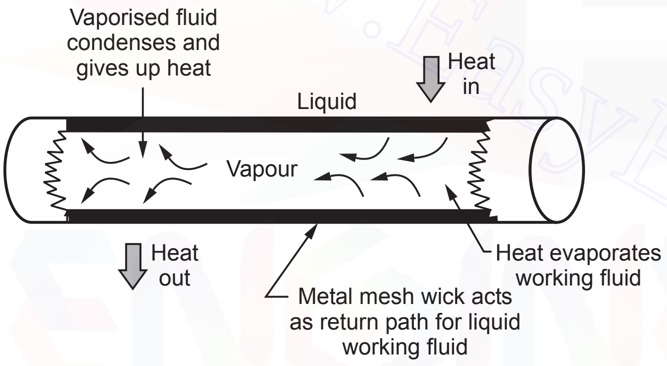

Fig. 4.10 Heat pipe

- The heat pipe comprises of three elements - a sealed container, a capillary wick structure and a working fluid.
- The capillary wick structure is integrally fabricated into the interior surface of the container tube and sealed under vacuum.
- Thermal energy applied to the external surface of the heat pipe is in equilibrium with its own vapour as the container tube is sealed under vacuum.
- Thermal energy applied to the external surface of the heat pipe causes the working fluid near the surface to evaporate instantaneously.
- Vapour thus formed absorbs the latent heat of vapourisation and this part of the heat pipe becomes an evaporator region.
- The vapour then travels to the other end the pipe where the thermal energy is removed causing the vapour to condense into liquid again, thereby giving up the latent heat of the condensation.
- This part of the heat pipe works as the condenser region. The condensed liquid then flows back to the evaporated region. A figure of heat pipe is shown in Fig. 4.10.

4.6.6 Waste Heat Boilers

- Waste heat boilers are ordinarily water tube boilers in which the hot exhaust gases from gas turbines, incinerators, etc., pass over a number of parallel tubes containing water.
- The water is vaporized in the tubes and collected in a steam drum from which it is drawn off for use as heating or processing steam.
- Because the exhaust gases are usually in the medium temperature range and in order to conserve space, a more compact boiler can be produced if the water tubes are finned in order to increase the effective heat transfer area on the gas side.
- The Fig. 4.11 shows a mud drum, a set of tubes over which the hot gases make a double pass, and a steam drum which collects the steam generated above the water surface.
- The pressure of a pure vapor in the presence of its liquid is a function of the temperature of the liquid from which it is evaporated.
- The steam tables tabulate this relationship between saturation pressure and temperature.
- If the waste heat in the exhaust gases is insufficient for generating the required amount of process steam, auxiliary burners which burn fuel in the waste heat boiler or an after-burner in the exhaust gases flue are added. Waste heat boilers are built in capacities from 25 m^3 almost 30,000 m^3 / min. of exhaust gas.

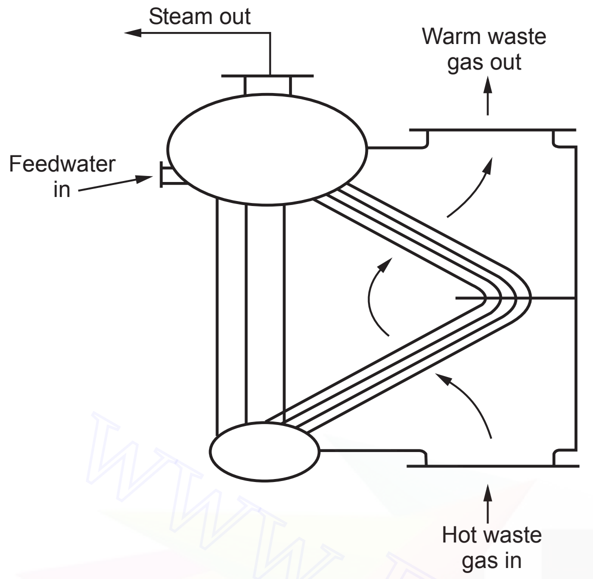

Fig. 4.11 Two-pass water tube waste heat recovery boiler

- Typical applications of waste heat boilers are to recover energy from the exhausts of gas turbines, reciprocating engines, incinerators, and furnaces.

4.6.7 Heat Pumps

- In the various commercial options previously discussed, we find waste heat being transferred from a hot fluid to a fluid at a lower temperature.
- Heat must flow spontaneously "downhill", that is from a system at high temperature to one at a lower temperature.
- When energy is repeatedly transferred or transformed, it becomes less and less available for use.
- Eventually that energy has such low intensity (resides in a medium at such low temperature) that it is no longer available at all to perform a useful function.
- It has been taken as a general rule of thumb in industrial operations that fluids with temperatures less than 120 °C (or, better, 150 °C to provide a safe margin), as limit for waste heat recovery because of the risk of condensation of corrosive liquids. However, as fuel costs continue to rise, even such waste heat can be used economically for space heating and other low temperature applications.
- It is possible to reverse the direction of spontaneous energy flow by the use of a thermodynamic system known as a heat pump.
- The majority of heat pumps work on the principle of the vapour compression cycle.
- In this cycle, the circulating substance is physically separated from the source (waste heat, with a temperature of Tin) and user (heat to be used in the process, Tout) streams, and is re-used in a cyclical fashion, therefore called 'closed cycle'. In the heat pump, the following processes take place :
- In the evaporator the heat is extracted from the heat source to boil the circulating substance.
- The circulating substance is compressed by the compressor, raising its pressure and temperature;
- The low temperature vapor is compressed by a compressor, which requires external work. The work done on the vapor raises its pressure and temperature to a level where its energy becomes available for use.
- The heat is delivered to the condenser.
- The pressure of the circulating substance (working fluid) is reduced back to the evaporator condition in the throttling valve, where the cycle repeats.
- The heat pump was developed as a space heating system where low temperature energy from the ambient air, water, or earth is raised to heating system temperatures by doing compression work with an electric motor-driven compressor. The arrangement of a heat pump is shown in Fig. 4.12.
- The heat pumps have the ability to upgrade heat to a value more than twice that of the energy consumed by the device.
- The potential for application of heat pump is growing and number of industries have been benefited by recovering low grade waste heat by upgrading it and using it in the main process stream.
- Heat pump applications are most promising when both the heating and cooling capabilities can be used in combination.

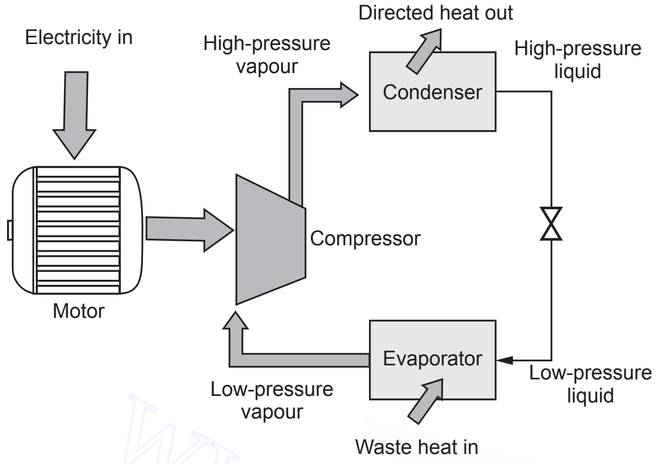

Fig. 4.12. Heat pump

- One such example of this is a plastics factory where chilled water from a heat is used to cool injection-moulding machines whilst the heat output from the heat pump is used to provide factory or office heating.
- Other examples of heat pump installation include product drying, maintaining dry atmosphere for storage and drying compressed air.

4.7 Two Marks Questions with Answers

Q.1 What is meant by duty of a heat exchanger ?

Ans. : It is a device which transfers heat from hot fluid to cold fluid.

Q.2 What is major advantage of waste heat recovery in industry ?

Ans. : It reduces pollution and increases thermal efficiency of the power plant

Q.3 At what temperature the heat recovery system is more effective ?

Ans. : The heat recovery system is more effective when the gas temperature is around 400 ℃

Q.4 Which device is used for billet reheating furnace ?

Ans. : Recuperator is used in billet reheating furnace

Q.5 In counter current mode which device is efficient ?

Ans. : Recuperator will be more efficient if the flow path of hot and cold fluids

Q.6 What are the major limitation of metallic recuperator ?

Ans. : The major limitation of recuperator is reduced life for handling temperature more than 1000 °C

Q.7 What are the direct and indirect benefits of waste heat recovery systems ?

Ans. : Direct Benefits

1. Efficiency of process is increased
2. Reduction in process cost.

Indirect Benefits :

1. Reduction in pollution
2. Reduction in equipment sizes
3. Reduction in auxiliary energy consumption

Q.8 List the points to be considered for developments of WHRS.

Ans. :

1. Sources and uses of waste heat
2. Upset conditions occurring in the plant due to heat recovery
3. Availability of space
4. Any other constraint, such as dew point occurring in an equipments etc.

Q.9 Write any three commercial waste heat recovery devices.

Ans. :

1. Recuperator 2. Economizers

3. WHRSG 4. Heat pumps

Q.10 Define 'heat wheels.

Ans. :

- A heat wheel is a sizable porous disk, fabricated with material having a fairly high heat capacity.
- Which rotates between two side-by-side ducts: one a cold gas duct, the other a hot gas duct.

- The axis of the disk is located parallel to, and on the partition between, the two ducts.

Q.11 Write the major applications of a 'heat wheel.

Ans. :

1. Where heat exchange between large masses of air having small temperature differences is required.
2. Heating and ventilation systems and recovery of heat from dryer exhaust air are typical applications.

Q.12 Define a 'heat pipe'.

Ans. :

1. A heat pipe can transfer up to 100 times more thermal energy than copper, the best known conductor.
2. In other words, heat pipe is thermal energy absorbing and transferring system and have no moving parts and requires minimum maintenance.

Q.13 Write five applications of 'heat pipe'.

Ans. :

1. Cooling of closed rooms with outside air
2. Preheating of boiler feed water with waste heat recovery from flue gases in the heat pipe economizers.
3. Drying, curing and baking ovens
5. Process to space heating
6. Process to Process
7. HVAC applications make up air.
8. Preheating of boiler combustion air
9. Recovery of waste heat from furnaces
10. Reheating of fresh air for hot air driers

Q.14 What is the principle of heat pump ?

Ans. :

- By nature heat must flow spontaneously from a system at high temperature to one at a lower temperature.
- Heat pump reverses the direction of spontaneous energy flow by the use of a thermodynamic system.

Q.15 Give three examples of low temperature air to air heat recovery devices.

Ans. :

a) Heat wheel b) Heat pipe c) Heat pump

Review Questions

1. *Explain with block diagram congeneration power plant.*
2. *State the working principle of congeneration.*
3. *Explain waste heat recovery from recuperators.*
4. *Explain heat pipe and heat pump.*
5. *Explain heat recovery from boiler.*

❑❑❑

UNIT - V

5

Refrigeration and Air Conditioning

Syllabus

Vapour compression refrigeration cycle, Effect of Superheat and Sub-cooling, Performance calculations, Working principle of air cycle, vapour absorption system and Thermoelectric refrigeration. Air conditioning systems, concept of RSHF, GSHF and ESHF, Cooling load calculations. Cooling towers - concept and types.

Contents

5.1 Introduction

- Refrigeration is a process of providing and maintaining temperature below to the atmosphere.
- The medium used for refrigeration process is called as refrigerants.
- The device which is used for the process of refrigeration is called as refrigerator.
- In air cycle refrigeration, air is used as working fluid.
- The basic elements of air refrigeration systems are compressor, condenser, expansion valve and evaporator.

5.2 History of Refrigeration

- In older days, refrigeration was achieved by natural means by using the ice or by evaporative cooling.
- In earlier days ice was transported from colder region or it is generally harvested in winter.
- There are some methods of refrigeration which were used in olden days.

5.2.1 Nocturnal Cooling

- In India, this method is used for making the ice.
- In this method ice was prepared by keeping the layer of water in shallow earthen tray and then these trays were exposed to night sky.
- This method was very popular in India.

5.2.2 Evaporative Cooling

- In this process temperature of the system was reduced by evaporation of water.
- This method is used in India for centuries to obtain cold water in summer season by the use of earthen pots.
- Now a days, desert bags are used in hot and dry areas to provide the cooling in summer days.

5.2.3 Cooling by Salt Solution

- The common salt (NaCl) when added to a water. It dissolves in water and absorbs the heat of solution from water.
- This reduces the temperature of solution (water + salt).
- Now a days, desert bogs are used in hot and dry areas to provide the cooling in summer days.

5.3 Ton of Refrigeration

- The unit of refrigeration is expressed in tonne of refrigeration.
- Tonne of refrigeration is defined as amount of refrigerating effect produced by melting of one tonne of ice from and at 0° C within 24 hrs.

$$1 \text{ TR} = 1000 \times 335 \text{ kJ} \qquad \ldots\ldots (h_{fg})_{ice} = 335 \text{ kJ / kg}$$

$$= \frac{1000 \times 335}{24 \times 60} = 232.6 \text{ kJ / min.}$$

But in actual practice 1TR = 210 kJ / min or 3.5 kW.

5.4 Coefficient of Performance

- The performance of refrigeration system is always expressed in COP.
- It is defined as ratio of refrigerating effect to work of compressor.

$$\text{COP} = \frac{\text{Refrigerating Effect (R.R)}}{\text{Work Done (W.D)}}$$

- Relative COP can be defined as ratio of Actual C.O.P. to theoretical COP.

$$\text{Relative COP} = \frac{\text{Actual COP}}{\text{Theoretical COP}}$$

5.5 Simple Vapour Compression System

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- Fig. 5.1 shows the vapour compression system.
- It mainly consists of four major components
  - i) Compressor : Compression will takes place.
  - ii) Condensor : Condensation will takes place.

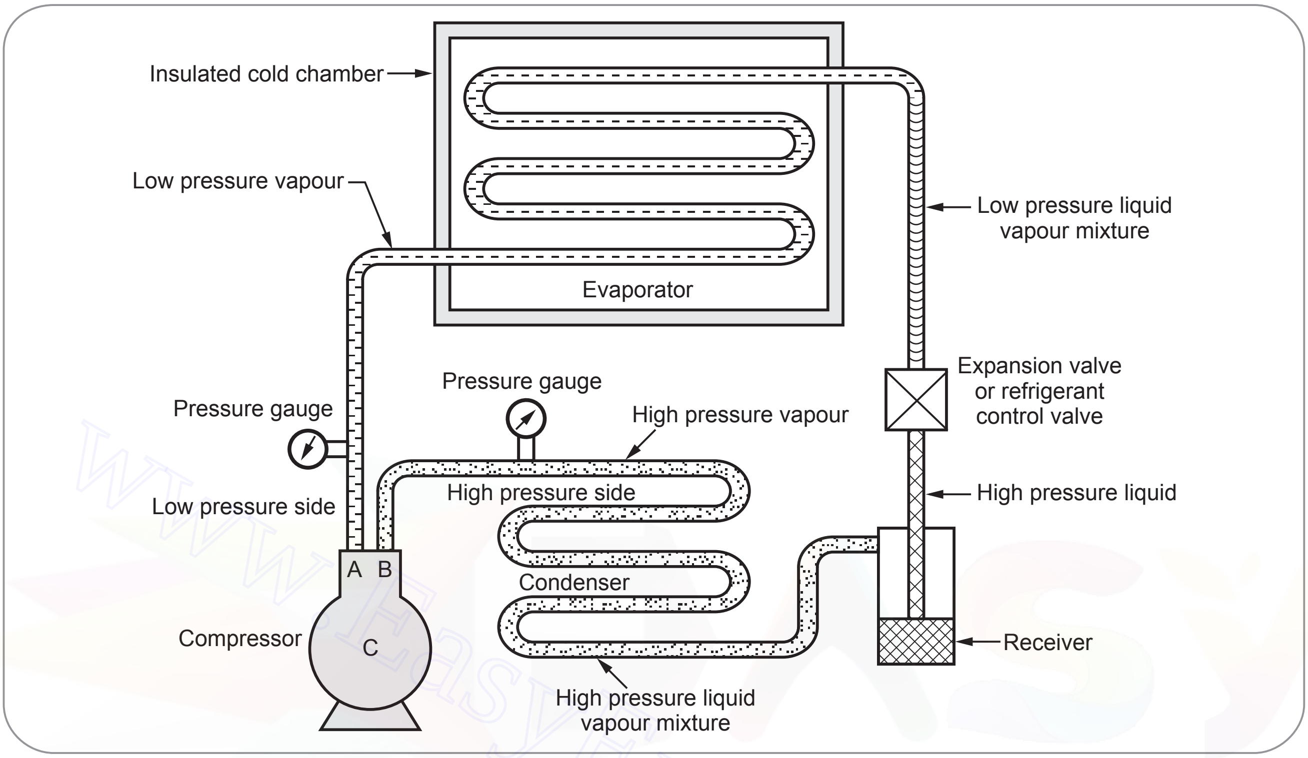


Fig. 5.1 Simple VCC

iii) Expansion device : Expansion will takes place.

iv) Evaporator : Evaporation will takes place.

i) Compressor :

- The low pressure low temperature vapour refrigerant from the evaporator enters into the compressor during suction stroke.
- In the compressor, compression will takes place i.e. low pressure low temperature vapour refrigerants are compressed to high pressure and high temperature.
- This compression can be considered as isentropic compression.
- At the outlet of compressor we get high pressure and high temperature vapour refrigerant.

ii) Condenser :

- High pressure, high temperature vapour refrigerant from the compressor enters into condenser where the condensation will takes place.
- In the condenser, change of phase of refrigerant will takes place. i.e. vapour refrigerants are converted into liquid refrigerant by giving latent heat to the surrounding.

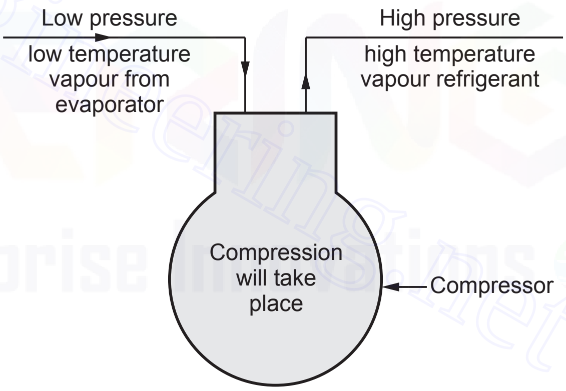


Fig. 5.2 Mechanism of compression

- Generally condensing medium is air or water.
- During the condensation process temperature will remain constant because of latent heating.
- The liquid refrigerant from the outlet of the condenser is stored in the vessel known as receiver. (Refer Fig. 5.3)

iii) Expansion device/valve :

- The expansion valve is also called as throttle valve where the expansion will takes place.
- The liquid refrigerant which is at high pressure and high temperature passes through the expansion valve.

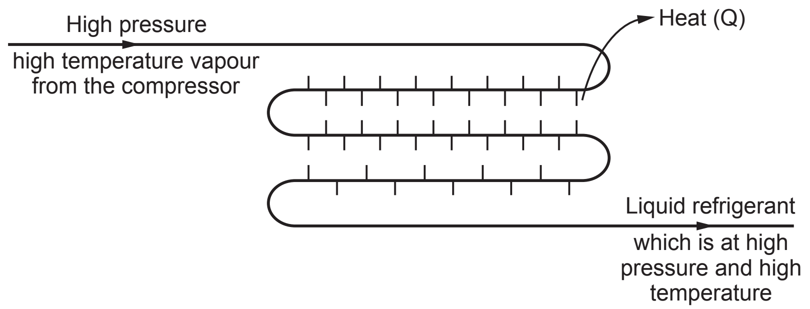


Fig. 5.3 Mechanism of condensation

- The throttling process is pressure relieving process. During this process enthalpy will remain constant.
- Some of the liquid refrigerant evaporates as it passes through the expansionn value and hence at the outlet of expansion valve we get low pressure low temperatue vapour + liquid (mixture) refrigerants.

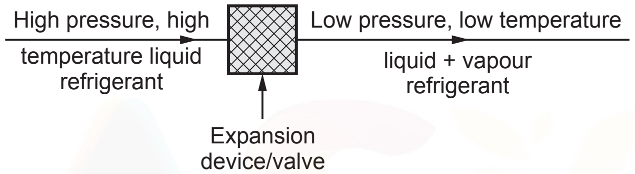


Fig. 5.4 Mechanism of expension

iv) Evaporator

- In the evaporator evaporation will takes place.
- Low pressure, low temperature liquid+vapour refrigerant enters into evaporator where it gets evaporated.
- Finaly from the outlet of the evaporator we will get low pressure, low temperature vapour refrigerants.
- In the evaporating, the liquid + vapour refrigerant absorbs latent heat of vaporisation from the medium (air, water) which is to be cooled.
- Hence we can say during evaporation heat is added and during condensation heat is rejected.

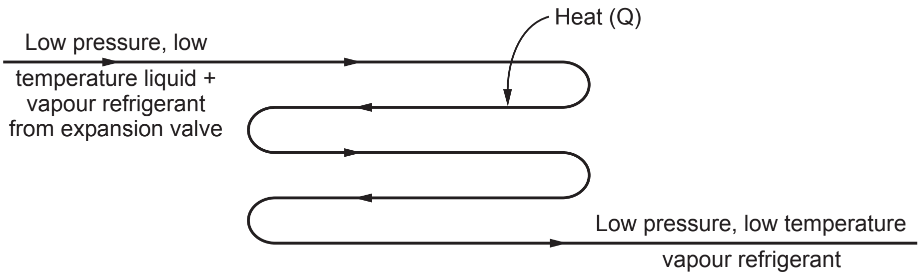


Fig. 5.5 Mechanism of evaporation

5.6 Cases of Vapour Compression System with p-h and T-S Diameter

5.6.1 Theoretical VCC with Dry Saturated Vapour after Compression

- **Processes 1-2** is compression process.
- Initially vapour refrigerants are at low pressure (p_1) and low temperature (T_1) gets compressed isentropically to a high pressure (p_2) and high temperature (T_2) .

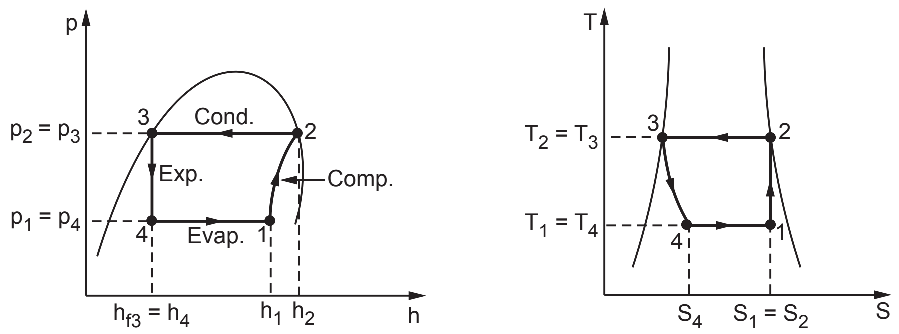


Fig. 5.6 p-h and T- S diagram for dry vapour after compression

- The compression is isentropic the entropy during compression remains constan as shown in T-S diagram i.e. $(S_1 = S_2)$.
- Work done during isentropic compression per kg of refrigerant is given by

$$\boxed{W_c = h_2 - h_1}$$

Where,

h_1 - Enthalpy of refrigerrant at temperature T_1

h_2 - Enthalpy of refrigerrant at temperature T_1

- **Process 2-3** is condensinag process.
- High pressure and high temperature vapour refrigerant gets condensed in condenser.
- During this process latent heat is given out to the surrounding and hence vapour refrigerant gets converted into liquid refrigerant i.e. change of phase of refrigerant takes place.
- From p-h and T-S diagram we can say during this process pressure and temperature remains constant i.e. $p_2 = p_3$ and $T_2 = T_3$.
- **Process 3-4** is an expansion process.
- Liquid refrigerant which is at pressure $p_3 = p_2$ and temperature $T_3 = T_2$ is expanded in expansion valve to low pressure $(p_4 = p_1)$ and low temperature $(T_4 = T_1)$.
- During throttling process no heat is added or rejected by liquid refrigerant.
- **Process 4-1** is an evaporation process.
- In evaporation liquid-vapour mixture of refrigerant is evaporated and changed into low pressure, low temperature vapour refrigerant.
- During this process heat which is absorbed by refrrigerants is called as refrigerating effect. The process of evaporation continues upto point 1 which is the suetion of compressor.
- Heat absorbed or refrigerating effect can be calculated as

$$\boxed{R_E = h_1 - h_4} \quad \text{or} \quad \boxed{R_E = h_1 - h_{f3}}$$

where, h_{f3} = Sensible heat at temperature T_3.

- **Calculation of coefficient of performance (COP)**
- COP is the ratio of refrigerating effect to the work done.

$$\text{C.O.P.} = \frac{\text{Refrigerating effect (RE)}}{\text{Work done}}$$

$$\boxed{\text{COP} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_{f3}}{h_2 - h_1}}$$

- The ratio of C.O.P. of vapour compression cycle to C.O.P. of Carnot cycle is known as performance index (P.I.) or refrigeration efficiency (η_R)
- **Procedure to solve the numerical on dry compression**
- In this type the point '1' is inside the saturation curve. So it is required to find enthalpy at point 1.

$$\therefore \quad \mathbf{h_1 = h_{f1} + x_1 h_{fg1}}$$

In some cases value of latent heat (h_{fg}) is not given then we can use the following relation.

$$\therefore \quad h_{fg1} = h_{g1} - h_{f1}$$

- The same thing is appplicable to find the entropy

$$\therefore \quad S_1 = s_{f1} + x_1 s_{fg1}$$

and $$S_{fg1} = s_{g1} - S_{f1}$$

- COP can be found out,

$$COP = \frac{h_1 - f_{f3}}{h_2 - h_1}$$

Solved Numericals

Ex. 5.1 : *Vapour compression refrigerator works between the pressure limit of 60 bar and 25 bar. The working fluid is just dry at the end of the compression and there is no under cooling of liquid before the expansion valve Determine - i) COP of the cycle and ii) Capaciy of the refrigerator if the fluid flow is at rate of 7 kg/min.*

Pressure	Temp	Enthalpy (kg/kg)		Entropy (kg/kgK)	
(bar)	(K)	Liquid	Vapour	Liquid	Vapour
60	295	151.96	293.29	0.554	1.0332
25	261	56.32	332.58	0.226	1.2464

Sol. : Given data :

$p_2 = p_3 = 60$ bar $\quad h_{g1} = 322.58$ kg / kg

$S_{g1} = 1.2464$ kg/kg K $\quad p_1 = p_4 = 25$ bar

$h_{g2} = h_2 = 293.39$ kg / kg $\quad S_2 = S_{g2} = 1.0332$ kg/kg K

$T_2 = T_3 = 295$ K $\quad h_{f3} = h_4 = 151.96$ kg / kg

$S_{f1} = 0.226$ kg/kg K $\quad T_1 = T_4 = 261$ K

$h_{f1} = 56.32$ kg / kg

- To find entropy at point '1'

$$S_1 = S_{f1} + x_1 S_{fg1}$$

$$S_1 = S_{f1} + x_1 (S_{g1} - S_{f1}) \qquad \ldots S_{fg1} = S_{g1} - S_{f1}$$

$$= 0.226 + x_1 (1.2464 - 0.226)$$

$$S_1 = 0.226 + 1.0204\, x_1 \qquad \ldots(1)$$

- Entropy at point '2'

$$S_2 = S_{g2}$$

But process 1-2 is isentropic compression

$$\therefore \quad S_1 = S_2$$

Putting value of S_{g2} in equation (1).

$$1.0332 = 0.226 + 1.0204\, x_1$$

$$\therefore \quad \mathbf{x_1 = 0.791}$$

- Now to find enthalpy at point '1'.

$$h_1 = h_{f1} + x_1 h_{fg1}$$

$$h_1 = h_{f1} + x_1 (h_{g1} - h_{f1}) \qquad \ldots h_{fg1} = h_{g1} - h_{f1}$$

$$h_1 = 56.32 + 0.791\ (332.58 - 56.32)$$

$$\mathbf{h_1 = 266.93\ kJ/kg}$$

- COP of the cycle,

$$COP = \frac{h_1 - f_{f3}}{h_2 - h_1}$$

$$= \frac{266.93 - 151.96}{293.29 - 266.93}$$

$$= \mathbf{4.36}$$

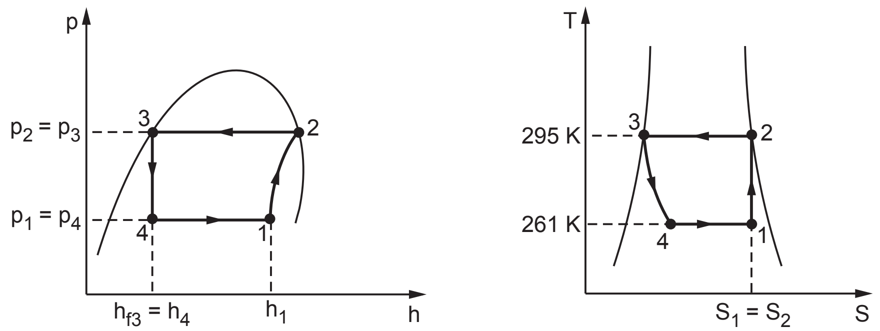


Fig. 4.7

- Capacity of refrigerator,

$$\text{R.E.} = h_1 - h_{f3}$$

$$= 266.93 - 151.96$$

$$= \mathbf{114.97\ kg/kg}$$

We have given fluid flow rate 7 kg/min

$$\therefore \text{Heat extracted} = m \times \text{R.E.} = 7 \times 114.97$$

$$= \mathbf{804.79\ kg/min}$$

$$\therefore \quad \text{Capacity} = \frac{804.79}{210} = \mathbf{3.83\ TR}$$

Ex. 5.2 : *An ammonia refrigerator produces 30 tones of ice at 0 °C from water at 20 °C in a day. The temperature range of working cycle is 25 °C to –15 °C. The vapour is dry saturated at the end of compression. Assuming actual COP of 60 % of theoretical. Calculate the power required to drive the compressor. Latent heat of ice is 335 kJ/kg. Use the following properties of ammonia.*

Temperature	Enthalpy kJ/kg		Entropy of	
	Liquid	Vapour	Liquid	Vapour
25	100.04	1319.22	0.3473	4.4852
– 15 °C	– 54.58	1304.99	– 2.134	5.0585

Sol. : Process 1 - 2 is isentropic compression

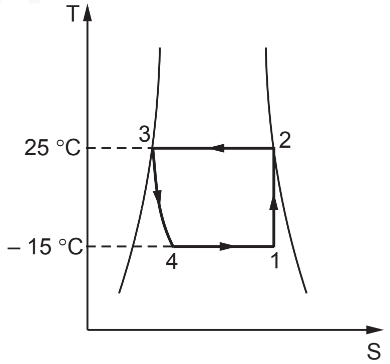


Fig. 5.8

$$\therefore \quad S_1 = S_2$$

$$\therefore \quad S_{f1} + x_1 S_{fg1} = S_{g2}$$

But, $$S_{fg1} = (S_{g1} - S_{f1})$$

$$\therefore \quad S_{f1} + x_1 (s_{g1} - S_{f1}) = S_{g2}$$

$$\therefore -2.134 + x_1 (5.0585 - 2.134) = 4.4352$$

$$\therefore \quad \mathbf{x_1 = 0.92}$$

Now, $$h_1 = h_{f1} + x_1 (h_{g1} - h_{f1})$$

$$= -54.58 + 0.92\,[(1304.99 - (-54.58)]$$

$$= \mathbf{1196.43\ kJ/kg}$$

$$h_2 = h_{g2} = 1319.22\ \text{kJ/kg}$$

$$h_3 = h_{f3} = 100.04$$

Process 3 - 4 is throttling process

$$\therefore \quad h_3 = h_4$$

$$\text{R.E.} = h_1 - h_4$$

$$= 1196.43 - 100.04$$

$$= 1096.18\ \text{kJ/kg}$$

Compressor work,

$$W_C = h_2 - h_1$$

$$= 1319.22 - 1196.43$$

$$= 122.79\ \text{kJ/kg}$$

$$\text{COP} = \frac{\text{Refrigerating effect}}{\text{Work}}$$

$$= \frac{1096.18}{122.79} = 8.92$$

$$\text{Actual COP} = 0.6 \times 8.92 = 5.356$$

Heat extracting per kg of ice,

$$= C_{PW}(20 - 0) + \text{Latent heat of ice}$$

$$= 4.187\,(20) + 335$$

$$= 418.74\ \text{kJ/kg of ice}$$

$$\text{Mass of ice/sec} = \frac{30 \times 1000}{24 \times 60 \times 60}$$

$$= 0.3472\ \text{kg/sec}$$

Actual heat extracted /sec

$$= 418.74 \times 0.7472$$

$$= 145.39\ \text{kJ/sec}$$

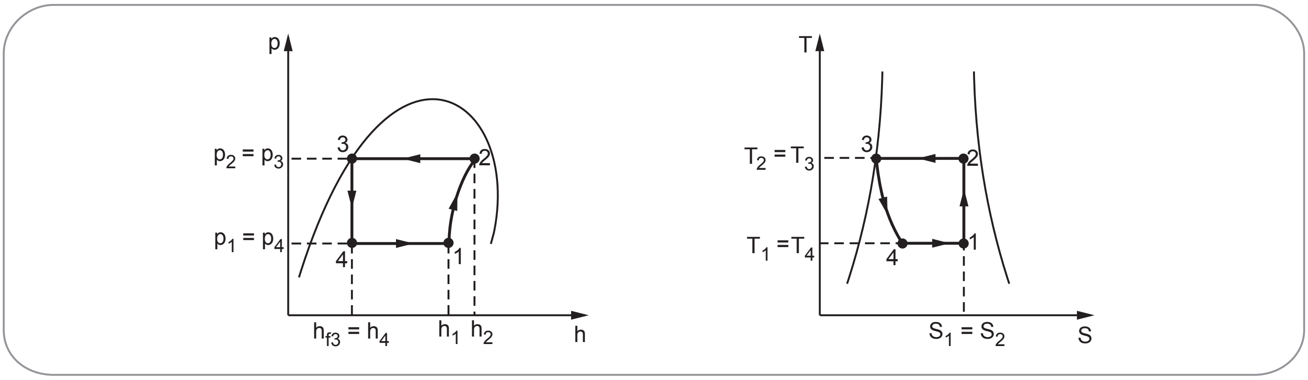


Fig. 5.9 p-h and T-S diagram for wet compression

$$\text{Actual COP} = \frac{\text{Actual heat extracted / sec}}{\text{Actual work / sec}}$$

$$\text{Actual work/sec} = \frac{145.39}{5.348}$$

$$\text{Work/sec} = 27.18 \text{ kW}$$

$$\text{Power} = 27.18 \text{ kW}$$

5.6.2 Theoretical VCC with Wet Vapour after Compression

- In case of wet compressions the point 2 is inside the saturation curve. (Refer Fig. 5.9 below)
- The wet compression is not desirable as liquid droplets in the refrigerant would enter the compressor and damage the valve and other moving parts.
- Wet compression also reduces the heat transfer rate of condenser.
- The enthalpy at point 2 i.e. h_2 can be found out with the help of dryness fraction at that point.
- COP can be found out by following formula.

$$\boxed{COP = \frac{h_1 - h_{f3}}{h_2 - h_1}}$$

- **Procedure to solve the numerical on wet compression.**

The wet compression point '1' and point '2' are inside the saturation curve.

To find entropy at point 1 and 2, following equations are used.

$$S_1 = S_{f1} + \frac{x_1 h_{fg1}}{T_1} \quad \text{...(i)}$$

Similarly
$$S_2 = S_{f2} + \frac{x_2 h_{fg2}}{T_2} \quad \text{..(ii)}$$

In some cases we have to find dryness fraction x_1 or x_2, in such time we can equate two equations (i) and (ii) as process 1-2 is isentropic compression i.e. $S_1 = S_2$.

- To find enthaly at point '1' and at point '2'

$$h_1 = h_{f1} + x_1 h_{fg1}$$

$$h_1 = h_{f1} + x_1 (h_{g1} - h_{f1})$$

Similarly

$$h_2 = h_{f2} + x_2 h_{fg2}$$

$$h_2 = h_{f2} + x_2 (h_{g2} - h_{f2})$$

- To calculate COP,

$$COP = \frac{h_1 - f_{f3}}{h_2 - h_1}$$

Solved Numericals

Ex. 5.3 : *Determine the theoretical COP for* CO_2 *machine working between the temperature range of 25 °C and –5 °C. The dryness fraction if* CO_2 *gas during the suction stroke is 0.6. How many tonnes of ice would machine working between the same limits and having relative COP of 45 % make in 24 hrs ? The water for the ice is supplied at 15 °C and compressor takes 9 kg/min of* CO_2. *Take* c_{pw} *= 4.18 kg/kg °C and latent heat of ice 335 kg/kg.*

Temp.	Heat kg/kg		Latent heat	Entropy (kg/kgK)	
(°C)	Liquid	Vapour	(kJ/kg)	Liquid	Vapor
25	81.17	202.5	121.34	0.251	0.644
–5	–7.53	236.8	245.2	– 0.042	0.841

Sol. : Given data :

$T_1 = T_4 = -5\,°C = 268$ K $\qquad h_{f1} = -7.53$ kJ/kg

$T_3 = T_2 = 25\,°C = 298$ K $\qquad h_{fg1} = 245.2$ kJ/kg

$x_1 = 0.6 \qquad S_{f1} = -0.042$ kJ/kg K

$(COP)_{relative} = 45\ \%\qquad S_{f2} = 0.251$ kJ/kg K

$m = 9$ kg/min $\qquad h_{f2} = 81.17$ kJ/kg

$C_{pw} = 4.18$ kJ/kg °C $\qquad h_{f2} = 121.34$ kJ/kg

$(h_{fg})_{ice} = 335$ kJ/kg

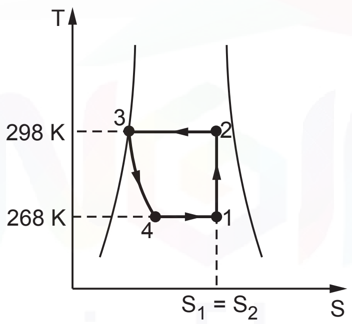


Fig. 5.10

- Entropy at point '1' = Entropy at point '2'

$$S_1 = S_2$$

∴ We can write,

$$S_{f1} + \frac{x_1 h_{fg1}}{T_1} = S_{f2} + \frac{x_2 h_{fg2}}{T_2}$$

$$-0.042 + \frac{0.6 \times 245.2}{(268)} = 0.251 + \frac{x_2 \times 121.34}{(298)}$$

$$\therefore \qquad x_2 = \frac{(0.507 - 0.251)}{0.407}$$

$$\mathbf{x_2 = 0.63}$$

- To find out enthalpy at point '1' and '2'.

$$h_1 = h_{f1} + x_1 h_{fg1}$$

$$= -7.53 + (0.6 \times 245.2)$$

$$\mathbf{h_1 = 139.6}\ \text{kJ/kg}$$

Similarly $\qquad h_2 = h_{f2} + x_2 h_{fg2}$

$$= 81.17 + (0.63 \times 121.34)$$

$$\mathbf{h_2 = 157.6}\ \text{kJ/kg}$$

- Theoretical COP

$$COP = \frac{\text{Refrigerating effect}}{\text{Work done}}$$

$$= \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

$$= \frac{139.6 - 81.17}{157.6 - 139.6}$$

$$= \mathbf{3.24}$$

To find out relative COP

$$(COP)_{relaive} = \frac{(COP)_{actual}}{(COP)_{theoretical}}$$

$$0.45 = \frac{(COP)_{actual}}{3.24}$$

$$\therefore \quad (COP)_{actual} = 0.45 \times 3.24$$

$$= \mathbf{1.458}$$

Heat extracted per minute $= (COP)_{actual} \times [m(h_2 - h_1)]$

$$= 1.458 \times [9(157.6 - 139.6)]$$

$$= \mathbf{236.19\ kJ/min}$$

Heat extracted per day $= 236.19 \times 60 \times 24$

$$= 340122.2\ \text{kJ/day}$$

Heat extracted per kg of water

$$= 1 \times C_{pw}(15 - 0) + (h_{fg})_{ice}$$

$$= 1 \times 4.18(15) + 335 = \mathbf{337.7\ kJ/kg}$$

Mass of ice per day $= \dfrac{340122.2}{397.7 \times 1000}$

$$= \mathbf{0.85\ tonne}$$

Ex. 5.4 : *A refrigerating plant works between temperature limits of – 5 °C and 25 °C. The working fluid is ammonia with dryness fraction of 0.62 at the entry to compressor. If the machine has a relative COP of 55 % calculate the amount of ice formed during a period of 24 hrs. The ice is to be formed at 0 °C from water at 15 °C and 6.4 kg of ammonia is circulated per minute. Take specific heat of water = 4.187 kJ/kg K and latent heat of ice = 335 kJ/kg. Show process on T-S chart.*

Temperature	Enthalpy kJ/kg		Entropy kJ/kg K	
	h_f	h_g	S_k	S_g
25 °C	298.8	1166.15	1.07345	3.9121
– 5 °C	158.5	1279.85	0.6298	4.7738

Sol. : $x_1 = 0.62$

Now we have to find condition of vapour at point 2

$$S_{f2} + x_2 S_{fg2} = S_{f1} + x_1 S_{fg1}$$

$$1.07842 + x_2 \times 3.9121 = 0.62985 + 0.62 \times 4.77385$$

$$\mathbf{x_2 = 0.64}$$

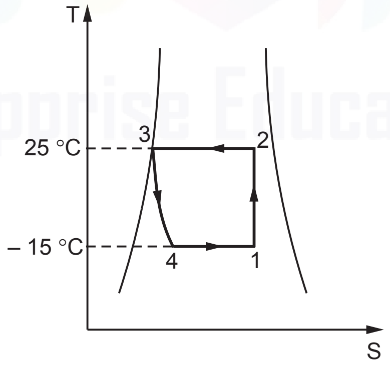


Fig. 5.11

Enthalpy at point (1) and (2) can be calculated as

$$h_1 = h_{f1} + x_1 h_{fg1}$$
$$= 158.5 + 0.62 \times 1279.85$$
$$= 952.01 \text{ kJ/kg}$$
$$h_2 = 298.8 + 0.64 \times 1166.15$$
$$= 1045.13 \text{ kJ/kg}$$
$$h_3 = 298.8 \text{ kJ/kg}$$
$$h_3 = h_4$$

… throttling process enthalpy remains same

Refrigerating effect can be calculated as,

$$\text{R.E.} = h_1 - h_4$$
$$= 952.01 - 298.8$$
$$= 653.21 \text{ kJ/kg}$$

Work of compressor

$$W_C = h_2 - h_1$$
$$= 1045.13 - 952.01$$
$$= 93.126 \text{ kJ/kg}$$

$$\text{Ideal COP} = \frac{\text{R.E.}}{\text{Work done}}$$
$$= \frac{653.21}{93.12} = 7.01$$

$$\text{Actual COP} = 0.55 \times 7.01 = 3.85$$

Heat extracted/kg of ice

$$= C_{Pw}(15 - 0) + \text{Latent heat of ice}$$
$$= 4.187\ (15) + 335$$
$$= 398.8 \text{ kJ/kg of ice}$$

$$\text{Mass of refrigerant per second} = \frac{6.4}{60} = 0.106 \text{ kg/sec}$$

Heat extracted per second = R.E. × Mass flow rate

$$= 653.21 \times 0.106$$
$$= 69.67 \text{ kJ/sec}$$

$$\text{Mass of ice produced/second} = \frac{69.67}{398.8} = 0.174 \text{ kg/sec}$$

$$\text{Ice produced in 24 hrs.} = \frac{0.1747 \times 24 \times 3600}{1000} = 15 \text{ tons}$$

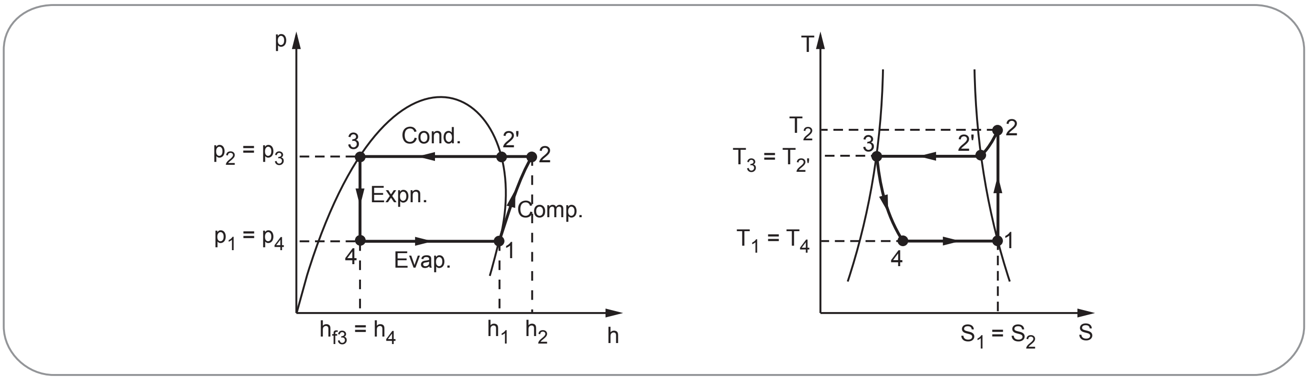


Fig. 5.12 : p-h and T-S diagram for superheated vapour after compression

5.6.3 Superheated Vapour after Compression

- In this cycle enthalpy at point 2 can be found out with the help of degree of superheat and by using following relation. (Refer Fig. 5.12)

$$h_2 = h'_2 + C_p \times (\text{Degree of superheat})$$

$$\boxed{h_2 = h'_2 + C_p \times (T_2 - T'_2)}$$

- In superheating process increase in refrigerating effect is less as compared to work done in compressor.
- Hence net effect of superheating is to have low coefficient of performance.
- COP can be found out by using following relation.

$$COP = \frac{\text{Refrigerating effect}}{\text{Workdone}}$$

$$\boxed{COP = \frac{h_1 - h_{f3}}{h_2 - h_1}}$$

- **Procedure to solve the numericals on superheating**

i) In the numericals of suprheating we have to find the enthalpy at point '2'. It can be found out by using relation

$$\mathbf{h_2 = h'_2 + C_p(T_2 - T'_2)}$$

ii) But in some cases, it is required to find out temperature at point (T_2). It can be found out by using relation

$$\mathbf{S_2 = S'_2 + 2.3\, C_p \log\left(\frac{T_2}{T'_2}\right)}$$

In some cases we have to find out the value of C_p (when T_2 and T'_2 is given in numerical)

iii) To find out the mass flow rate

$$\mathbf{m = \frac{Q}{h_2 - h_{f3}}} \quad \text{OR} \quad \mathbf{m = \frac{Q}{h_1 - h_{f3}}}$$

iv) Theoretical piston displacement of compressor

$$\mathbf{= m \times v_1}$$

v) Theoretical power of compressor

$$\mathbf{P = m(h_2 - h_1)kW}$$

vi) Refrigerating effect

$$\mathbf{R.E. = h_1 - h_{f3}}$$

vii) $$\mathbf{COP = \frac{h_1 - h_{f3}}{h_2 - h_1}}$$

Solved Numericals

Ex. 5.5 : *A VCR uses methyl chloride (R-40) and operates between temp limits of – 10 °C and 45 °C at the entry of compressor the refrigerent is dry saturated and after compression it acquires a temp of 70 °C find the C.O.P. of refrigerator.*

Sat. temp.	Enthalpy (kJ/kg)		Entropy (kJ/kg K)	
in °C	Liquid	Vapour	Liquid	Vapour
– 10	45.4	460.7	0.183	1.637
45	133.0	483.6	0.485	1.587

Sol. : (Refer Fig. 5.13)

h_1 = 460.7 kJ/kg $\quad S_1 = S_2$ = 1.637 kJ/kg K

h'_2 = 483.6 kJ/kg $\quad S'_2$ = 1.587 kJ/kg K

h_{f3} = 133.0 kJ/kg $\quad S_{f3}$ = 0.485 kJ/kg K

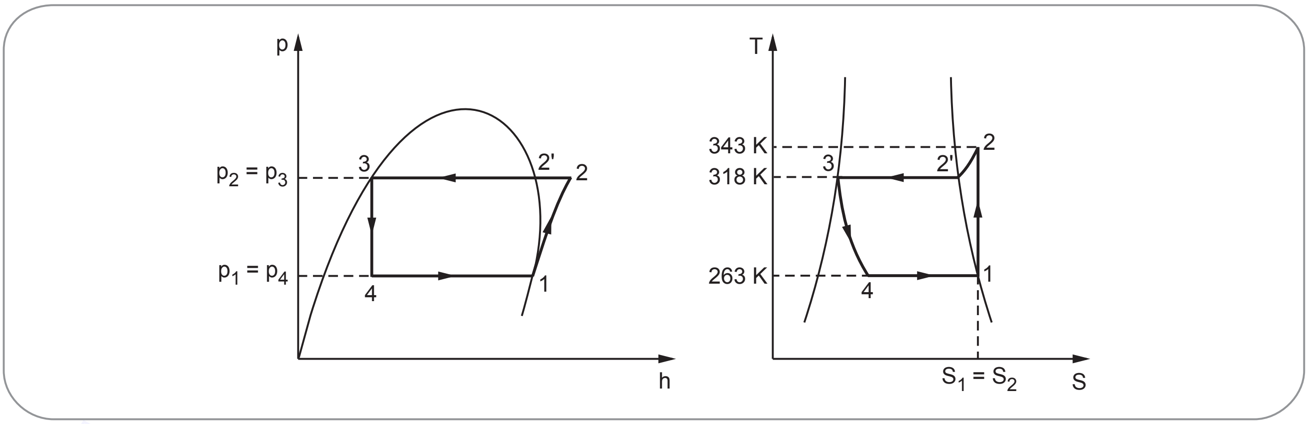


Fig. 5.13

h_{f1} = 45.4 kJ/kg $\quad S_{f1}$ = 0.183 kJ/ kg K

To find out entropy at 2,

$$S_2 = S_2' + 2.3\, C_P \log\left(\frac{T_2}{T_2'}\right)$$

$$1.637 = 1.587 + 2.3\, C_P \log\left(\frac{343}{318}\right)$$

$$C_P = 0.984$$

Enthalpy at point 2

$$h_2 = h_2' + C_P \times (\text{Degree of Superheat})$$

$$= h_2' + C_P\,(T_2 - T_2')$$

$$= 483 + 0.984\,(343 - 318)$$

$$= 507 \text{ kJ/kg}$$

$$\text{COP} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{460.7 - 133}{507 - 460.7} = \mathbf{7.077}$$

Ex. 5.6 *A simple F-12 heat pump for space heating operates between 15 °C and 50 °C. The heat required to be pumped is 200 MJ/hr. Determine i) Moss flow rate of refrigerant ii) Discharge temperature assuming C_p = 0.8 kJ/kg iii) Theoretical power required to run the compressor iv) COP of system.*

Temp.	*Pressure*	*Sp. enthalpy (kJ/kg)*		*Sp. entropy (kJ/kgK)*	
(°C)	*bar*	h_f	h_g	s_f	s_g
15	*4.91*	*50.1*	*193.8*	*0.1915*	*0.6902*
50	*12.19*	*84.9*	*206.5*	*0.3037*	*0.6797*

Sol. : Given data :

(Refer Fig. 5.14)

$$T_1 = T_4 = 15\ °C = 288\ K$$

$$T_3 = T_2' = 50\ °C = 323\ K$$

$$Q = 200 \text{ MJ/hrs}$$

$$= \frac{200 \times 10^3}{3600} = 55.5 \text{ kJ/sec}$$

$$h_{g1} = h_1 = 193.8 \text{ kJ/kg}$$

$$h_2' = 206.5 \text{ kJ/kg}$$

$$h_{f3} = 84.9 \text{ kJ/kg}$$

$$S_{g1} = 0.6902 \text{ kJ/kg K } (S_{g1} = S_1)$$

$$S_{g2} = 0.6797 \text{ kJ/kg K } (S_{g2} = S_2')$$

$$C_p = 0.8 \text{ kJ/kg}$$

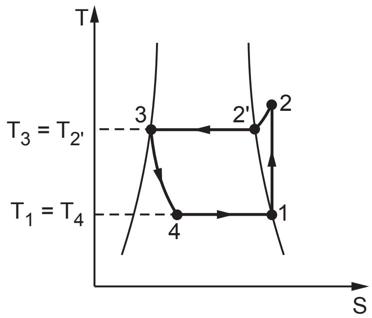


Fig. 5.14

i) Mass flow rate of refrigerant

$$m = \frac{Q}{h_1 - h_{f3}} = \frac{55.55}{193.8 - 84.9}$$

$$\mathbf{m = 0.5101\ kg/sec}$$

ii) Discharge temperature,

Considering isentropic compression 1-2

$$\therefore \quad S_1 = S_2$$

$$\therefore \quad S_2 = S_2 + 2.3\, C_p \log\left(\frac{T_2}{T_2'}\right)$$

$$0.6902 = 0.6797 + 2.3 \times 0.8 \log\left(\frac{T_2}{323}\right)$$

$$\log\left(\frac{T_2}{323}\right) = \frac{0.6902 - 0.6797}{2.3 \times 0.8}$$

$$\log\left(\frac{T_2}{323}\right) = 0.00570$$

Taking antilog

$$\left(\frac{T_2}{323}\right) = 1.0132$$

$$\therefore \quad \mathbf{T_2 = 327.26\ K}$$

iii) Theoretical power required to run compressor.

$$P = m(h_2 - h_1)$$

To find value of h_2.

$$h_2 = h_2' + C_p\,(T_2 - T_2')$$

$$= 206.5 + 0.8\,(327.26 - 323)$$

$$= 209.90 \text{ kJ/kg}$$

$$P = 0.5101(209.90 - 193.8)$$

$$\mathbf{P = 8.21\ kW}$$

iv) $\text{COP} = \dfrac{\text{Refrigerating effect}}{\text{Power used}} = \dfrac{55.55}{8.21} = \mathbf{6.76}$

Ex. 5.7 *Simple saturated VCC using ammonia has a capacity of 25 TR. Evaporator and condenser temperature are –5 °C and 40 °C respectively. Calculate mass flowrate of refrigerant, COP of the system, heat rejected in condenser. What will be the dimension bore and stroke of single acting 4 cylinder compression running at 350 rpm. L/D = 1.1, volumetric efficiency = 0.7 use the following properties sp. heat of ammonia vapour is 2.1897 kJ/kg K.*

Temp (°C)	Pressure bar	V_g m^3/kg	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kg K)	s_g (kJ/kg K)
– 5	3.5571	0.346	176.9	1456.1	0.9154	5.6856
40	15.57	--	390.6	1490.4	1.6437	5.1558

Sol. : Given data :

(Refer Fig. 5.15)

Q = 25 TR No. of cylinder = 4

$= 25 \times 3.52$ $\eta_{vol} = 0.7$

= 88 kW L/D = 1.1

$T_1 = T_4 = -5$ °C $C_p = 2.1897$ kJ/kg

$T_3 = T_2' = 40$ °C N = 350 rpm

$h_1 = h_{g1} = 1456.1$ kJ/kg $S_1 = S_{g1} = 5.6856$ kJ/kg

$h_2' = h_{g2}' = 1490.4$ kJ/kg $S_2' = S_{g2}' = 5.1558$ kJ/kg

$h_3 = h_{f3} = h_4 = 390.6$ kJ/kg

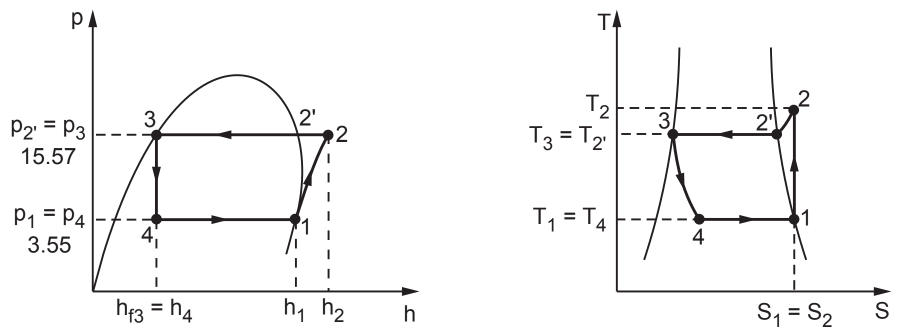


Fig. 5.15

- Mass flow rate of refrigerant,

$$m = \frac{Q}{h_1 - h_{f3}}$$

$$m = \frac{88}{(1456.1 - 390.6)}$$

$$\mathbf{m = 0.0825\ kg/sec}$$

Considering isentropic compression (1 - 2)

$$S_1 = S_2$$

$$S_2 = S_2' + 2.3\, C_p \log\left(\frac{T_2}{T_2'}\right)$$

$$5.6856 = 5.1558 + 2.3 \times 2.1897 \ \log\left(\frac{T_2}{313}\right)$$

$$\frac{5.6856 - 5.1558}{(2.3 \times 2.1897)} = \log\left(\frac{T_2}{313}\right)$$

$$0.1051 = \log\left(\frac{T_2}{313}\right)$$

Taking antilog,

$$1.274 = \left(\frac{T_2}{313}\right)$$

$$\therefore \quad \mathbf{T_2 = 398.78\ K}$$

The find out the enthalpy at point '2'

$$h_2 = h_2' + C_p\,(T_2 - T_2') = 1490.4 + 2.1897\ (398.78 - 313)$$

$$\mathbf{h_2 = 1678.232\ kJ/kg}$$

- COP of system

$$COP = \frac{Q}{W} = \frac{88}{m(h_2 - h_1)} = \frac{88}{0.0825\ (1678.23 - 145.1)} = \mathbf{4.801}$$

- Heat rejected in the condenser,

$$Q_{condensor} = m\,(h_2 - h_3) = 0.0825(1678.23 - 390.6) = \mathbf{106.22\ kW}$$

- Volume flow rate

$$v = m \times v_{s1} = 0.0825 \times 0.346 = \mathbf{0.0285\ m^3/sec}$$

$$v = \frac{\pi}{4} D^2 \times L \times \frac{N}{60} \times \eta_{vol} \times \text{no.of cylinder}$$

$$0.0285 = \frac{\pi}{4} D^2 \times (1.10) \times \frac{350}{60} \times 0.7 \times 4$$

$$D^3 = 2.0197 \times 10^{-3}$$

$$\mathbf{D = 0.126\ m}$$

$$L = D \times 1.1 = 0.126 \times 1.1 = \mathbf{0.1386\ m}$$

Ex. 5.8 *A 2 TR air conditioner based on simple saturate vapour compression cycle operates between 12 ℃ and 50 ℃ Determine i) COP ii) Power per tonn of refrigeration iii) Mass of circulation of refrigerant in kg/hrs. iv) Dryness fraction after throttling. (Solve by using property table for refrigerant R-134 a)*

Sol. : Given data

$$\text{Capacity} = 2\text{TR} = 2 \times 3.52 = 7.04\ \text{kW}$$

$$T_1 = T_4 = 12\ °C = 285\ K$$

$$T_3 = T_2' = 50\ °C = 323\ K$$

From property table (R – 134 a)

$$h_1 = h_g = 405.51\ \text{kJ/kg}$$

$$h_2' = 423.63\ \text{kJ/kg}$$

$$h_3 = h_{f3} = h_4 = 271.59\ \text{kJ/kg}$$

$$S_1 = 1.7215\ \text{kJ/kg K}$$

$$S_2' = 1.7078\ \text{kJ/kg K}$$

$$C_p = 1.218\ \text{kJ/kg}$$

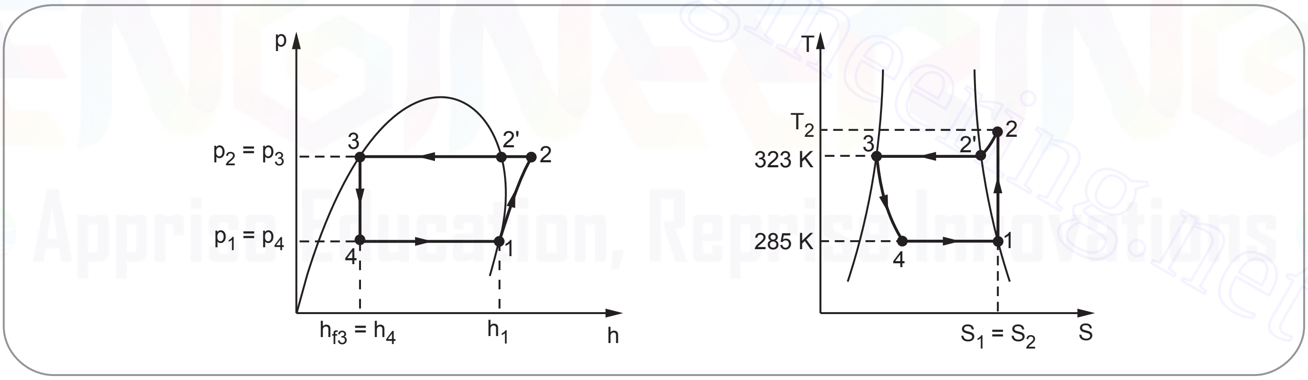


Fig. 5.16

- Process 1-2 is isentropic compression.

$$\therefore \quad S_1 = S_2$$

$$S_2 = S_2' + 2.3\ C_p \log\left(\frac{T_2}{T_2'}\right)$$

$$1.7215 = 1.7078 + 2.3 \times 1.218 \log\left(\frac{T_2}{323}\right)$$

$$\frac{1.7215 - 1.7078}{2.3 \times 1.218} = \log\left(\frac{T_2}{323}\right)$$

$$0.00489 = \log\left(\frac{T_2}{323}\right)$$

Taking antilog

$$1.01132 = \left(\frac{T_2}{323}\right)$$

$$\therefore \quad \mathbf{T_2 = 326.65\ K}$$

- To find enthalpy at point '2'.

$$\therefore \quad h_2 = h_2' + C_p(T_2 - T_2')$$

$$= 423.63 + 1.218\ (326.65 - 323)$$

$$\mathbf{h_2 = 428.07\ kJ/kg}$$

$$COP = \frac{h_1 - h_4}{h_2 - h_1} = \frac{405.51 - 271.59}{428.07 - 405.51} = 5.93$$

- Mass flow rate of refrigerant

$$\dot{m} = \frac{Q}{(h_1 - h_{f3})}$$

$$\dot{m} = \frac{7.04}{(405.51 - 271.59)}$$

$$\mathbf{\dot{m} = 0.0525\ kg/sec}$$

- To find out the power,

$$P = \dot{m}(h_2 - h_1)$$

$$= 0.525\ (428.07 - 405.51)$$

$$\mathbf{P = 1.184\ kW}$$

- In problem we have given 2TR

$$\therefore \text{To find per tonne of refrigration} = \frac{\text{Power}}{2TR} = \frac{1.184}{2}$$

$$= 0.592 \frac{kW}{TR}.$$

- To find dryness fraction after trottling,

$$h_4 = h_{f4} + x_4(h_1 - h_{f4})$$

where, $h_{f4} = 216.384$ kJ/kg

$$271.59 = 216.384 + x_4\ (405.51 - 216.384)$$

$$\mathbf{x_4 = 0.291}$$

Ex. 5.9 *A Freon-12 simple vapour compression system operating at a condenser temperature of 40 °C and evaporator temperature of 0 ºC develops 15 TR using p-h chart for Freon 12, Determine.*

i) Mass flow rate of refrigerant

ii) Theoretical piston displacement of compressor and piston displacement per TR.

iii) Theoretical horse power of compressor

iv) Heat rejected in condenser.

v) Cornot COP and actual COP.

Sol. :

Given data	**From the p-h chart**
$T_3 = T_2 = 40$ ºC	$h_1 = 351$ kJ/kg
$T_1 = T_4 = 0$ ºC	$h_2 = 370$ kJ/kg
Capacity = 15 TR	$hf_3 = h_4 = 238$ kJ/kg
$= 15 \times 3.52$	$v_{s1} = 0.55$
= 52.8 kW	

- Mass flow rate of refrigerant.

$$m = \frac{Q}{h_1 - h_{f3}} \text{ or } m = \frac{Q}{h_1 - h_4}$$

$$\therefore \quad m = \frac{52.8}{(351 - 238)}$$

$$\mathbf{m = 0.467\ kg/sec}$$

Thererical piston displacement of compressor,

$$V = m \times v_{s1} = 0.467 \times 0.55$$

$$= \mathbf{0.256\ m^3/sec}$$

- Piston displacement per TR

We have given 15 TR

$$\therefore \quad v = \frac{V}{15}$$

$$\therefore \quad v = \frac{0.256}{15} = \mathbf{0.01713\ m^3/sec}\ TR$$

- Power consumption,

$$P = m(h_2 - h_1)$$

$$= 0.467\ (370 - 351) = \mathbf{8.873\ kW}$$

• Theoretical horse power of compressor,

$$= \frac{8.873 \times 10^3}{746} = \mathbf{11.894\ HP}$$

• Horse power per TR

$$= \frac{11.894}{15} = \mathbf{0.79\ HP/TR}$$

• Heat rejected in condenser

$$= m(h_2 - h_3) = 0.467(370 - 238)$$

$$= \mathbf{61.64\ kW}$$

$$(COP)_{cycle} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{351 - 238}{370 - 351} = \mathbf{5.94}$$

$$(COP)_{carnot} = \frac{T_1}{T_2 - T_1} = \frac{(0 + 273)}{(40 + 273) - (0 + 273)}$$

$$= \mathbf{6.8}$$

Ex. 5.10 *In simple vapour compression cycle following properties of refrigerant R-12 at various points*

compressor inlet : $h_1 = 183.2\ kJ/kg$, $V_1 = 0.0767\ m^3/kg$

Compressor discharge : $h_2 = 222.6\ kJ/kg$, $V_2 = 0.00164\ m^3/kg$

Condenser exit : $h_{f\,3} = 84.9\ kJ/kg$, $V_3 = 0.00083\ m^3/kg$

The piston displacement for compressor is 1.5 liter per stroke and η_{vol} *80 %. The speed of compressor is 1650 rpm.*

Find :

i) Power rating of compressor (kW)

ii) Refrigerating effect (kW)

Sol. :

Power rating of compressor.

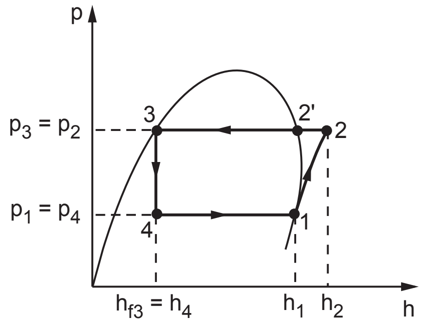


Fig. 5.17

$$\text{Compressor discharge} = \frac{\pi}{4} D^2 \times L \times N \times \eta_{vol}$$

We have given piston displacement volume = 1.5 litre

i.e. $$\frac{\pi}{4} D^2 \times L = 1.5 \text{ litre}$$

$$= 1.5 \times 1000 \times 10^{-6}\ m^3/stroke$$

$$= 0.0015\ m^3/rev$$

$$\text{Compressor discharge} = 0.0015 \times 1650 \times 0.8$$

$$= 1.98\ m^3/\text{min}$$

• Mass flow rate of compressor,

$$m = \frac{\text{Compressor discharge}}{v_1}$$

$$= \frac{1.98}{0.0767}$$

$$= 25.81 \text{ kg/min} = 0.430 \text{ kg/ sec}$$

• Power rating of compressor

$$P = m(h_2 - h_1)$$

$$= 0.430\ (222.6 - 183.2)$$

$$= \mathbf{16.95\ kW}$$

$$\text{Refrigerating effect} = m(h_1 - h_4)$$

$$= 0.430\ (183.2 - 84.9)$$

$$= \mathbf{42.2\ kW}$$

Ex. 5.11 *An ammonia refrigerator operates between – 16 °C and 50 °C. The vapour is dry-saurated at the entry of the compressor and compession is isentropic. Assuming there is no under cooling determine.*

i) The mass flow rate and power input per kW.

ii) COP of system (Refer P-h chart).

Sol. : Given data :

$$T_1 = T_4 = -16\ °C = -16 + 273 = 257\ K$$

$$T_3 = T_2' = 50\ °C = 323\ K.$$

From p-h-chart

$$h_1 = 1418 \text{ kJ/kg}$$

$$h_2 = 1785 \text{ kJ/kg}$$

$$h_{f3} = h_4 = 450 \text{ kJ/kg}$$

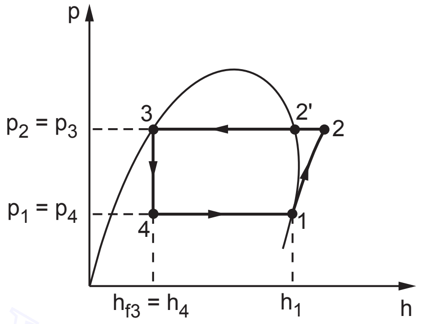


Fig. 5.18

- To find mass flow rate per kW.

Refrigeration capacity $= m(h_1 - h_4) = m(1418 - 450)$

$$\therefore \quad \mathbf{m = 1.033 \times 10^{-3} \text{ kg/sec}}$$

- Power required.

$$P = m(h_2 - h_1) = 1.033 \times 10^{-3} \; (1785 - 1418)$$

$$= \mathbf{0.397 \text{ kW}}$$

$$COP = \frac{h_1 - h_4}{h_2 - h_1} = \frac{1418 - 450}{1785 - 1418} = \mathbf{2.63}$$

Ex. 5.12 *An ammonia refrigerator operates between evaporating and condensing temperatures of – 16 °C and 50 °C respectively. The vapour is dry saturated at the compressor inlet, the compression process is isentropic and there is no undercooling of the condensate. Calculate :*

i) The refrigerating effect per kg.

ii) The mass flow and power input per kW of refrigeration and

iii) The C.O.P. **AU : May-15, Marks 16**

Sol. :

Given Data :

$$T_E = -16 + 273 \text{ K} = 275 \text{ K}, \; T_C = 50 + 273 = 323 \text{ K}$$

To find :

i) Refrigeration effect per kg.

ii) Mass flow and power input per kW of refrigeration.

iii) C.O.P.

From Refrigeration table .

T	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
–16	107.83	1317.45	1425.28	0.4397	5.1242	5.5639
50	421.94	1052.98	1474.92	1.5148	3.2584	4.7732

$$h_1 = h_g = 1425.28 \text{ kJ/kg}$$

For finding 'h_2' we required 'T_2'

$$S_1 = S_2$$

$$S_g = S_g + c_{pg} \times \log_e \left[\frac{T_2}{T_{2'}}\right]$$

$$5.5639 = 4.7732 + 2.8 \log_e \left[\frac{T_2}{50 + 273}\right]$$

$$0.7907 = 2.8 \log_e \left[\frac{T_2}{323}\right]$$

$$0.2823 = \log_e \left[\frac{T_2}{323}\right]$$

$$T_2 = 428.35 \text{ K} \quad T_2 = 155.35°C$$

$$\therefore \quad h_2 = h_g + c_{pg}[T_2 - T_2']$$

$$= 1474.92 + 2.8[155.35 - 50]$$

$$h_2 = 1769.9 \text{ kJ/kg}$$

$$h_3 = h_4 = 107.83 \text{ kJ/kg}$$

$\therefore$ **i) RE**

$$RE = \dot{m}\,(h_1 - h_4) = 1(1425.28 - 107.83)$$

$$\mathbf{RE = 1317.45 \text{ kJ/kg}}$$

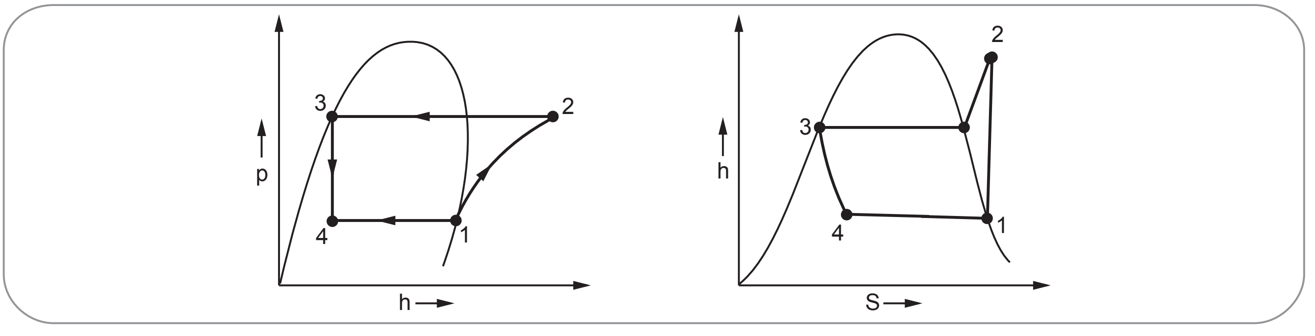


Fig. 5.19

ii) **Mass flow and power input per 'kW' of RE**

$$RE = \dot{m}(h_1 - h_4) \quad \dot{m} = \frac{RE}{(h_1 - h_4)}$$

$$= \frac{1}{(1425.28 - 107.83)}$$

$$\dot{m} = \mathbf{7.590 \times 10^{-4}\ kg/s}$$

$$\therefore \quad P = \dot{m}(h_2 - h_1) = 7.590 \times 10^{-4}(1769.9 - 1425.28)$$

$$P = \mathbf{0.2615\ kW}$$

iii) **C.O.P.**

$$\text{C.O.P.} = \frac{RE}{W_c} = \frac{(h_1 - h_4)}{(h_2 - h_1)} = \frac{(1425.28 - 107.83)}{(1769.9 - 1425.28)}$$

$$\text{C.O.P.} = \mathbf{3.822}$$

Ex. 5.13 *A F_{12} vapour compression refrigeration system has a condensing temperature of 50 °C and evaporating temperature of 0°C. The refrigeration capaciy is 7 tons. The liquid leaving the condenser is saturated liquid and compression is isentropic. Determine i) The refrigerant flow rate ii) The power required to run the compressor iii) The heat rejected in the plant and iv) COP of the system Use the properties of F_{12} as listed in the table.*

AU : May-18, Marks 13

Temp (°C))	Pressure (bar)	h_f (kJ/kg)	h_g (kJ.kg)	S_t (kJ.kg K)	S_R (kJ.kg K)
50	12.199	84.868	206.298	0.3034	0.6792
0	3.086	36.022	187.397	0.1418	0.6960

Sol. :

Given data :

$$h_3 = h_4 = 80.868 \text{ kJ/kg}$$
$$h_1 = 187.39 \text{ kJ/kg.}$$
$$h_2 = 210 \text{ kJ/kg.}$$

Capacity = 7 TR

i) Refrigerant flow rate

$$m = \frac{\text{Refrigerating capacity}}{h_1 - h_4}$$

$$= \frac{7 \times 3.5}{187.397 - 84.868} = 0.24 \text{ kg/sec.}$$

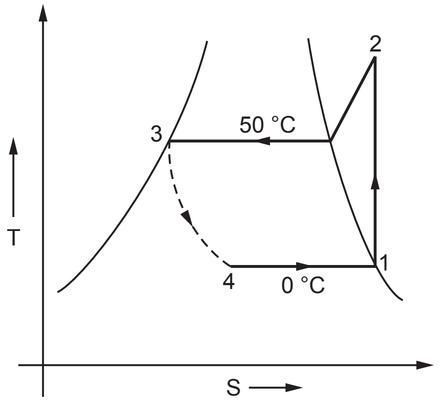


Fig. 5.20

ii) Compressor power required

$$p = m(h_2 - h_1) = 0.24(210 - 187.397)$$

$$p = 5.4 \text{ kW}$$

iii) Heat rejected in the plant

$$Q_R = m(h_2 - h_3) = 0.24(210 - 84.868)$$

$$p = 29.9 \text{ kW.}$$

iv) $$\text{COP} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{187.397 - 84.868}{210 - 187.397} = 4.54$$

5.6.4 Superheated Vapour before Compression

- In simple VCC at the entry of compressor there must be a dry saturated vapour.
- If some liquid refrigerant enters the suction line of compressor, due to wet compression. lubriating oil which is present in the compressor will be washed off and will caues more wear and tear of compressor.
- Hence the life of the compressor reduces.
- In order to increase the life of the compressor refrigerant vapour which is coming of the evaporator is allowed to stay for some more time in evaporator to ensure dry saturated vapour at evaporator exit.
- Thus we can define suprheating as, increasing the temperature of vapour refrigerant more than saturation temperature in evaporator.
- Superheating is shown in P-h and T-S diagram by process 1′ - 1

$$\text{COP} = \frac{\text{Refrigerating effect}}{\text{Work done}}$$

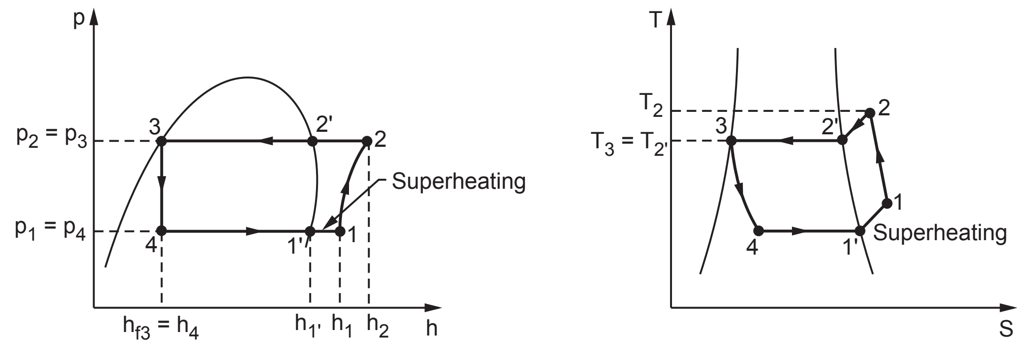


Fig. 5.21 p-h and T-S diagram for super heated vapour before compression

$$\therefore \qquad \boxed{COP = \frac{h_1 - h_{f3}}{h_2 - h_1}}$$

- **Procedure to solve numericals on superheating before compression**
- In such types of numericals, it is required to find the enthalpy at point '1'

$$\mathbf{h_1 = h_1' + C_p(T_1 - T_1')}$$

- Simillarly we can find the enthalpy at point '2'

$$\mathbf{h_2 = h_2' + C_p(T_2 - T_2')}$$

- To find entropy a point '1'

$$\mathbf{S_1 = S_1' + 2.3\,C_p \;\log\left(\frac{T_1}{T_1'}\right)}$$

- Sometimes it is required to find temperature at point '2' it can be found out by using following equation.

$$\mathbf{S_2 = S_2' + 2.3\,C_p \;\log\left(\frac{T_2}{T_2'}\right)}$$

- To find heat rejection in condenser.

$$(Q)_{reject} = m(h_2 - h_3)$$

Solved Numericals

Ex. 5.14 *A vapour compression refrigeration plant works between pressure limit of 5.3 bar and 2.1 bar. The vapour is superheated at the end of compression its temperatue being 37 ºC. The vapour is supperheated by 7 ºC before entering to the compressor. Take C_p = 0.63 kJ/kg K. Find COP.*

Pressure (bar)	Temp (ºC)	Liquid heat (kJ/kg)	Latent heat (kJ/kg)
5.3	+ 15.5	56.15	144.9
2.1	– 14.00	25.12	158.7

Sol. : Given data

$$p_2 = p_3 = 5.3 \text{ bar} \qquad h'_{f1} = 25.12 \text{ kJ/kg}$$

$$p_1 = p_4 = 2.1 \text{ bar} \qquad h'_{fg1} = 158.7 \text{ kJ/kg}$$

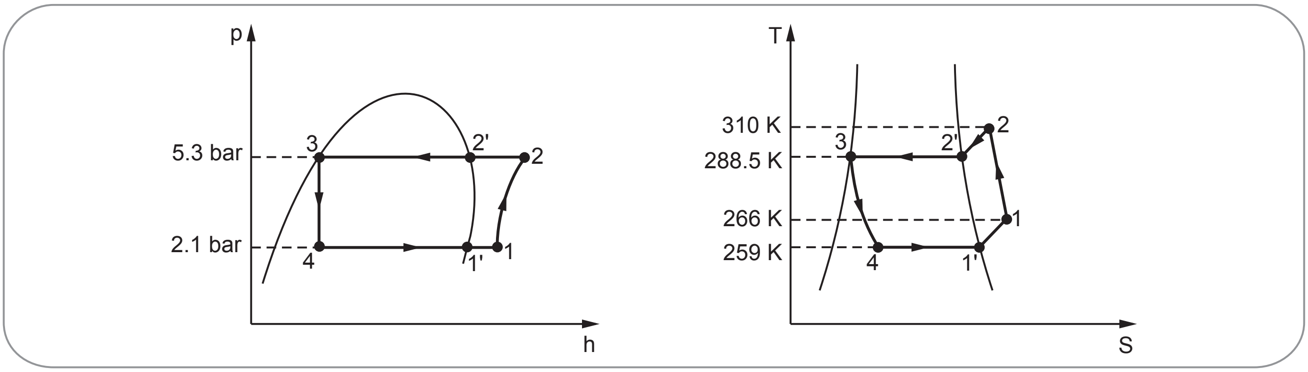


Fig. 5.22

$$T_2 = 37\ ^\circ C = 37 + 273 = 310\ K \qquad h'_{fg2} = 144.9\ kJ/kg$$

$$T_1 - T'_1 = 7\ ^\circ C \qquad h_{f3} = 56.15\ kJ/kg$$

$$T'_1 = -14\ ^\circ C = 259\ K$$

$$T'_2 = 15.5 + 273 = 288.5\ K$$

- To find enthalpy at point '1'

$$h_1 = h'_1 + C_p\,(T_1 - T'_1)$$

To find the value of h'_1

$$h'_1 = h'_{f1} + h'_{fg1} = 25.12 = 158.7 = 183.82\ kJ/kg$$

Now, $$h_1 = h'_1 + C_p\,(T_1 - T'_1) = 183.82 + 0.63\ (266 - 259)$$

$$\mathbf{h_1 = 188.23\ kJ/kg}$$

- Similarly to find out enthalpy at point '2'

$$h_2 = h'_2 + C_p\,(T_2 - T'_2).$$

Now to find.

$$h'_2 = h'_{f2} + h'_{fg2} = 56.15 + 144.9 = 201.05\ kJ/kg$$

Now,

$$h_2 = 201.05 + 0.63\ (310 - 288.5) = \mathbf{214.59\ kJ/kg}$$

- To find COP

$$COP = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{188.23 - 56.15}{214.6 - 186.97} = \mathbf{4.78}$$

Ex. 5.15 *A refrigerating machine using F-12 as working fluid works between temperature –18 ºC and 37 ºC. The enthalpy of liquid at 37 ºC is 78 kJ/kg,. The enthalpy of F-12 entering and leaving the compressor are 200 kJ/kg and 238 kJ/kg respectively. The rate of circulation of refregerant is 5 kg/min and efficiency of compressor is 0.90. Determine.*
i) Capacity of plant in TR ii) Power required to run plant

Sol. :

Given data :

$$T'_1 = T_4 = -18\ ^\circ C = 255\ K \qquad h_3 = 78\ kJ/kg$$

$$T_2' = T_3 = 37\ ^\circ C = 310\ K \qquad h_1 = 200\ kJ/kg$$

$$m = 5\ kg/min = 0.083\ kg/sec \qquad h_2 = 280\ kJ/kg$$

$$\eta_c = 0.90$$

- To find capacity of plant in TR

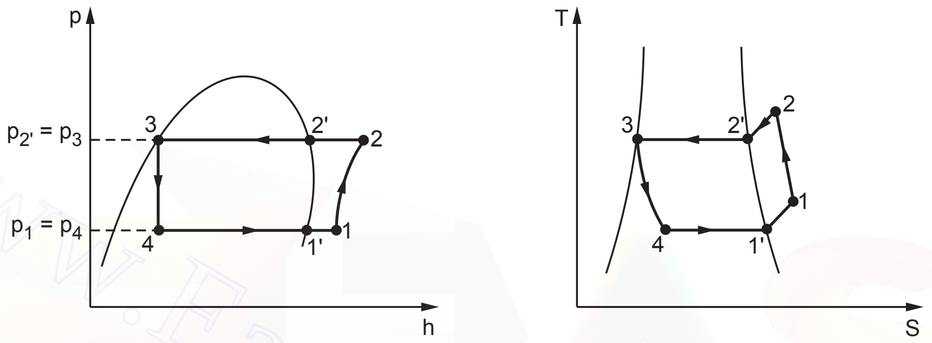


Fig. 5.23

$$= \frac{m(h_1 - h_4)}{3.5} = \frac{0.083(200 - 78)}{3.5} \qquad \text{.... } (h_3 = h_4)$$

$$= \mathbf{2.893\ TR}$$

- Power required to run plant

$$P = \frac{m(h_2 - h_1)}{\eta_c} = \frac{0.083(238 - 200)}{0.90} = \mathbf{3.5\ kW}$$

5.6.5 Undercooling or Subcooling of Refrigerant

- The process of cooling the refrigerant below the condensing temperature for a given pressure is known as undercooling or subcooling.
- The process 3'-3 represents the undercooling.
- The effect of subcooling is to increase the refrigerating effect and therefore COP of the system increases.

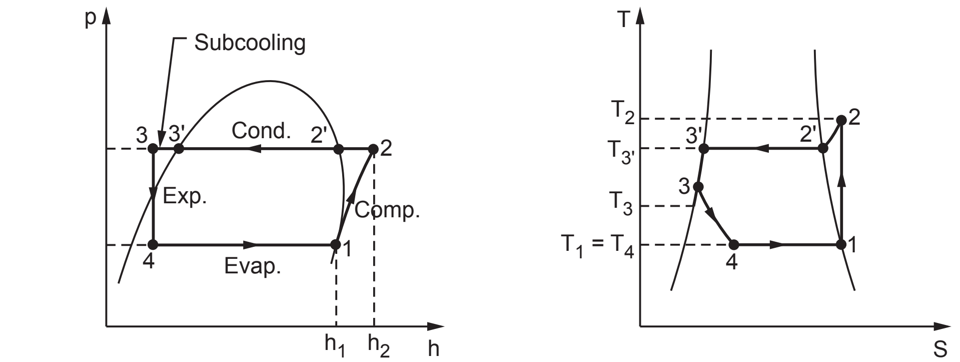


Fig. 5.24 p-h and T-S diagram for subcooling and under cooling

- In subcooling process refrigerating effect can be found out by using following relation.

$$\text{R.E.} = h_1 - h_{f3} \quad \text{or} \quad \text{R.E.} = h_1 = h_4$$

- In above case, value of h_{f3} can be found out as

$$h_{f3} = h'_{f3} - Cp \times (\text{Degree of subcool})$$

$$\therefore \quad \boxed{h_{f3} = h'_{f3} - C_p \times (T'_3 - T_3)}$$

$$\text{COP} = \text{Refrigerating effect / Work done}$$

$$\therefore \quad \boxed{\text{COP} = \frac{h_1 - h_{f3}}{h_2 - h_1}}$$

- **Procedure to solve numericals on subcooling**
- To find the enthalpy at point '2'

$$\mathbf{h_2 = h'_2 + C_{p_v} (T_2 - T'_2)}$$

- To find enthalpy at point '3'

$$\mathbf{h_3 = h'_{f3} - C_{p_l} (T'_3 - T_3)} \qquad \ldots(h_3 = h_{f3})$$

- To find the entropy at point '2'

$$\mathbf{S_2 = S'_2 + 2.3\, C_{p_v} \log\left(\frac{T_2}{T'_2}\right)}$$

- To find work during the compression

$$\mathbf{w = m(h_2 - h_1)}$$

- To find refrigerating effect

$$\mathbf{R.E. = m(h_1 - h_4)}$$

- To find COP of system

$$\text{COP} = \frac{\text{R.E.}}{\text{W.D.}} = \frac{(h_1 - h_4)}{(h_2 - h_1)}$$

Solved Numericals

Ex. 5.16 *A vapour compression refrigeration plant operates between evaporator and condenser temperature at – 15°C and 40°C respectively. The refrigerant is dry and saturated at the suction discharge temperature of refrigerant is 98°C. The bore and stroke of compressor are 85 mm each. It runs at 750 rpm with η_{vol} 82 %. The liquid enters the expansion value at 32°C. Calculate, i) COP ii) Mass of flow of refrigerant iii) Capacity. C_{Pl} = 1.62 and C_{Pv} = 0.96 kJ/kg K.*

Temperature °C	V_g m^3/kg	h_f (kJ/kg)	h_g (kJ/kg)	S_f (kJ/kg K)	S_g (kJ/kg K)
– 15	0.24	43.4	458.7	0.18	1.742
40	0.043	131	468.6	0.48	1567

Sol. : Given data :

$T_1 = T_4 = -15\ °C = 258\ K$ $\qquad$ $h_1 = h_{g1} = 458.7$ kJ/kg

$T_3' = T_2' = 40^\circ C = 313\ K \qquad h_2' = h_{g2}' = 468.6\ kJ/kg$

$T_2 = 98^\circ C = 371\ K \qquad h_{F3}' = 131.0\ kJ/kg$

$T_3 = 32^\circ C = 305\ K \qquad d = 85\ mm$

$L = 85\ mm \qquad N = 750\ rpm$

$\eta_{vol} = 82\ \% \quad C_{Pl} = 1.62$ and $C_{Pv} = 0.96\ kJ/kg\ K$

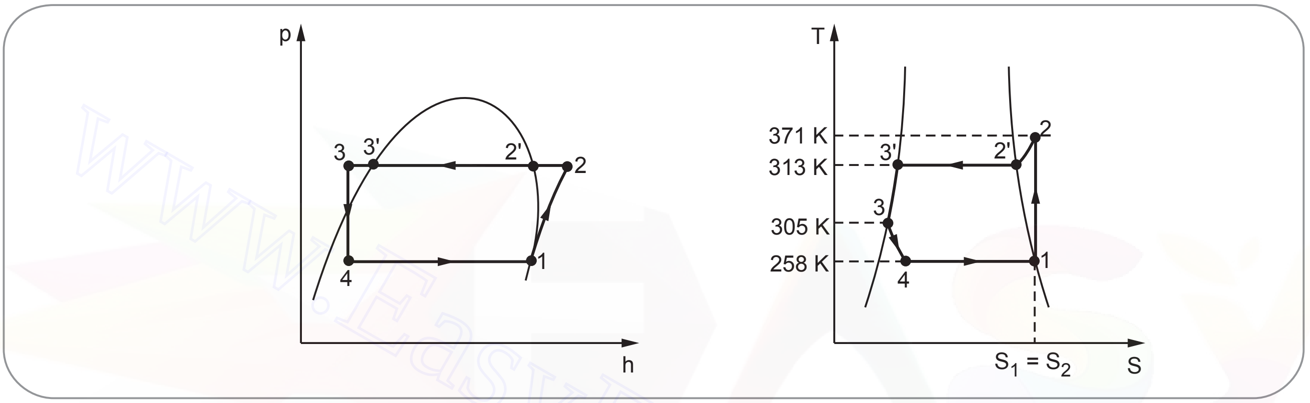


Fig. 5.25

- To find enthalpy at point '2'.

$$h_2 = h_{g2}' + C_{Pv}\ (T_2 - T_2') = 468.6 + 0.96\ (371 - 313)$$

$$\mathbf{h_2 = 524.28\ kJ/kg}$$

- To find enthalpy at point '3'.

$$h_3 = h_{f3}' - C_{Pl}\ (T_3' - T_3) = 131 - 1.62\ (313 - 305)$$

$$\mathbf{h_3 = 118.04\ kJ/kg}$$

- COP of the system

$$COP = \frac{R.E.}{\text{Work done}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_3}{h_2 - h_1} \qquad \ldots h_4 = h_3$$

$$= \frac{458.7 - 118.04}{524.28 - 458.7} = \mathbf{5.194}$$

- Volume flow rate of refrigerant

$$V = \frac{\pi}{4} D^2\ L \times \frac{N}{60} \times \eta_{vol} = \frac{\pi}{4}(0.085)^2 \times 0.085 \times \frac{750}{60} \times 0.82 = \mathbf{4.943 \times 10^{-3}\ m^3/sec}$$

$$m = \frac{V}{V_{g1}} = \frac{4.943 \times 10^{-3}}{0.24} = 0.020\ kg/sec$$

$$\text{Capacity} = m\ (h_1 - h_4)$$

$$= 0.020\ (458.7 - 118.04) = 7.153\ kW = \mathbf{2.03\ TR}$$

Ex. 5.17 *A vapour compression system operates between – 10° C and 45° C. The refrigerant is dry-saturated at the entry of the compressor and attains 102° C after compression. The temperature of liquid refrigerant at the entry of throttle valve is 35° C. Take C_{Pl} = 1.62 kJ/kg K and C_{Pv} = 1.06 kJ/kg K. Determine COP of system.*

Temperature °C	h_f (kJ/kg)	h_g (kJ/kg)	S_f (kJ/kg K)	S_g (kJ/kg K)
– 10	45.4	460.7	0.183	1.762
45	133	483.6	0.485	1.587

Sol. : Given data :

$$T_1 = T_4 = -10\text{ °C} = 263\text{ K} \qquad h_1 = h_{g1} = 460.7\text{ kJ/kg}$$

$$T_3' = T_2' = 45°\text{ C} = 318\text{ K} \qquad S_1 = 1.762\text{ kJ/kg K}$$

$$T_3 = 308\text{ K} \qquad h_{f3}' = 133\text{ kJ/kg}$$

$$C_{Pl} = 1.62\text{ kJ/kg K} \qquad S_{g2} = 1.587\text{ kJ/kg K}$$

$$T_2 = 102\text{ °C} = 102 + 273 = 375\text{ K}$$

- The process 1-2 is isentropic compression

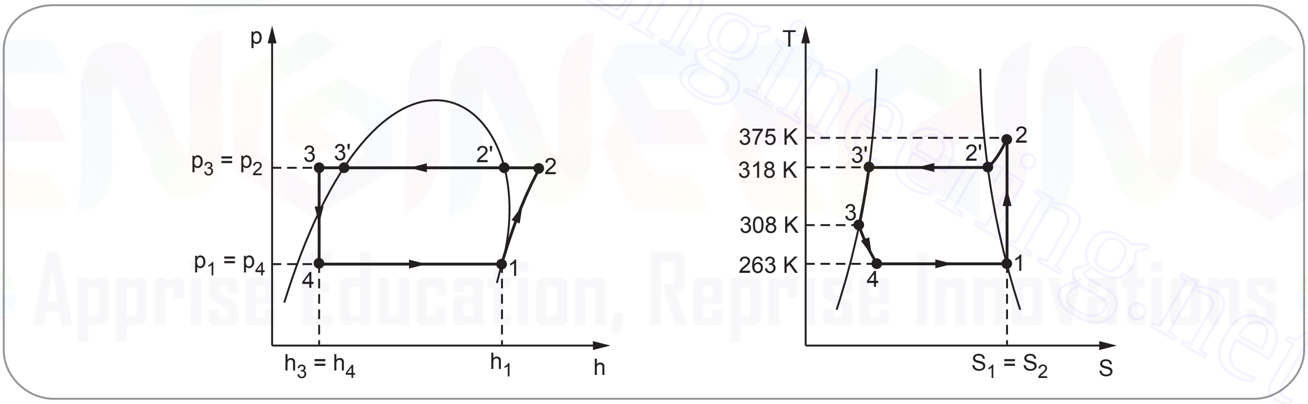


Fig. 5.26

$\therefore$ $$S_1 = S_2$$

In this numerical the value of C_{Pv} is not given

- To find the value of C_{Pv}

$$S_2 = S_2' + 2.3 \times C_{Pv} \times \log\left(\frac{T_2}{T_2'}\right)$$

But $$S_1 = S_2$$

$\therefore$ $$1.762 = 1.587 + 2.3 \times C_{Pv} \times \log\left(\frac{375}{318}\right)$$

$$C_{Pv} = \frac{1.762 - 1.587}{0.165} = \mathbf{1.06\ kJ/kg\ K}$$

- To find enthalpy at point '2'.

$$h_2 = h'_2 + C_{Pv}(T_2 - T'_2) = 483.6 + 1.06(375 - 318) = \mathbf{538\ kJ/kg}$$

- To find the value at h_3

$$h_3 = h'_{f3} - C_{Pl}(T'_3 - T_3) = 133 - 1.62(318 - 308) = \mathbf{149.2\ kJ/kg}$$

- The process 3-4 is throttling process

$$\therefore \quad h_3 = h_4 = 149.2 \text{ kJ/kg}$$

- COP of the system

$$\text{COP} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{460.7 - 149.2}{538 - 460.7} = \mathbf{4.03}$$

Ex. 5.18 *A food storage locker requires a refrigeration capacity of 60 kW. It works between a condenser temperature of 35° C and evaporator temperature of – 10° C. The refrigerant is ammonia. It is subcooled by 7° C before entering the expansion valve by dry saturated vapour leaving the evaporator. Assuming single cylinder single acting compressor operating at 1000 rpm with stroke equal to 1.2 times the bore determine Assume specific volume 0.417477 m^3/kg.*

i) Power required ii) The cylinder dimensions

Temperature °C	Enthalpy (kJ/kg)		Entropy (kJ/kg K)		Sp. heat (kJ/kg K)	
	Liquid	Vapour	Liquid	Vapour	Liquid	Vapour
– 10	154.056	1450.22	0.8296	5.7550	--	2.492
35	366.072	1488.57	1.5660	5.2086	4.556	2.903

Sol. : Given data :

$$\text{Capacity} = 60 \text{ kW} \qquad h_1 = 1450.22 \text{ kJ/kg}$$

$$T'_3 = T'_2 = 35° C = 308 \text{ K} \qquad S_1 = 5.7550 \text{ kJ/kg K}$$

$$T_1 = T_4 = -10° C = 263 \text{ K} \qquad S'_2 = 5.2086 \text{ kJ/kg K}$$

$$h'_2 = 14.88.57 \text{ kJ/kg}$$

$$T_3 = 35° C - 7° C = 28° C = 301 \text{ K} \qquad h'_{f3} = 366.072 \text{ kJ/kg}$$

$$N = 1000 \text{ rpm} \qquad L = 1.2 \times D$$

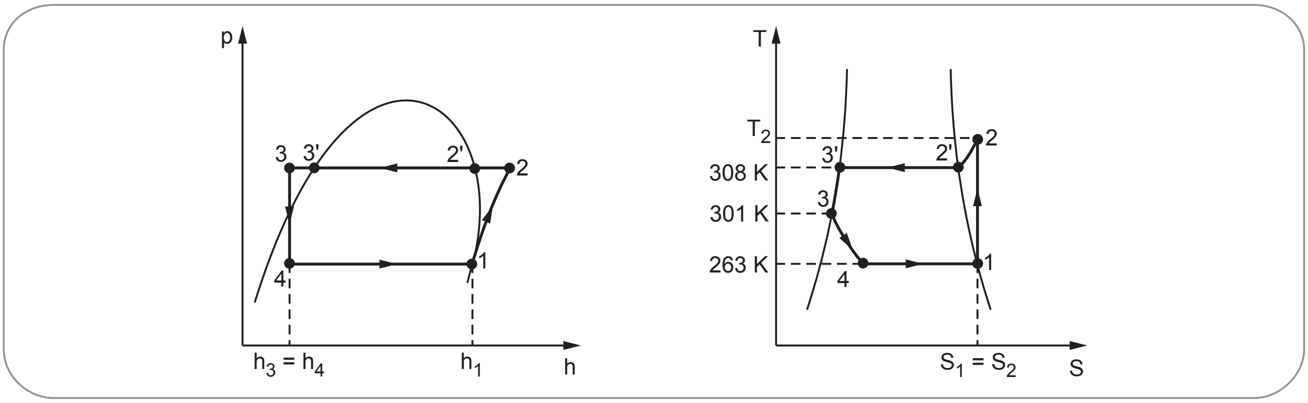


Fig. 5.27

- The process 1-2 is isentropic process,

$$\therefore \quad S_1 = S_2$$

$$S_2 = S'_2 + 2.3 \times C_{Pv} \log\left(\frac{T_2}{T'_2}\right)$$

$$\therefore \quad 5.755 = 5.2086 + 2.3 \times 2.903 \log\left(\frac{T_2}{308}\right)$$

$$\frac{5.755 - 5.2086}{2.3 \times 2.903} = \log\left(\frac{T_2}{308}\right)$$

$$0.08183 = \log\left(\frac{T_2}{308}\right)$$

Taking antilog

$$1.2073 = \left(\frac{T_2}{308}\right)$$

$$\mathbf{T_2 = 371.86\ K}$$

- To find enthalpy at point '2'.

$$h_2 = h'_2 + C_{Pv}\ (T_2 - T'_2) = 1488.57 + 2.903\ (371.86 - 308)$$

$$= \mathbf{1673.96\ kJ/kg}$$

- To find enthalpy at point 3

$$h_3 = h'_{f3} - C_P\ (T'_3 - T_3) = 366.07 - 4.556\ (308 - 301)$$

$$\mathbf{= 334.178\ kJ/kg}$$

- Mass of refrigerant flowing

$$m = \frac{Q}{h_1 - h_3} = \frac{Q}{h_1 - h_4} = \frac{60}{1450.22 - 334.178} = 0.0537\ \text{kg/sec}$$

- Power required,

$$P = m\,(h_2 - h_1) = 0.0537\ (1673.96 - 1450.22) = \mathbf{12.02\ kW}$$

- Cylinder dimension

$$m = \frac{\pi}{4} D^2 \times L \times \frac{N}{60} \times \text{(Specific) volume}$$

$$0.0537 = \frac{\pi}{4} D^2 \times 1.2\ D \times \frac{1000}{60} \times 0.417477$$

$$D^3 = \frac{0.04517 \times 4 \times 60}{\pi \times 1.2 \times 1000 \times 0.417477}$$

$$\therefore \quad \mathbf{D = 0.19\ m}$$

$$L = 1.2 \times D = 1.2 \times 0.19$$

$$\mathbf{L = 0.228 \ m}$$

Ex. 5.19 : *Standard ammonia at 2.5 bar enters a 160 mm × 150 mm (bore × stroke) twin cylinder, single acting compressor whose* η_v *is 79 % and speed is 250 rpm. The head pressure is 12 bar. The subcooled liquid ammonia at 22 ºC enters the expansion valve find i) Ammonia circulated in kg/min ii) The refrigeration in TR iii) COP of cycle. Assume* (C_{Pl} = *4.6006 and* C_{Pv} = *2.703 kJ / kg.K).*

Pressure (bar)	Temperature (°C)	Specific volume (m^3 / kg)	Specific enthalpy kJ/kg		Specific entropy kJ / kg K	
			L	V	L	V
2.5	– 15	0.5098	112.4	142.58	0.4572	5.5497
12	30	0.1107	323.08	1468.87	1.203	4.9842

Sol. : $p_1 = p_4 = 2.5$ bar, D = 160 mm = 0.16 m, L = 0.15 m

No. of cylinder = 2, $\eta_v = 0.79$, N = 250 rpm, $p_2 = p_3 = 12$ bar, $T_3 = 22$ ºC,

$T_1 = T_4 = 258$ K, $T'_2 = T'_3 = 30$ ºC, $\nu_1 = 0.5098$, $\nu'_2 = 0.1107$, $h_{f1} = 112.4$,

$h'_{f3} = 323.08$, $h_1 = 1426.58$, $h'_2 = 1468.87$, $S_{f1} = 0.4572$, $S_{f3} = 1.2037$,

$S_1 = S_2 = 5.5497$, $S'_2 = 4.9842$, $C_{Pl} = 4.6006$, $C_{Pv} = 2.703$ kJ /kg

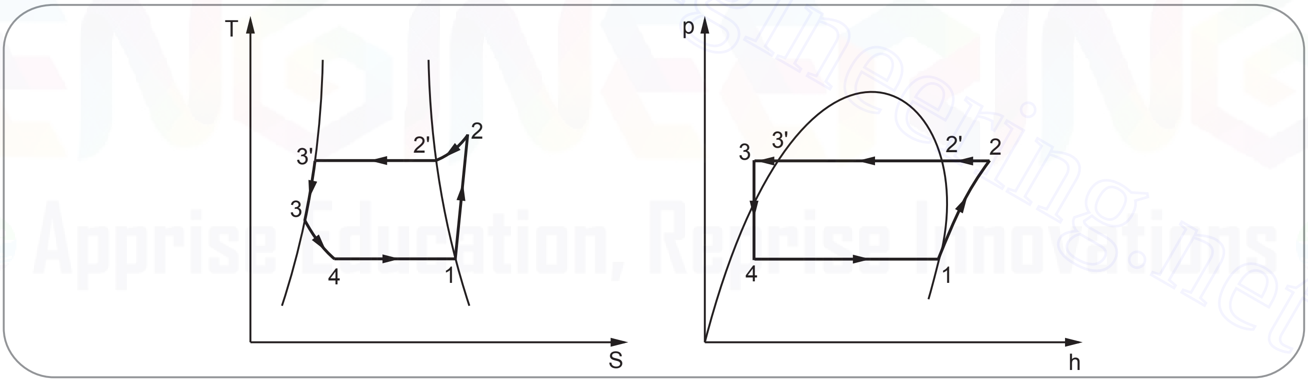


Fig. 5.28

Ammonia circulated (m_R)

Piston displacement / min = Area × Stroke × RPM × no. of cylinder

$$= \pi / 4 \ D^2 \ LN \times 2 = \pi / 4 \times (0.16)^2 \times (0.15) \times 250 \times 2$$

$$= 1.508 \ m^3 / \min \qquad \ldots (1)$$

We also know, piston displacement / min

$$= m_R \times V_1 \times 1 / \eta_v = m_R \times 0.5098 \times 1 / 0.79 = 0.6453 \ m_R \qquad \ldots (2)$$

Equate (1) and (2)

$$m_R = 1.508 / 0.6453 = 2.34 \ \text{kg/min}$$

Refrigeration in TR

$$h_{f3} = hf_3' - C_{Pl}(T_3' - T_3) = 286.23 \text{ kJ/kg}$$

Total RE $= m_R(h_1 - h_{f3}) = 2668.4$ kJ/min

Capacity $= 2668.4 / 210 = 12.7$ TR

To find COP

$$S_2 = S_2' + 2.3\, C_{Pv} \log\left(\frac{T_2}{T_2'}\right)$$

$$5.5497 = 4.9842 + 2.3 \times 2763 \log \frac{T_2}{T_2'}$$

$$\frac{T_2}{T_2'} = 1.227$$

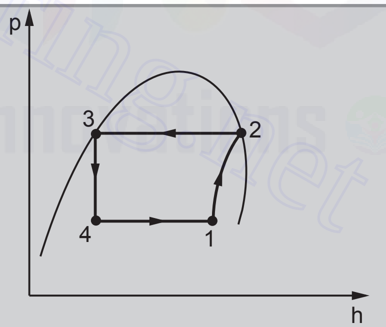
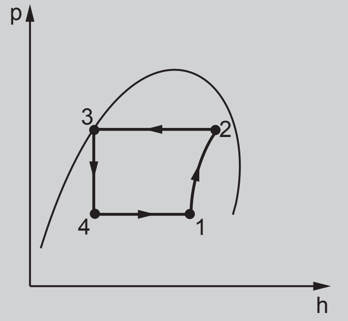
$$T_2 = T_2' \times 1.227 = 303 \times 1.227 = 371.78 = 371.78 - 273 = \mathbf{98.78\ ^\circ C}$$

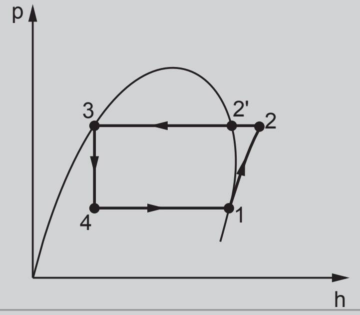
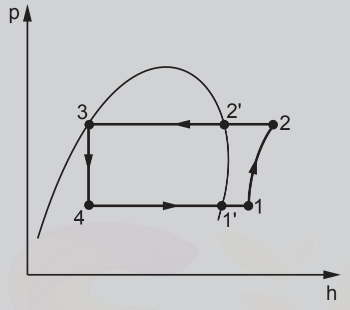
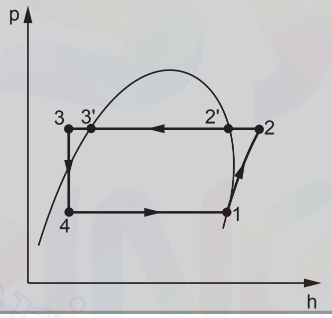
Enthalpy at (2)

$$h_2 = h_2' + C_{Pv}(T_2 - T_2') = 1658.9 \text{ kJ/kg}$$

$$\text{COP} = \frac{h_1 - h_{f3}}{h_2 - h_1} = 4.91$$

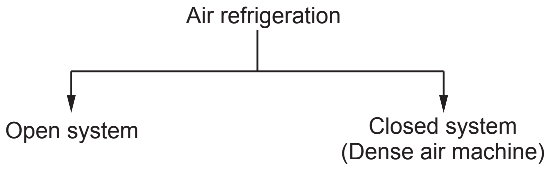
Table 5.1 - Review of all the processes with p - h representation

Dry compression	• $h_1 = h_{f1} + x_1\, h_{fg1}$ $h_1 = h_{f1} + x_1\,(h_{g1} - h_{f1})$ • $S_1 = S_{f1} + x_1\, S_{fg1}$ $S_1 = S_{f1} + x_1\,(S_{g1} - S_{f1})$	
Wet compression	• $h_1 = h_{f1} + x_1\, h_{fg1}$ $h_1 = h_{f1} + x_1\,(h_{g1} - h_{f1})$ • $h_2 = h_{f2} + x_2\, h_{fg2}$ $h_2 = h_{f2} + x_2\,(h_{g2} - h_{f2})$ • $S_1 = S_{f1} + \frac{x_1\, h_{fg1}}{T_1}$ • $S_2 = S_{f2} + \frac{x_2\, h_{fg2}}{T_2}$	

Superheating after compression or simple saturated VCC	• $h_2 = h_2' + C_P (T_2 - T_2')$ $S_2 = S_2' + 2.3\, C_P \log\left(\frac{T_2}{T_2'}\right)$	
Superheating before compression	• $h_1 = h_1' + C_P (T_1 - T_1')$ • $h_2 = h_2' + C_P (T_2 - T_2')$ • $S_1 = S_1' + 2.3\, C_P \log\left(\frac{T_1}{T_1'}\right)$ • $S_2 = S_2' + 2.3\, C_P \log\left(\frac{T_2}{T_2'}\right)$	
Subcooling	• $h_2 = h_2' + C_{Pv} (T_2 - T_2')$ $h_3 = h'_{f3} - C_{Pl} (T_3' - T_3)$ $S_2 = S_2' + 2.3\, C_{Pv} \log\left(\frac{T_2}{T_2'}\right)$	

5.7 Working Principle of Air Cycle

- An air refrigeration system uses air as a refrigerant.
- Now a days, this system is absolute because of its low Coefficient Of Performance (COP).
- This system is mostly applicable in an aircraft refrigeration system.
- In an air refrigeration system, refrigerant (air) remains in gaseous state throughout the cycle.
- Air refrigeration systems are devided in two systems.



Open System

- In this system refrigeration is obtained by three processes -

  i) Compression ii) Cooling iii) Expansion
- This system does not requires heat exchanger for refrigeration purpose.
- Thus it saves cost and weight of equipment.

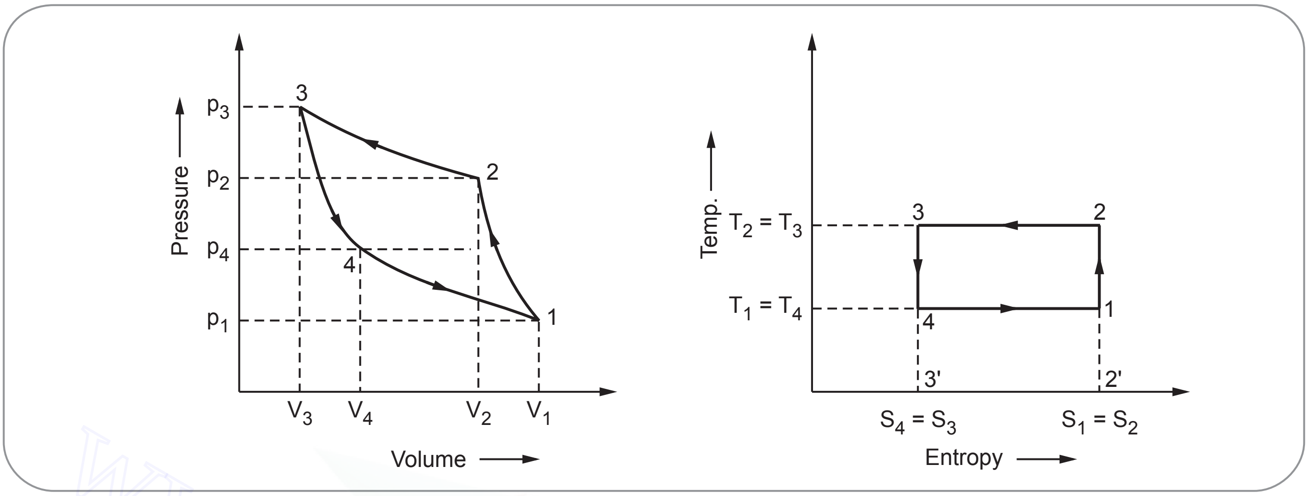


Fig. 5.29

Closed System

- In this system refrigerant is contained within components of the system.
- As compare to open system, closed system has higher C.O.P.
- It can work at suction pressure higher than atmospheric.

5.8 Reverse Carnot Cycle

- Carnot cycle is considered as reversed carnot cycle in refrigeration system.
- This cycle is only a theoretical cyclel but it serves as an ideal cycle.
- The refrigeration systems which works on reverse carnot cycle has maximum COP [Coefficient Of Performance].
- The carnot cycle is independent on physical properties of working medium.
- The reversed carnot cycle which work by considering air as a working medium is shown in Fig. 5.29.

Process 1-2 : Isentropic compression

- In this process, air is compressed isentropically.
- Because of compression pressure increases from p_1 to p_2 and temperature increases from T_1 to T_2.
- Where as the specific volume decreases from V_1 to V_2.

Process 2-3 : Isothermal compression process

- In this process, by maintaining temperature constant i.e. ($T_2 = T_3$) air is compressed.
- During this process pressure increases from p_2 to p_3 and specific volume decreases from V_2 to V_3.
- Heat rejected during this process can be calculated as,

$$Q_{2\text{-}3} = T_3 (S_2 - S_3) \quad \text{or}$$

$$Q_{2\text{-}3} = T_2 (S_2 - S_3) \qquad \ldots\ldots (\because T_2 = T_3).$$

Process 3-4 : Isentropic expansion process

- In this proces expansion of air is takes place isentropically
- During this process pressure and temperature decreases from p_3 to p_4 and T_3 to T_4 respectively.
- Where as specific volume increases from V_3 to V_4.

Process 4-1 : Isothermal expansion process

- In this process air is expanded isothermally ($T_1 = T_4$).
- During this process pressure decreases from p_4 to p_1 and temperature remains constant ($T_1 = T_4$).
- Specific volume gets increased from V_4 to V_1
- Heat absorbed by air during this process can be calculated as,

$$Q_{4\text{-}1} = T_4 (S_1 - S_4) \quad \text{or}$$

$$= T_4 (S_2 - S_3) \text{ or } T_1 (S_2 - S_3).$$

- Work done during the cycle,

$$W = \text{Heat rejected} - \text{Heat absorbed.}$$

$$= T_2 (S_2 - S_3) - T_1 (S_2 - S_3)$$

$$= (T_2 - T_1)(S_2 - S_3)$$

- Coefficient of reversed carnot cycle

$$COP = \frac{\text{Heat absorbed}}{\text{Work done}}$$

$$COP = \frac{T_1 (S_2 - S_3)}{(T_2 - T_1)(S_2 - S_3)}$$

$$\boxed{COP = \frac{T_1}{T_2 - T_1}}$$

- No any refrigerator has been made by using reversed carnot cycle because of following reasons.
- Isentropic process requires high speed while isothermal process is extremely slow speed.
- This variation in the speed is not practicable.
- COP of heat pump,

$$(COP)_{H.P.} = (COP)_R + 1 = \frac{T_1}{T_2 - T_1} + 1$$

$$\boxed{(COP)_{H.P.} = \frac{T_2}{T_2 - T_1}}$$

- COP or efficiency of Heat engine,

$$(COP)_{H.E.} = \frac{1}{(COP)_{H.P.}}$$

$$(COP)_{H.E.} = \frac{1}{\frac{T_2}{T_2 - T_1}}$$

$$\boxed{(COP)_{H.E.} = \frac{T_2 - T_1}{T_2}}$$

5.9 Block Diagram of Heat Engine, Heat Pump and Refrigerator

- The performance of heat engine is always expressed in terms of efficiency.
- Heat engine is a work producing device.

$$\eta_{H.E.} \text{ or } (COP)_{H.E.} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$\boxed{(COP)_{H.E.} = \frac{Q_2 - Q_1}{Q_2}}$$

- Refrigerator is a work absorbing device.
- Refrigerator is reversed heat engine which either cool or maintain the temperature of body.
- Refrigerator extract the heat (Q_1) from cold body which is at a temperature of (T_1) and supplied to hot body which is at a temperature of (T_2)

$$\boxed{(COP)_R = \frac{Q_1}{Q_2 - Q_1}}$$

- The performance of heat pump is expressed by the ratio of amount of heat supplied to hot body (Q_2) to amount of work required to be done on the system (Wp).
- This ratio is also called as energy performance ratio.

$$\boxed{(COP)_{HP} = \frac{Q_2}{Q_2 - Q_1}}$$

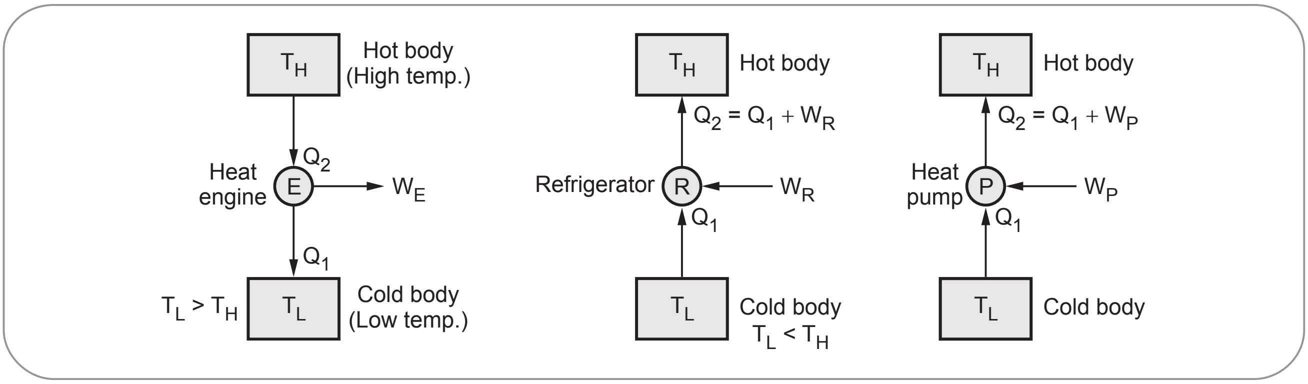


Fig. 5.30

5.9.1 Solved Examples

Ex. 5.20 *The COP of a refrigerator is 6, when it maintains the temperature of −3 °C in the evaporator. Determine the condenser temperature and refrigerating effect if the power required to run the refrigerator is 7.5 kW.*

Sol. : Given data : $(COP)_{ref.} = 6$,
$T_2 = -3\ °C = -3 + 273 = 270$ K, W = 7.5 kW

To find : i) Condenser temperature T_1 and
ii) Refrigerating effect.

Step - 1 : Calculate the condenser temperature T_1

COP of refrigerator is given by,

$$(COP)_{ref.} = \frac{T_2}{T_1 - T_2}$$

$$\therefore \quad 6 = \frac{270}{T_1 - 270}$$

$$\therefore \quad T_1 = 315 \text{ K} \qquad \text{... Ans.}$$

Step - 2 : Calculate the refrigerating effect

Power required to run the refrigerator = 7.5 kW = 7500 W = 7500 J/sec

We know that,

$$(COP)_{ref.} = \frac{Q_2}{W} = \frac{\text{Work output}}{\text{Work input}}$$

$$(COP)_{ref.} = \frac{\text{Refrigerating effect}}{\text{Input power}}$$

$$\therefore \quad 6 = \frac{\text{Refrigerating effect}}{7500}$$

$$\therefore \quad \text{Refrigerating effect} = 45000 \text{ J/sec.}$$

$$= 45000 \text{ W} = \mathbf{45\ kW} \qquad \text{... Ans.}$$

Ex. 5.21 *A reversible heat engine takes 900 kJ heat from a source at 700 K. The engine develops 350 kJ of network and rejects heat to two low temperature reservoirs at 600 K and 500 K. Determine engine thermal efficiency and heat rejected to each low temperature reservoir by using Clausius inequality.*

Sol. : Given data : $T_1 = 700$ K, $Q_1 = 900$ kJ,

$W_{net} = 350$ kJ, $T_{2\,(Reservoir\text{-}1)} = 600$ K,

$T_{2\,(Reservoir\text{-}2)} = 500$ K

To find : 1) η_E 2) Q_2

Thermal Efficiency

Step - 1 : Calculate thermal efficiency

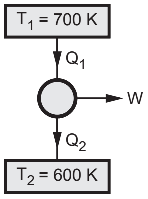


Fig. 5.31 (a)

For Reservoir - 1

$$\eta_E = 1 - \frac{T_2}{T_1} = 1 - \frac{600}{700} = 0.143$$

$$= \mathbf{14.30\ \%} \qquad \text{... Ans.}$$

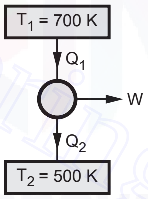


Fig. 5.31 (b)

For Reservoir - 2

$$\eta_E = 1 - \frac{T_2}{T_1} = 1 - \frac{500}{700} = 0.2357$$

$$= \mathbf{23.57\ \%}$$

Step - 2 : Calculate heat rejected (Q_2)

$$W_{net} = Q_1 - Q_2$$

$$\therefore \quad Q_2 = Q_1 - W_{net} = 900 - 350$$

$$\therefore \quad \boxed{Q_2 = 550 \text{ kJ}} \qquad \text{... Ans.}$$

Ex. 5.22 *A reversible heat pump is used to maintain a temperature of 0 °C in a refrigerator when it rejects the heat to the surroundings at 27 °C. Determine COP of the machine and work input required if the heat removal rate is 25 kW. If the required input to run the pump is developed by reversible engine which receives heat at 673 K and rejects heat to the atmosphere, determine overall COP of the system.*

Sol. : Given data : $T_1 = 27 + 273 = 300$ K,

$T_2 = 0 + 273 = 273$ K

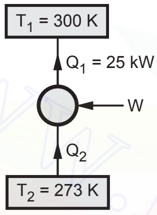


Fig. 5.32

Device : Heat pump, $Q_1 = 25 \text{ kW} = 25 \times 10^3$ W

To find : **i) COP** **ii)** W_{net}

Step - 1 : Calculate COP

$$COP = \frac{T_2}{T_1 - T_2}$$

$$\mathbf{COP} = \frac{293}{300 - 273} = \mathbf{10.11} \qquad \textbf{... Ans.}$$

Step - 2 : Calculate work input

We have $Q_1 = 25 \times 10^3$ W

$$Cop = \frac{Q_1}{Q_1 - Q_2} = \frac{Q}{W_{net}}$$

$$\therefore \quad \mathbf{W_{net}} = \frac{Q_1}{Cop} = \frac{25 \times 10^3}{10.11}$$

$$= \mathbf{2472.80 \ W} \qquad \textbf{... Ans.}$$

Ex. 5.23 *A reversible heat engine working as a refrigerator absorbs heat from low temperature reservoir of 650 kJ, when work input is 250 kJ.*

i) Find its COP and heat transferred to the surrounding.

ii) If the same device works as a heat engine, find out its thermal efficiency.

iii) If the same device work as a heat pump, estimate the COP.

Sol. : Given data : $Q_2 = 650$ kJ, W = 250 kJ

To find : **i) COP of refrigeratorii)** Q_1 **ii)** $(Cop)_{HP}$ **iv)** η_e

Step 1 : Calculate the $\mathbf{(COP)_R}$

$$\mathbf{(COP)_R} = \frac{Q_2}{W} = \frac{650}{250} = \mathbf{2.6} \qquad \textbf{... Ans.}$$

Step - 2 : Calculate heat transfer $\mathbf{(Q_1)}$

$$\mathbf{Q_1} = W + Q_2 = 650 + 250$$

$$= \mathbf{900 \ kJ} \qquad \textbf{... Ans.}$$

Step - 3 : Calculate Cop of heat engine

$$\mathbf{(COP)_{HP}} = \frac{Q_1}{W} = \frac{900}{250} = \mathbf{3.6} \qquad \textbf{... Ans.}$$

Step - 4 : Calculate efficiency of engine (η_e)

$$\eta_e = \frac{W}{Q_1} = \frac{250}{900}$$

$$= 0.278 = \mathbf{27.8 \ \%} \qquad \textbf{... Ans.}$$

Ex 5.24 *The efficiency of a Carnot engine rejecting heat to a cooling pond at 28 °C is 30 %. If the cooling pond receives 1050 kJ/min. What is the power developed by the cycle in kW. Also find the temperature of the source.*

Sol. : Given data : $T_2 = 28\,°C = 28 + 273 = 301$ K, $\eta_{Carnot} = 0.3$, $Q_2 = 1050$ kJ/min.

To find : i) Power developed W ii) T_1

Step - 1 : Calculate the source temperature $\mathbf{T_1}$

Refer Fig. 5.33.

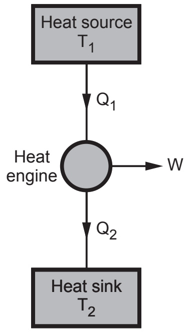


Fig. 5.33

We know that,

$$\eta_{Carnot} = 1 - \frac{T_2}{T_1}$$

$$\therefore \quad 0.3 = 1 - \frac{301}{T_1}$$

$$\therefore \quad \mathbf{T_1} = \mathbf{430 \ K} \qquad \textbf{... Ans.}$$

Step - 2 : Calculate the heat rejecting from heat source and power

Carnot efficiency is also given by,

$$\eta_{Carnot} = 1 - \frac{Q_2}{Q_1}$$

$$\therefore \quad 0.3 = 1 - \frac{1050}{Q_1}$$

$$\therefore \quad Q_1 = 1500 \text{ kJ/min.}$$

Power developed by the cycle is,

$$\text{Power} = W = Q_1 - Q_2 = 1500 - 1050$$

$$\therefore \quad W = 450 \text{ kJ/min} = 7.5 \text{ kJ/sec.}$$

$$\therefore \quad \mathbf{W = 7.5 \ kW} \qquad \textbf{... Ans.}$$

Ex. 5.25 *A heat pump is used to maintain an auditorium hall at 25 °C, when the atmospheric temperature is 10 °C. The heat leakage from the hall is 1500 kJ/min. Calculate the power required to run the actual heat pump, if the COP of the actual heat pump is 30 % of the COP of the Carnot heat pump working between the same temperature limits.*

Sol. : Given data : T_1 = 25 °C = 25 + 273 = 298 K,

T_2 = 10 °C = 10 + 273 = 283 K,

Q_1 = 1500 kJ/min = 25 kJ/sec = 25 kW,

$(COP)_{actual} = 0.3 \ (COP)_{Carnot}$

To find : Power

Step - 1 : Calculate the power required to run the actual heat pump

Refer Fig. 5.34.

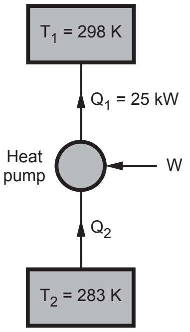


Fig. 5.34

$$(COP)_{Carnot\ hp} = \frac{T_1}{T_1 - T_2}$$

$$= \frac{298}{298 - 283} = 9.867$$

$$\therefore (COP)_{actual\ hp} = 0.3 \times (COP)_{Carnot\ hp}$$

$$= 0.3 \times 19.867$$

$$= 5.96$$

$$(COP)_{heat\ pump} = \frac{Q_1}{Q_1 - Q_2} = \frac{Q_1}{W}$$

$$\therefore \quad 5.96 = \frac{25}{W}$$

$$\mathbf{W = 4.195 \ kW} \qquad \textbf{... Ans.}$$

Ex. 5.26 *A reversible heat engine operates on Carnot cycle between source and sink temperatures of 225 °C and 25 °C. If the heat engine receives 40 kW from the source, find the net workdone, heat rejected to sink and efficiency of the engine.*

Sol. : Given data : T_1 = 225 °C = 225 + 273 = 498 K, T_2 = 25 °C = 25 + 273 = 298 K, Q_1 = 40 kW

To find : W, Q_2 and η_{Carnot}.

Step - 1 : Calculate the efficiency of heat engine

Refer Fig. 5.35.

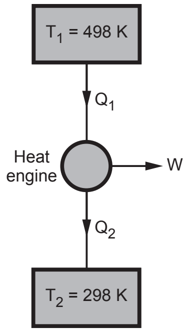


Fig. 5.35

Efficiency of Carnot cycle is,

$$\eta_{Carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{298}{498}$$

$$\therefore \quad \boldsymbol{\eta_{Carnot} = 0.4 = 40 \ \%} \qquad \textbf{... Ans.}$$

Step - 2 : Calculate the net workdone and heat rejected to the sink

We know that,

$$\eta_{Carnot} = \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1}$$

$$\therefore \quad 0.4 = \frac{W}{40}$$

$$\therefore \quad \mathbf{W = 16\ kW} \qquad \text{... Ans.}$$

But, $\quad W = Q_1 - Q_2$

$$\therefore \quad 16 = 40 - Q_2$$

$$\therefore \quad \mathbf{Q_2 = 24\ kW} \qquad \text{... Ans.}$$

Ex. 5.27 *A Carnot engine operates between source and sink temperatures of 230 ºC and 30 ºC. If the engine receives 450 kJ of heat from the source, determine*
i) Workdone ii) Heat rejected iii) η_{Carnot}
iv) Also find COP if it operates as a heat pump and when it operates as a refrigerator ?

Sol. : Given data : T_1 = 230 ºC = 230 + 273 = 503 K, T_2 = 30 ºC = 30 + 273 = 303 K, Q_1 = 450 kJ

To find : i) W ii) Q_2 iii) η_{Carnot} iv) $(COP)_{hp}$ and $(COP)_{ref.}$

Step - 1 : Calculate the efficiency and workdone of cycle

We know that,

$$\eta_{Carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{303}{503}$$

$$\therefore \quad \eta_{Carnot} = 0.398 = \mathbf{39.8\ \%} \qquad \text{... Ans.}$$

Efficiency is also given by,

$$\eta_{Carnot} = \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1}$$

$$0.398 = \frac{W}{450}$$

$$\therefore \quad \mathbf{W = 179.1\ kJ} \qquad \text{... Ans.}$$

But, $\quad W = Q_1 - Q_2$

$$\therefore \quad 179.1 = 450 - Q_2$$

$$\therefore \quad \mathbf{Q_2 = 270.9\ kJ} \qquad \text{... Ans.}$$

Step - 3 : Calculate the $(COP)_{hp}$ and $(COP)_{ref.}$

When the engine operates like heat pump,

$$(COP)_{hp} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{450}{450 - 270.9}$$

$$\therefore \quad \mathbf{(COP)_{hp} = 2.515} \qquad \text{... Ans.}$$

When it operates like refrigerator,

$$(COP)_{ref.} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{270.9}{450 - 270.9}$$

$$\therefore \quad \mathbf{(COP)_{ref.} = 1.515} \qquad \text{... Ans.}$$

Ex. 5.28 *An inventor claims to have developed a refrigeration unit which maintains at – 5 ºC in the refrigerator which is in a room of surrounding temperature 28 ºC. It has COP of 9.5. Check whether his claim is right or not ?*

Sol. : Given data : T_2 = – 5 ºC = – 5 + 273 = 268 K, T_1 = 28 ºC = 28 + 273 = 301 K, $(COP)_{inv.}$ = 9.5

To find : Check whether inventor's claim is right or not.

Step - 1 : Calculate the $(COP)_{ref.}$ and check it

Refer Fig. 5.36.

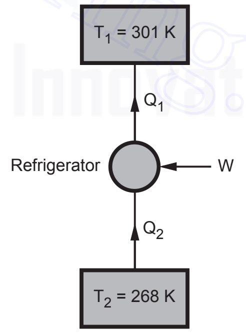


Fig. 5.36

We know that,

$$(COP)_{ref.} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

$$\therefore \quad (COP)_{ref.} = \frac{268}{301 - 268} = 8.121$$

$$\therefore \quad (COP)_{ref.} < (COP)_{inv.}$$

It means for given temperature range inventor's claim is invalid. ... **Ans.**

Ex. 5.29 *A fish freezing plant of 100 tons capacity is to be maintained at – 40 ºC for which the outside atmospheric temperature is 30 ºC. The actual COP of the refrigeration system is 10 % of the theoretical Carnot pump working between the same temperature limits. Calculate the power required to run the plant.*

Take, 1 ton of refrigeration = 3.5 kW

Sol. : Given data : $T_1 = 30\ ºC = 30 + 273 = 303\ K$,

$T_2 = -40\ ºC = -40 + 273 = 233\ K$,

$(COP)_A = 0.1\ (COP)_T$,

$Q_1 = 100\ \text{tons} = 100 \times 3.5 = 350\ kW$

To find : Power (W)

Step - 1 : Calculate the power required to run the plant

Refer Fig. 5.37.

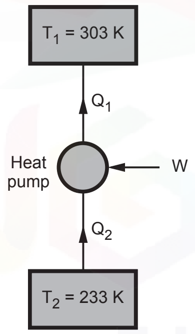


Fig. 5.37

COP of Carnot or theoretical heat pump is given by,

$$(COP)_T = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$$

$$\therefore \quad (COP)_T = \frac{303}{303 - 233} = 4.33$$

$$(COP)_A = 0.1\ (COP)_T \quad \text{... (Given)}$$

$$\therefore \quad (COP)_A = 0.1 \times 4.33 = 0.433$$

Now, $$(COP)_A = \frac{Q_1}{Q_1 - Q_2} = \frac{Q_1}{W}$$

$$\therefore \quad 0.433 = \frac{350}{W}$$

$$\therefore \quad \mathbf{W = 808.31\ kW} \quad \textbf{... Ans.}$$

Ex. 5.30 *The power input required for a grinding mill is 30 MJ/min. A heat source at 420 ºC is available for supplying the energy and the surrounding is at 30 ºC. If the actual engine is 25 % as efficient as a Carnot engine working between the same temperature limits. Calculate the energy supplied by the source per sec.*

Sol. :

Given data : Power input to grinding mill

$$W = 30\ MJ/min = \frac{30 \times 10^3}{60}\ \frac{kJ}{sec.} = 500\ kW,$$

$T_1 = 420\ ºC = 420 + 273 = 693\ K$,

$T_2 = 30\ ºC = 30 + 273 = 303\ K$, $(\eta)_A = 0.25\ (\eta)_T$

To find : Q_1

Step - 1 : Calculate the enrgy supplied by the source

Refer Fig. 5.38.

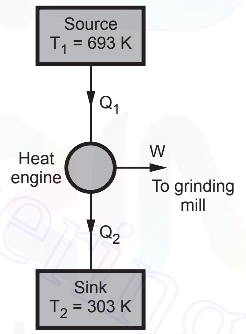


Fig. 5.38

Efficiency of Carnot engine is,

$$\eta_{Carnot} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

$$\therefore \quad \eta_{Carnot} = \frac{693 - 303}{693} = 0.563 = 56.3\ \%$$

But, $$\eta_A = 0.25\ \eta_{Carnot} = 0.25 \times 0.563$$

$$\therefore \quad \eta_A = 0.141$$

Now, $$\eta_A = \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1}$$

$$\therefore \quad 0.141 = \frac{500}{Q_1}$$

$$\therefore \quad \mathbf{Q_1 = 3546.1\ kJ/sec. = 3546.1\ kW} \quad \textbf{... Ans.}$$

Ex. 5.31 *An inventor claims to have invented refrigeration machine operating between −23 °C and 27 °C. It consumes 1 kW electrical power and gives 20000 kJ of refrigerating effect in one hour. Comment on his claim.*

Sol. : Given data : $T_1 = 27\ °C = 27 + 273 = 300\ K$,

$T_2 = -23\ °C = -23 + 273 = 250\ K$, $W = 1\ kW$,

$$Q_2 = 20000\ kJ/hr = \frac{20000}{3600}\frac{kJ}{sec.} = 5.55\ kW$$

To find : Check whether inventor's claim is right or not.

Step - 1 : Calculate the actual and theoretical COP of refrigerator

$$(COP)_T = (COP)_{Carnot} = \frac{Q_2}{Q_1 - Q_2}$$

$$= \frac{T_2}{T_1 - T_2} = \frac{250}{300-250}$$

$$\therefore\quad (COP)_{Carnot} = 5$$

$$(COP)_{Actual} = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2}{W} = \frac{5.55}{1}$$

$$\therefore\quad (COP)_{Actual} = 5.55$$

It means COP of Carnot refrigerator is less than the COP of actual refrigerator. Hence, inventor's claim is invalid. **... Ans.**

Ex. 5.32 *The Carnot engine is operating between source and sink temperature T_1 and T_2 respectively. It has efficiency of 30 %. If the sink temperature is reduced by 25 °C then the efficiency is increased to 35 %. Find the temperatures T_1 and T_2.*

Sol. : Given data : $\eta = 0.3 = 1 - \frac{T_2}{T_1}$, $\eta' = 0.35$ when $T_2' = T_2 - 25$ and $T_1' = T_1$

To find : T_1 and T_2.

Step - 1 : Calculate the source and sink temperatures T_1 and T_2

It is given that, $$\eta = 0.3 = 1 - \frac{T_2}{T_1} \qquad \text{... (i)}$$

and $$\eta' = 0.35 = 1 - \frac{T_2'}{T_1'}$$

$$\therefore\quad 0.35 = 1 - \frac{(T_2 - 25)}{T_1} \qquad \text{... (ii)}$$

Dividing equation (i) and (ii),

$$\frac{\eta'}{\eta} = \frac{0.35}{0.3} = \frac{1 - \dfrac{(T_2 - 25)}{T_1}}{1 - \left(\dfrac{T_2}{T_1}\right)} = \frac{T_1 - T_2 + 25}{T_1 - T_2}$$

$$\therefore\quad 1.167 = \frac{T_1 - T_2 + 25}{T_1 - T_2}$$

$$\therefore\quad 1.167\ T_1 - 1.167\ T_2 = T_1 - T_2 + 25$$

$$\therefore\quad 0.167\ T_1 = 0.167\ T_2 + 25$$

$$\therefore\quad T_1 = T_2 + 149.7 \qquad \text{... (iii)}$$

Substituting this value in equation (i),

$$\therefore\quad 0.3 = 1 - \frac{T_2}{T_1} = 1 - \frac{T_2}{(T_2 + 149.7)}$$

$$\therefore\quad \frac{T_2}{T_2 + 149.7} = 1 - 0.3 = 0.7$$

$$\therefore\quad \mathbf{T_2 = 349.3\ K} \qquad \textbf{... Ans.}$$

and $$T_1 = T_2 + 149.7 = 349.3 + 149.7$$

$$\therefore\quad \mathbf{T_1 = 499\ K} \qquad \textbf{... Ans.}$$

Ex. 5.33 *A reversible heat engine operates between two reservoirs at temperature of 600 °C and 40 °C. The engine drives a reversible refrigerator, which operates between reservoir at temperature 40 °C and −20 °C. The heat transfer to the heat engine is 2000 kJ and the net work output of the combined engine refrigerator plant is 360 kJ.*

Sol. : Given data : $T_1 = 600\ °C = 600 + 273 = 873\ K$, $T_2 = 40\ °C = 40 + 273 = 313\ K$,

$T_3 = -20\ °C = -20 + 273 = 253\ K$,

$T_4 = T_2 = 313\ K$, $Q_1 = 2000\ kJ$, $W_1 = 360\ kJ$

To find :

i) Heat transfer to the refrigerator and reservoir and

ii) Heat transfer to the refrigerator and reservoir at

$$\eta_A = 0.4\,\eta_T \text{ and } (COP)_A = 0.4\ (COP)_T$$

Step - 1 : Calculate heat transfer to the refrigerator and reservoir

Refer Fig. 5.39.

$$W = W_1 + W_2 \qquad \text{... (i)}$$

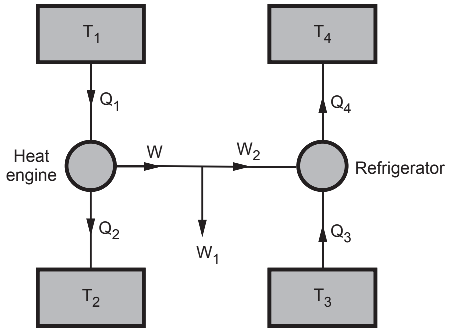


Fig. 5.39

But, $\eta_T = 1 - \dfrac{T_2}{T_1}$

$\therefore \quad \eta_T = 1 - \dfrac{313}{873}$

$\therefore \quad \eta_T = 0.6415$

Now, $\eta_T = \dfrac{W}{Q_1}$

$\therefore \quad 0.6415 = \dfrac{W}{2000}$

$\therefore \quad W = 1283 \text{ kJ}$

Substituting in equation (i),

$$1283 = 360 + W_2$$

$\therefore \quad \mathbf{W_2 = 923 \text{ kJ}}$ **... Ans.**

But, $\eta_T = \dfrac{W}{Q_1} = \dfrac{Q_1 - Q_2}{Q_1} = 1 - \dfrac{Q_2}{Q_1}$

$\therefore \quad 0.6415 = 1 - \dfrac{Q_2}{2000}$

$\therefore \quad Q_2 = 717 \text{ kJ}$

We know that,

$$(COP)_{ref.} = \frac{Q_3}{W_2} = \frac{Q_3}{Q_4 - Q_3} = \frac{T_3}{T_4 - T_3}$$

$\therefore \quad (COP)_{ref.} = \dfrac{253}{313 - 253} = 4.22$

But, $(COP)_{ref.} = \dfrac{Q_3}{W_2}$

$\therefore \quad 4.22 = \dfrac{Q_3}{923}$

$\therefore \quad Q_3 = 3895.06 \text{ kJ}$

and $W_2 = Q_4 - Q_3$

$\therefore \quad 923 = Q_4 - 3895.06$

$\therefore \quad Q_4 = 4818.06 \text{ kJ}$

$\therefore$ Total heat transfer to the reservoir

$= Q_2 + Q_4 = 717 + 4818.06 =$ **5535.06 kJ** **... Ans.**

Step - 2 : Calculate heat transfer to the refrigerator and reservoir at 40 % of their maximum values

$$\eta_A = 0.4\ \eta_T = 0.4 \times 0.6415 = 0.2566$$

Now, $\eta_A = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{Q_2}{Q_1}$

$\therefore \quad 0.2566 = 1 - \dfrac{Q_2}{2000}$

$\therefore \quad \mathbf{Q_2 = 1486.8 \text{ kJ}}$ **... Ans.**

and $\eta_A = \dfrac{W}{Q_1}$

$\therefore \quad 0.2566 = \dfrac{W}{2000}$

$\therefore \quad W = 513.2 \text{ kJ}$

But, $W = W_1 + W_2$

$\therefore \quad 513.2 = 360 + W_2$

$\therefore \quad W_2 = 153.2 \text{ kJ}$

Similarly, $(COP)_A = 0.4\ (COP)_T = 0.4 \times 4.22 = 1.688$

Now, $(COP)_A = \dfrac{Q_3}{W_2}$

$\therefore \quad 16.88 = \dfrac{Q_3}{153.2}$

$\therefore \quad Q_3 = 258.6 \text{ kJ}$

But, $W_2 = Q_4 - Q_3$

$$153.2 = Q_4 - 258.6$$

$\therefore \quad Q_4 = 411.8 \text{ kJ}$

$\therefore$ Total heat transfer to the reservoir $= Q_2 + Q_4 = 1486.8 + 411.8 =$ **1898.6 kJ** **... Ans.**

5.10 Bell - Coleman Cycle / Modified Reverse Carnot Cycle

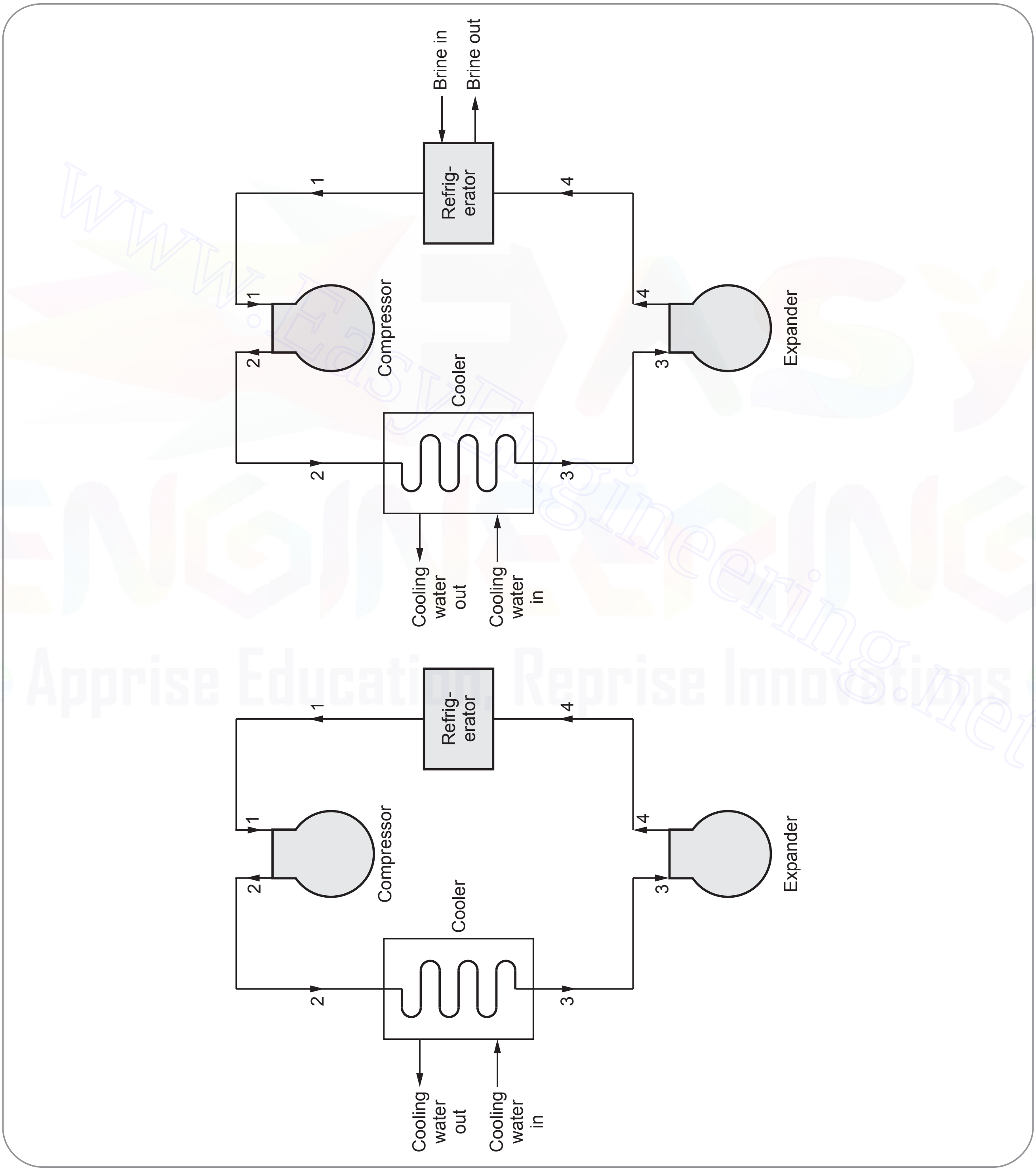


Fig. 5.40

- Bell coleman cycle is also called as Brayton or Joul cycle.
- In this cycle the isothermal processes of carnot cycle are replaced by constant pressure process.
- This cycle cosists of compressor, coolar, expander and refrigerator.
- The four processes of this cycle are described below,

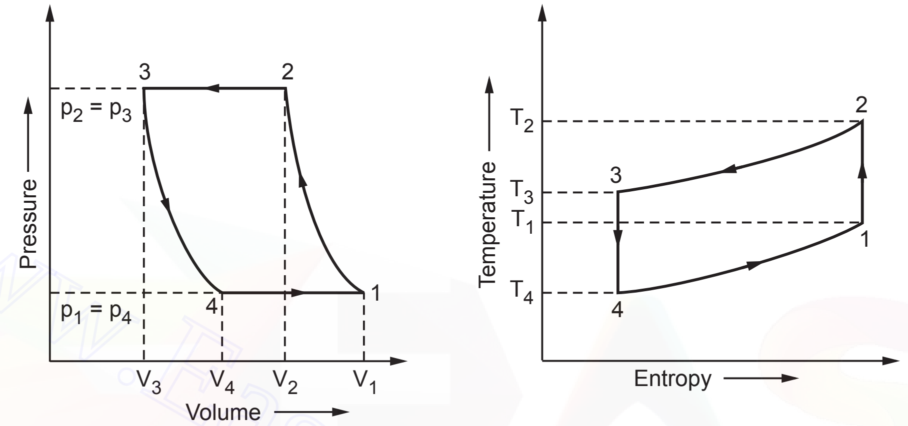


Fig. 5.41

Process 1-2 : Isentropic compression

- In this process, compression of air takes place isentropically.
- During this process no heat is added or removed.
- The pressure and temperature increases from p_1 to p_2 and T_1 to T_2.
- Where as the specific volume decreases from V_1 to V_2.

Process 2-3 : Constant pressure cooling process

- In this process hot air from the compressor is passed into cooler where it is cooled at constant pressure ($p_2 = p_3$).
- During this process temperature is reduced to T_2 to T_3 and specific volume reduces from V_2 to V_3.
- Heat rejected during this process,

$$Q_R = C_p (T_2 - T_3)$$

Process 3-4 : Isentropic expansion process

- The cooled air from coolar is drawn into the expander for the expansion process.
- During this process pressure and temperature decreases from p_3 to p_4 and T_3 to T_4.
- Where as the specific volume increases from V_3 to V_4.

Process 4-1 : Constant pressure expansion process

- The cold air from expander is passes into the refrigerator. where it is expanded at constant pressure ($p_4 = p_1$).
- The temperature of air increases from T_4 to T_1.
- Specific volume changes from V_4 to V_1.

- Heat absorbed during this process,

$$Q_A = C_p(T_1 - T_4)$$

- Work done by the cycle,

$$W = \text{Heat rejected} - \text{heat absorbed}$$

$$= C_p(T_2 - T_3) - C_p(T_1 - T_4)$$

- Coefficient of performance of cycle,

$$COP = \frac{\text{Heat absorbed}}{\text{Work done}}$$

$$= \frac{C_p(T_1 - T_4)}{C_p(T_2 - T_3) - C_p(T_1 - T_4)}$$

$$COP = \frac{(T_1 - T_4)}{(T_2 - T_3) - (T_1 - T_4)}$$

$$= \frac{T_4\left(\frac{T_1}{T_4} - 1\right)}{T_3\left(\frac{T_2}{T_3} - 1\right) - T_4\left(\frac{T_1}{T_4} - 1\right)} \quad \ldots (5.1)$$

- Process 1-2 is isentropic process,

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \ldots (5.2)$$

- Process 3-4 is isentropic expansion.

$$\frac{T_1}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} \quad \ldots (5.3)$$

But, $p_2 = p_3$ and $p_1 = p_4$

We can write,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \text{ or } \frac{T_2}{T_3} = \frac{T_1}{T_4} \quad \ldots (5.4)$$

Put the above value in equation (5.1)

We get,

$$COP = \frac{T_4}{T_3 - T_4}$$

$$COP = \frac{1}{\frac{T_3}{T_4} - 1}$$

$$COP = \frac{1}{\left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} - 1}$$

$$COP = \frac{1}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}$$

$$\boxed{COP = \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}} - 1}} \quad \ldots r_p \text{ - compression ratio or expansion ratio}$$

- But sometimes compression and expansion processes are takes place as per the law pV^n = Constant.
- Work done by compressor during process 1-2

$$W_1 = \frac{n}{n-1}\left(p_2 V_2 - p_1 V_1\right)$$

$$= \frac{n}{n-1}\left(R T_2 - RT_1\right)$$

- Work done by expander during process 3-4.

$$W_1 = \frac{n}{n-1}\left(p_3 V_3 - p_4 V_4\right)$$

$$= \frac{n}{n-1}\left(R T_3 - RT_4\right)$$

- Net work done can be find out as,

$$W = \frac{n}{n-1} \times R\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right] \quad \ldots (5.5)$$

$$COP = \frac{\text{Heat absorbed}}{\text{Work done}}$$

$$= \frac{C_p\left(T_1 - T_4\right)}{\frac{n}{n-1} \times R\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right]} \quad \ldots (5.6)$$

We know, $C_P - C_V = R = C_V(r - 1)$

Put value of R in equation (5.6)

$$COP = \frac{C_p\left(T_1 - T_4\right)}{\frac{n}{n-1} \times (\gamma - 1) C_v\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right]}$$

$$\text{COP} = \frac{\gamma(T_1 - T_4)}{\frac{n}{n-1}(\gamma-1)[(T_2 - T_1) - (T_3 - T_4)]}$$

$$\boxed{\text{COP} = \frac{T_1 - T_4}{\frac{n}{n-1} \times \frac{\gamma-1}{\gamma}[(T_2 - T_1) - (T_3 - T_4)]}} \quad \ldots (5.7)$$

- For isentropic procss n = r. then equation (5.7) will become :

$$\boxed{\text{COP} = \frac{T_1 - T_4}{(T_2 - T_3) - (T_1 - T_4)}}$$

5.10.1 Advantages of Bell-Coleman Cycle

- It uses air as a working medium which is freely available.
- As the air is non flammable this system is safe.
- The weight of air refrigeration system per ton of refrigeration is low compared with other refrigeration system.

5.10.2 Dis-advantages of Bell-Coleman Cycle

- COP of system is low as compare to VCC.
- For 1TR large volume of air is required which results in large size of compressor and expander.

5.10.3 Solved Examples

Ex. 5.34 *In refrigeration plant working on Bell - coleman cycle, air is compressed to 1 bar from 7 bar. Its initial temperature is 10 °C. After compression air is cooled upto 20 °C in cooler before expanding back to pressure of 1 bar. Determine theoretical COP of the plant and net refrigerating effect, Take* $C_P = 1.005$ *kJ/kg K and* $C_V = 0.718$ *kJ/kg K.*

Sol : Given data :

$p_1 = 1 \text{ bar}, p_2 = 7 \text{ bar}$

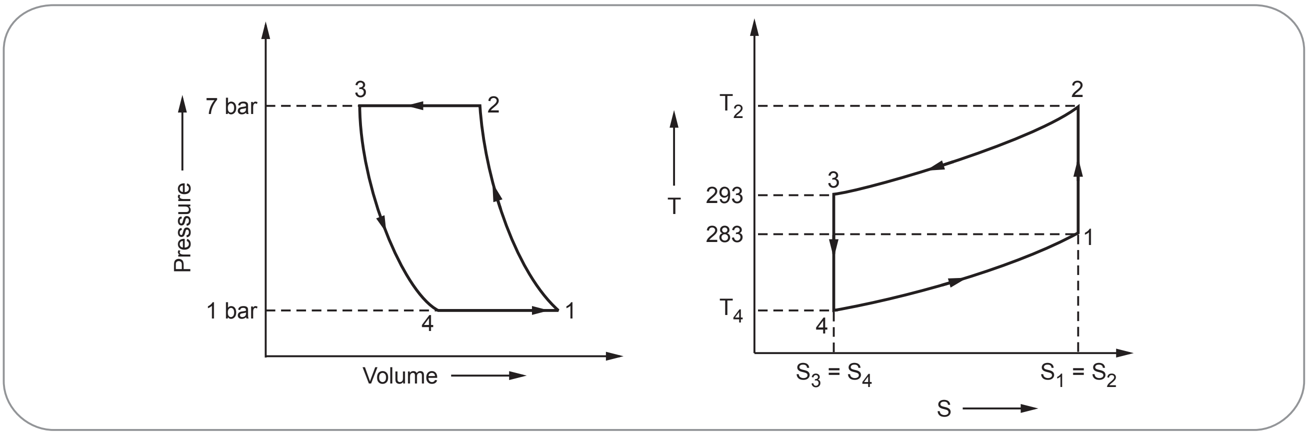


Fig. 5.42

$T_1 = 10\,°\text{C} = 10 + 273 = 283 \text{ K}$

$T_3 = 20\,^\circ C = 20 + 273 = 293 \text{ K}$

$C_P = 1.005 \text{ kJ/kg K}$

$C_P = 0.718 \text{ kJ/kg K}$

We know,

$$\gamma = \frac{C_p}{C_v} = \frac{1.005}{0.718} = 1.4$$

The process 1-2 is isentropic compression process,

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$T_2 = 283 \times \left(\frac{7}{1}\right)^{\frac{1.4 - 1}{1.4}}$$

$$T_2 = 493.44 \text{ K}$$

Process 3-4 is isentropic expansion

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{293}{T_4} = \left(\frac{7}{1}\right)^{\frac{1.4 - 1}{1.4}}$$

$$\frac{293}{T_4} = 1.743$$

$$\mathbf{T_4 = 168.03}$$

Theoretical COP of the plant,

$$\text{COP} = \frac{T_4}{T_3 - T_4} = \frac{168.03}{293 - 168.03} = 1.34$$

Net refrigerating effect,

$$\text{RE} = C_p\left(T_1 - T_4\right) = 1.005\ (283 - 168.03) = 115.54 \text{ kJ/kg}$$

Ex. 5.35 *A refrigerator working on Bell - coleman cycle operates between pressure limit of 1.06 and 8.7 bar. Air is drawn from cold chamber at 10 ºC. The air coming out of the compressor is cooled to 30 ºC before entering to expansion cylinder. Expansion and compression follows the law $pV^{1.33} = C$. Determine COP of the system. [$C_p = 1\ kJ/kg\ K$] [$\gamma = 1.4$]*

Sol. : Given data :

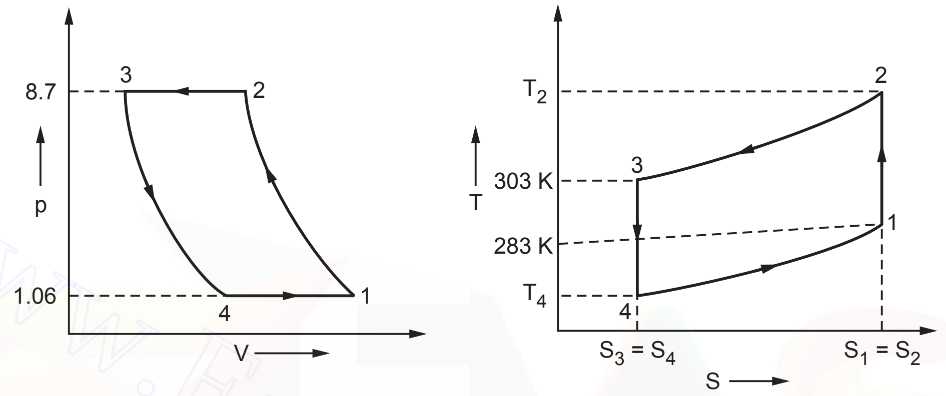


Fig. 5.43

$p_1 = 1.06$ bar, $p_2 = 8.7$ bar

$T_1 = 10\,^\circ C = 10 + 273 = 283$ K

$T_3 = 30\,^\circ C = 30 + 273 = 303$ K

$C_p = 1$ kJ/kg K

$\gamma = 1.4$

The compression follows the law $pV^{1.33} = C$.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = 283 \times \left(\frac{8.7}{1.06}\right)^{\frac{1.33-1}{1.33}}$$

$$T_2 = 477.11 \text{ K}$$

Similarly expansion follows the law $pV^{1.35} = C$.

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}}$$

$$\frac{303}{T_4} = \left(\frac{8\cdot 7}{1\cdot 06}\right)^{\frac{1\cdot 33 - 1}{1\cdot 33}}$$

$$\frac{303}{T_4} = 1.685$$

$$\mathbf{T_4 = 179.72\ K}$$

To find out net work done, (W_{net})

$$W_{net} = W_C - W_E$$

$$W_{net} = \frac{n}{n-1}\left(p_2V_2 - p_1V_1\right) - \frac{n}{n-1}\left(p_3V_3 - p_4V_4\right)$$

$$= \frac{n}{n-1} R\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right]$$

To find value of 'R'

$$R = C_p - C_v \quad \text{and } r = \frac{C_p}{C_v}$$

$$C_v = \frac{C_p}{\gamma} = \frac{1}{1.4} = 0.70 \text{ kJ/kg K}$$

$$R = C_p - C_v = 1 - 0.70 = 0.3 \text{ kJ/kg K}$$

Now,

$$W_{net} = \frac{1\cdot 33}{1\cdot 33 - 1} \times 0\cdot 3\left[\left(477\cdot 11 - 283\right) - \left(303 - 179\cdot 72\right)\right]$$

$$= 85.63 \text{ kJ/kg K}$$

$$COP = \frac{\text{Heat extracted}}{\text{Work done}} = \frac{C_p\left(T_1 - T_4\right)}{(W)_{net}} = \frac{1\left(283 - 179\cdot 72\right)}{85\cdot 63}$$

$$= 1.20$$

Ex. 5.36 *An air refrigerator works between pressure limit of 1 bar and 5 bar. The temperature of air entering the compressor is 15 ºC and entering the expansion cylinder is 30 ºC*

The expansion follows the law $pV^{1.25} = C$

The compression follows the law $pV^{1.35} = C$

Take $C_p = 1\ kJ/kg\ K$ *and* $C_v = 0.7 kJ/kg\ K$ *find.*

i) COP of the cycle

ii) If air circulation through system is 25 kg/min find refrigerating capacity of system.

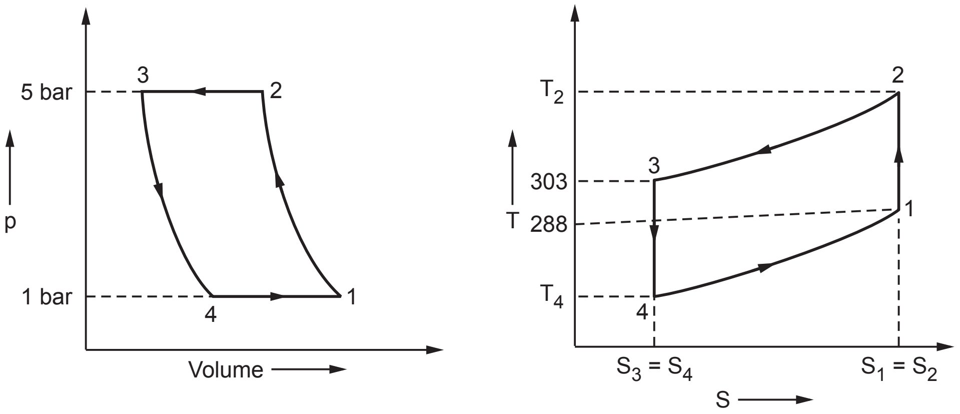


Fig. 5.44

Sol. : Given data :

$p_1 = 1$ bar $p_2 = 5$ bar

$T_1 = 15\,°C = 15 + 273 = 288$ K, $T_3 = 30\,°C = 30 + 273 = 303$ K

$C_p = 1$ kJ/kg K $C_v = 0 \cdot 7$ kJ/kg K

Compression follows the law $pV^{1.35} = C$.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{1.35 - 1}{1.35}}$$

$$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{1.35 - 1}{1.35}}$$

$$T_2 = 288 \times \left(\frac{5}{1}\right)^{\frac{1.35 - 1}{1.35}}$$

$$T_2 = 437.12 \text{ K}$$

Expansion follows the law $pV^{1.25} = C$.

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{1.25 - 1}{1.25}}$$

$$\frac{303}{T_4} = \left(\frac{5}{1}\right)^{\frac{1.25 - 1}{1.25}}$$

$$T_4 = 219.60 \text{ K}$$

Now, To find out the refrigerating effect.

$$\text{R.E.} = C_p\,(T_1 - T_4) = 1\,(288 - 219 \cdot 60) = 68.39 \text{ kJ/kg}$$

$$R = C_p - C_v = 1 - 0 \cdot 7 = 0.3 \text{ kJ/kg K}$$

To find out net work done,

$$W_{net} = \left[\frac{1\cdot35}{1\cdot35-1}\times R\,(T_2 - T_1) - \frac{1\cdot25}{1\cdot25-1}\times R\,(T_3 - T_4)\right]$$

$$= R\left[\frac{1\cdot35}{0\cdot35}\,(T_2 - T_1) - \frac{1\cdot25}{0\cdot25}\,(T_3 - T_4)\right]$$

$$= 0.3\left[\frac{1\cdot35}{0\cdot35}\,(437\cdot12 - 288) - \frac{1\cdot25}{0\cdot25}\,(303 - 219\cdot60)\right] = 47.45 \text{ kJ/kg}$$

$$\text{COP} = \frac{\text{R.E.}}{W_{net}} = \frac{68\cdot39}{47\cdot45} = 1.44$$

$$\text{Capacity of referigerating system} = \frac{\text{RE} \times m_a \text{ (kg/sec)}}{3.5} = \frac{68\cdot39 \times \frac{25}{60}}{3.5} = 8.14 \text{ tons.}$$

Ex. 5.37 *5 TR refrigerating machine based on Bell coleman cycle operates between 5 bar and 20 bar. The air temperature after heat rejection to the surrounding is 37 ºC and air temperature at compressor inlet is 7 ºC. The η_C and η_E = 48 % respectively determine i) COP, ii) Power per TR, iii) Mass of circulation of refrigerant in kg/hr, iv) carnot COP for same entry temperature to compressor and expander. (Take C_p = 1 kJ/kg K , C_v = 0.718 kJ/kg K).*

Sol. : Given data :

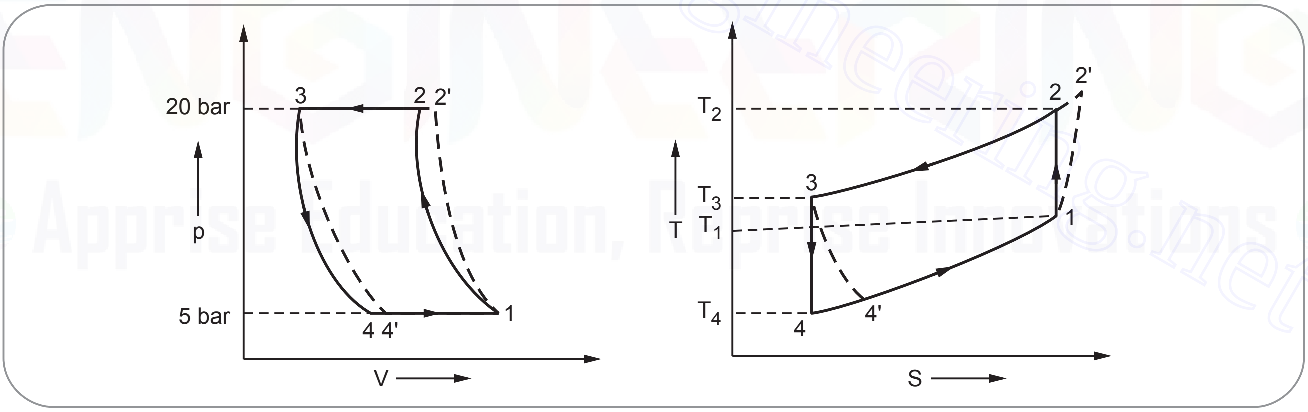


Fig. 5.45

Refrigerating capacity = 5 TR = 5 × 3·52 = 17·6 kW

p_1 = 5 bar p_2 = 20 bar

T_3 = 37 °C = 37 + 273 = 310 K T_1 = 7 °C = 7 + 273 = 280 K

$\eta_C = \eta_E$ = 48 %

We know process 1-2 is isentropic compression

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{280} = \left(\frac{20}{5}\right)^{\frac{1\cdot4-1}{1\cdot4}}$$

$$\mathbf{T_2 = 416 \cdot 82\ K}$$

Similarly process 3-4 is isentropic expansion

$$\left(\frac{T_3}{T_4}\right) = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{310}{T_4} = \left(\frac{20}{5}\right)^{\frac{1 \cdot 4 - 1}{1 \cdot 4}}$$

$$\frac{310}{T_4} = 1.488$$

$$\mathbf{T_4 = 208 \cdot 24}$$

But, isentropic efficiency of compressor,

$$\eta_C = \frac{T_2 - T_1}{T_2' - T_1}$$

$$\therefore \quad 0 \cdot 48 = \frac{416 \cdot 82 - 280}{T_2' - 280}$$

$$\mathbf{T_2' = 565 \cdot 04\ K}$$

Similarly isentropic efficiency of expander

$$\eta_E = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$\therefore \quad 0 \cdot 48 = \frac{310 - T_4'}{310 - 208 \cdot 24}$$

$$\mathbf{T_4' = 261 \cdot 15\ K}$$

Net work done by the system,

$$(W)_{net} = C_p \left[(T_2' - T_1) - (T_3 - T_4') \right]$$

$$= 1 \left[(565 \cdot 04 - 280) - (310 - 261 \cdot 15) \right]$$

$$= 236 \cdot 19\ kJ/kg$$

Refrigerating effect,

$$R.E. = C_p (T_1 - T_4')$$

$$= 1\ [280 - 261 \cdot 15] = 18 \cdot 85\ kJ/kg$$

$$COP = \frac{R \cdot E}{W \cdot D} = \frac{18 \cdot 85}{236 \cdot 19} = \mathbf{0 \cdot 079}$$

Refrigerating capacity = $\dot{m}_r \times$ R. E.

$$17 \cdot 6 = \dot{m}_r \times 18 \cdot 85$$

$$\text{Power per TR} = \frac{\dot{m}_r \times \text{Work done}}{\text{Refrigerating capacity}}$$

$$= \frac{0 \cdot 93 \times 236 \cdot 19}{5} = 44 \cdot 10\ kW / TR$$

$$(COP)_{carnot} = \frac{T_1}{T_2 - T_1}$$

$$\therefore \quad = \frac{280}{416 \cdot 82 - 280} = \mathbf{2 \cdot 046}$$

Ex. 5.38 *A refrigeration system based on Bell - coleman cycle working between the pressure limit of 4 bar and 16 bar extracts 125 M J/hr. The air enters the compressor at 5 °C and enters the expander at 23 °C. The compressor and expander mechanical efficiencies are 82 % and 87 % respectively determine.*

i) Power required to run the unit.

ii) Mass circulation rate of air.

iii) COP

iv) carnot COP, Take $C_p = 1 \cdot 005$ kJ/kg K
$C_v = 0.718$ kJ/kg K).

Sol. : Given data :

p_1 = 4 bar, p_2 = 16 bar

$T_1 = 5\ °C = 5 + 273 = 278$ K

$T_3 = 23\ °C = 23 + 273 = 296$ K

Heat extract = 125 MJ/hr

$$= \frac{125 \times 10^6}{3600} = 34722.2\ J/sec$$

$$= 34 \cdot 72\ kJ/sec$$

We have given values of C_p and C_v

$$\therefore \quad \gamma = \frac{C_p}{C_r} = \frac{1.005}{0.718} = 1.399 \approx 1.4$$

Process 1-2 is isentropic compression,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{T_2}{278} = \left(\frac{16}{4}\right)^{\frac{1 \cdot 4 - 1}{1 \cdot 4}}$$

$$\mathbf{T_2 = 413.84\ K}$$

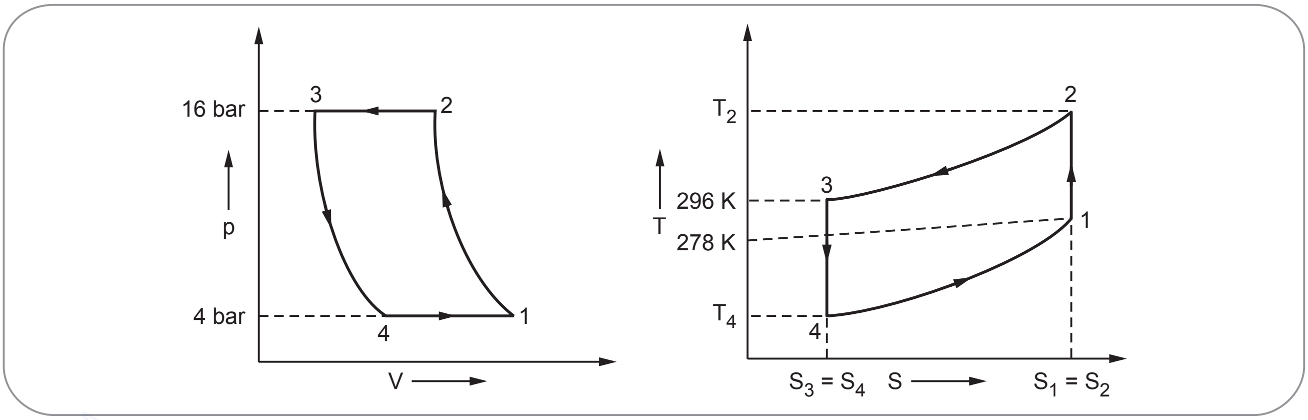


Fig. 5.46

Similarly, process 3-4 is isentropic expansion,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{296}{T_4} = \left(\frac{16}{4}\right)^{\frac{1.4 - 1}{1.4}}$$

$$\mathbf{T_4 = 198.84\ K}$$

Net work done,

$$(W)_{net} = C_p [(T_2 - T_1) - (T_3 - T_4)] = 1.005[(413.84 - 278) - (296 - 198.84)] = 38.87 \text{ kJ/kg}$$

$$\text{R.E.} = C_p (T_1 - T_4) = 1.005 (278 - 198.84) = 79.55 \text{ kJ/kg}$$

$$\text{COP} = \frac{\text{R.E}}{\text{W.D}} = \frac{79.55}{38.87} = 2.04$$

$$\text{carnot COP} = \frac{T_1}{T_2 - T_1} = \frac{278}{413.84 - 278} = \mathbf{2.04}$$

To find out mass of refrigerant ($\dot{m}_r$)

$$\dot{m}_r \times \text{R. E.} = \text{Heat extracted}$$

$$\dot{m}_r \times 79.55 = 34.72$$

$$\dot{m}_r = 0.436 \text{ kg/sec.}$$

Power required to run the unit,

$$P = (P)_{\text{compressor}} - (P)_{\text{Expander}} = \left[\frac{\dot{m}_r C_P (T_2 - T_1)}{\eta_{\text{mech.comp.}}} - \frac{\dot{m}_r C_P (T_3 - T_4)}{\eta_{\text{mech.Exp.}}}\right]$$

$$= \left[\frac{0.436 \times 1.005 \times (413.84 - 278)}{0.82}\right] - \left[\frac{0.436 \times 1.005 \times (296 - 198.84)}{0.87}\right]$$

$$= 72.58 - 48.93 = \mathbf{23.64\ kW}$$

5.11 Vapour Absorption System

- The principle of vapour absorption was first introduced by Michael Faraday in 1824.
- Vapour absorption refrigeration system is one of the oldest methods of producing refrigerating effect.
- In vapour absorption system, compressor is replaced by an absorber, pump, generator and pressure reducing valve.
- The vapour absorption system require at least two fluids.
- One fluid acts as refrigerant while other will acts as an absorber.
- The vapour absorption system can be built in the capacities above 1000 TR.

5.12 Working of Simple Vapour Absorption System

AU : May-16, 18

- It consists of following parts

  i) Absorber ii) Pump

  iii) Generator iv) Pressure reducing valve
- These four components replaces the compressor which is present in VCC.
- Remaining components are

  v) Condenser vi) Receiver

  vii) Expansion valve and viii) Evaporator
- In this system, low pressure ammonia vapours leave the evaporator.
- These vapours enter into absorber.
- These ammonia vapours are absorbed by the water and forms a solution known as aqua ammonia.
- Water has strong affinity of ammonia vapours.
- Some form of cooling arrangement is made in absorber to remove the heat of solution.
- The strong solution which is formed in absorber is pumped to generator by liquid pump.
- In the generator strong solution of ammonia is heated by external source.
- Due to heating process ammonia vapours are formed and the vapours move to the condenser leaving behind the hot weak ammonia solution in generator.
- This weak ammonia solution flows back to absorber through pressure reducing valve.
- High pressure ammonia vapours are condensed in condenser where the change of phase of ammonia vapour takes place.

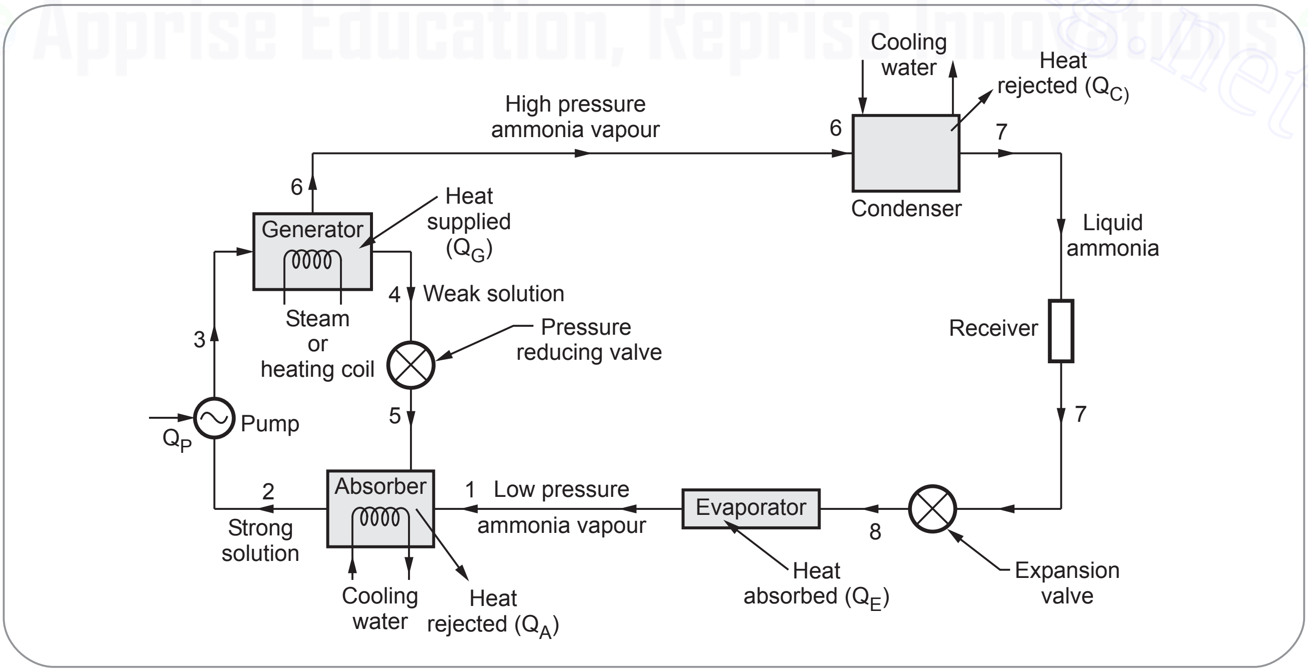


Fig. 5.47: Simple vapour absorption system

- This liquid ammonia is passed to expansion valve through the receiver and then it moves to evaporator.
- In this way simple vapour absorption cycle completes.

5.13 Practical Vapour Absorption System

AU : May-18

- Simple vapour absorption system is not very economical and in actual practice COP of system is very low.
- Therefore to make it practical some modifications are made.
- Following components are added to improve the performance of cycle.
  i) Analyzer ii) Rectifier iii) Heat exchanger
- Analyzer is provided to remove unwanted water particles associated with ammonia vapour before going to condenser.
- If these unwanted water particles are not removed they will enter in the expansion valve where they freeze and choke the pipe line.
- Analyzer consists of series of trays mounted above the generator.
- In some cases, vapour leaving the analyzer still contains some amount of water particles.
- Rectifier is cooling coil provided to remove moisture so that almost all water particles will be removed leaving only dehydrated ammonia vapour to enter into condenser.
- Heat exchanger is provided between pump and generator.
- Its function is to cool the weak hot solution returning from generator to absorber.
- The heat removed from weak solution raises the temperature of strong solution leaving the pump and going to analyser and generator.

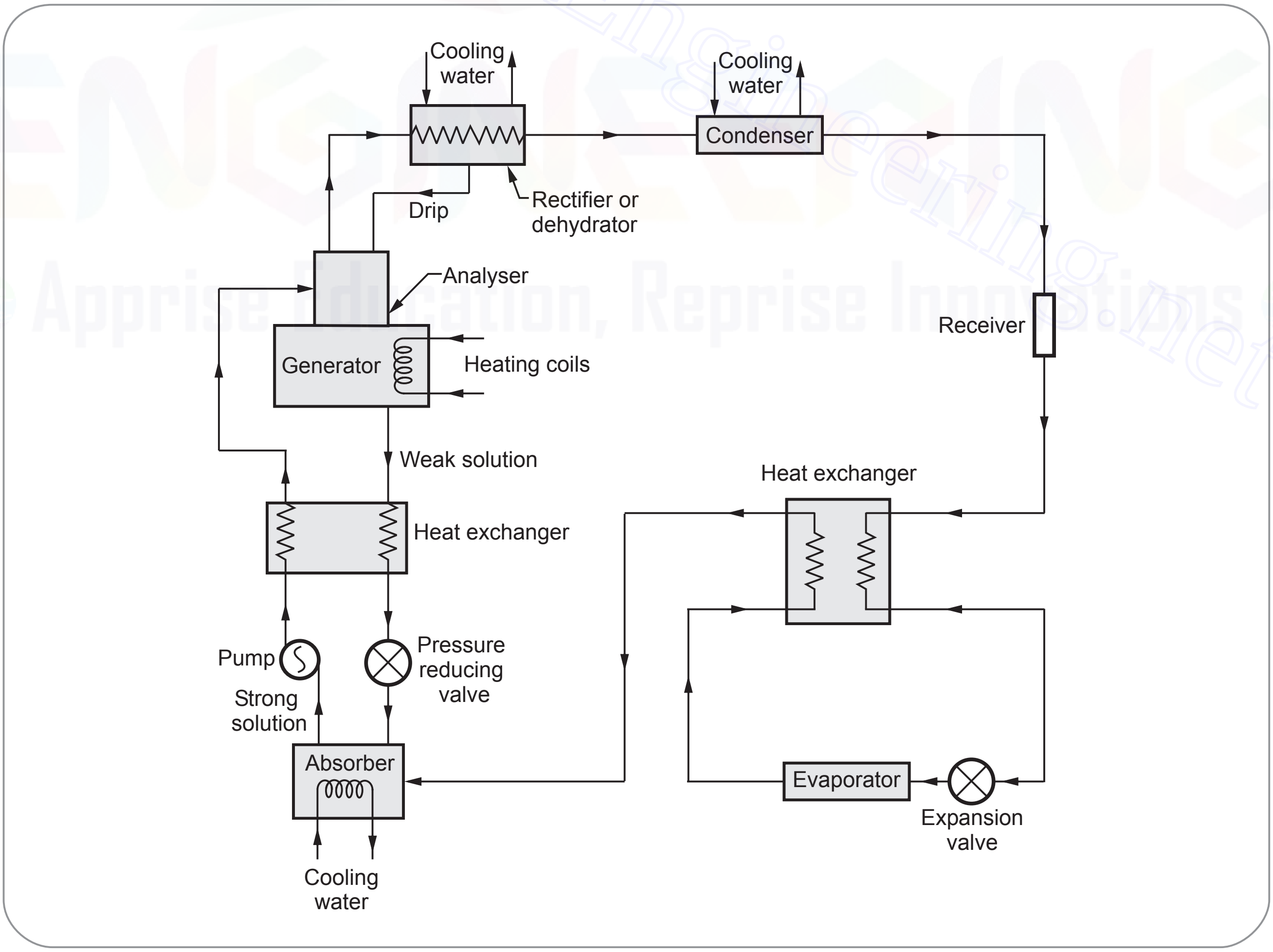


Fig. 5.48 : Practical vapour absorption system

- The heat exchanger which is provided between condenser and evaporator is also called as liquid subcooler.
- COP can be determined by following formula.

$$\text{COP} = \frac{\text{Heat absorbed in evaporator}}{\text{Work done by pump + Heat supplied in generator}}$$

5.14 Performance Evaluation of Vapour Absorption System

- Q_G - Heat given to the refrigerant in Generator
- Q_C - Heat discharged to atmosphere
- Q_E - Heat absorbed by refrigerant in evaporator
- Q_P - Heat added to refrigerant due to pumpwork.

Neglecting the heat due to pumpwork.

- According to I law of Thermodynamics,

We can write,

$$Q_C = Q_G + Q_E \qquad \text{...(i)}$$

Now,

- T_G - Temperature at which heat is given to generator
- T_C - Temperature at which heat is discharged to atmosphere
- T_E - Temperature at which heat is added to evaporator

$$\frac{Q_G}{T_G} + \frac{Q_E}{T_E} = \frac{Q_C}{T_C} \qquad \text{...(ii)}$$

But from equation (i) we can write,

$$\frac{Q_G}{T_G} + \frac{Q_E}{T_E} = \frac{Q_G + Q_E}{T_C}$$

Rearranging above equation,

$$\frac{Q_G}{T_G} - \frac{Q_G}{T_C} = \frac{Q_E}{T_C} - \frac{Q_E}{T_E}$$

$$Q_G\left(\frac{1}{T_G} - \frac{1}{T_C}\right) = Q_E\left(\frac{1}{T_C} - \frac{1}{T_E}\right)$$

$$Q_G\left(\frac{T_C - T_G}{T_G \times T_C}\right) = Q_E\left(\frac{T_E - T_C}{T_C \times T_E}\right)$$

$$\therefore \quad Q_G = Q_E\left[\frac{T_E - T_C}{T_C \times T_E}\right]\left[\frac{T_G \times T_C}{T_C - T_G}\right]$$

$$Q_G = Q_E\left[\frac{T_C - T_E}{T_C \times T_E}\right]\left[\frac{T_G \times T_C}{T_G - T_C}\right]$$

$$Q_G = Q_E\left[\frac{T_C - T_E}{T_E}\right]\left[\frac{T_G}{T_G - T_C}\right] \qquad \text{...(iii)}$$

- Maximum COP of the system is given by,

$$(\text{COP})_{max} = \frac{Q_E}{Q_G} = \frac{Q_E}{Q_E\left[\frac{T_C - T_E}{T_E}\right]\left[\frac{T_G}{T_G - T_C}\right]}$$

$$\therefore \quad \boxed{(\text{COP})_{max} = \left[\frac{T_E}{T_C - T_E}\right]\left[\frac{T_G - T_C}{T_G}\right]}$$

- COP of Carnot Refrigerator $= \dfrac{T_E}{T_C - T_E}$
- Efficiency of Carnot Engine $= \dfrac{T_G - T_C}{T_G}$
- Maximum COP can be written as,

$$\therefore \quad \boxed{(\text{COP})_{max} = (\text{COP})_{carnot} \times \eta_{carnot}}$$

5.14.1 Solved Numericals on VAS

Ex. 5.39 : *In a vapour absorption system, heating, cooling and refrigeration temperature are 115° C, 30° C and – 20° C respectively. Find the C.O.P. of the system. In case of heating temperature is increased to 200° C and refrigeration temperature and cooling temperature remaining same, find the new COP.*

Sol. :

Given Data : (I - Case)

$$T_G = 115\ °C = 115 + 273 = 388\ K$$

$$T_C = 30°\ C = 30 + 273 = 303\ K$$

$$T_E = -20°\ C = -20 + 273 = 253\ K$$

We have, $$\text{COP} = \left[\frac{T_E}{T_C - T_E}\right]\left[\frac{T_G - T_C}{T_G}\right]$$

$$= \left[\frac{253}{303 - 253}\right]\left[\frac{388 - 303}{388}\right] = 1.10$$

Given Data : (II - Case)

$$T_{G1} = 200°\ C = 200 + 273 = 473\ K$$

$$T_{E1} = -20°\ C = -20 + 273 = 253\ K$$

$$T_{C1} = 30^\circ C = 30 + 273 = 303\ K$$

$$COP = \left[\frac{T_{E1}}{T_{C1} - T_{E1}}\right]\left[\frac{T_{G1} - T_{C1}}{T_{G1}}\right]$$

$$= \left[\frac{253}{303 - 253}\right]\left[\frac{473 - 303}{473}\right] = 1.81$$

Ex. 5.40 : *In an absorption system, heating, cooling and refrigeration takes place at temperature of 115° C, 30° C and – 20° C respectively. Find Theoretical COP of the system. If the generator temperature increased to 200° C and eveporator temperature decreased to – 40° C. Find % change in COP of system.*

Sol. : Given Data : (I - Case)

$$T_G = 115\ ^\circ C = 115 + 273 = 388\ K$$

$$T_C = 30^\circ C = 30 + 273 = 303\ K$$

$$T_E = -20^\circ C = -20 + 273 = 253\ K$$

We have, $$COP = \left[\frac{T_E}{T_C - T_E}\right]\left[\frac{T_G - T_C}{T_G}\right]$$

$$= \left[\frac{253}{303 - 253}\right]\left[\frac{388 - 303}{388}\right] = 1.1081$$

Given Data : (II - Case)

$$T_{G1} = 200^\circ C = 200 + 273 = 473\ K$$

$$T_{C1} = 30^\circ C = 30 + 273 = 303\ K$$

$$T_{E1} = -40^\circ C = -40 + 273 = 233\ K$$

$$(COP)_1 = \left[\frac{T_{E1}}{T_{C1} - T_{E1}}\right]\left[\frac{T_{G1} - T_{C1}}{T_{G1}}\right]$$

$$(COP)_1 = \left[\frac{233}{303 - 233}\right]\left[\frac{473 - 303}{473}\right] = 1.1963$$

$$\%\ \text{change in COP} = \frac{COP_1 - COP}{COP} \times 100$$

$$= \frac{1.1963 - 1.1081}{1.1081} \times 100$$

$$= 7.96\ \%\ \text{(increased)}$$

Ex. 5.41 : *In a vapour absorption refrigeration system, the refrigeration temperature is – 15°. The generator is operated by solar heat where temperature reached is 110° C. The temperature of heat sink is 55° C. What is the maximum possible COP ?*

Sol. : Given Data :

$$T_G = 110\ ^\circ C = 110 + 273 = 383\ K$$

$$T_C = 55^\circ C = 55 + 273 = 328\ K$$

$$T_E = -15^\circ C = -15 + 273 = 258\ K$$

$$(COP)_{max} = \left[\frac{T_E}{T_C - T_E}\right]\left[\frac{T_G - T_C}{T_G}\right]$$

$$= \left[\frac{258}{328 - 258}\right]\left[\frac{383 - 328}{383}\right]$$

$$\mathbf{(COP)_{max} = 0.5292}$$

Ex. 5.42 : *In an aqua - ammonia absorption system, heat is supplied to the generator by condensing the steam at 20 kPa and 0.95 dry. The evaporator is maintained at – 5° C. The cooling water available for condenser at 30° C. Find ideal COP. If actual COP of the system is 80 % of ideal COP. Find mass of the steam needed in kg/hr to produce refrigeration load of 60 TR.*

Sol. : Given Data :

$$p = 200\ kPa = 2\ bar$$

$$x = 0.95$$

$$T_E = -5^\circ C = -5 + 273 = 268\ K$$

$$T_C = 30^\circ C = 30 + 273 = 303\ K$$

$$[COP]_{actual} = 80\ \%\ [COP]_{ideal}$$

$$R.E. = 60\ TR$$

From steam table, at p = 2 bar

$$T_{sat} = T_G = 120^\circ C = 120 + 273 = 393\ K$$

$$h_{fg} = 2200\ kJ/kg$$

$$(COP)_{ideal} = \left[\frac{T_E}{T_C - T_E}\right]\left[\frac{T_G - T_C}{T_G}\right]$$

$$= \left[\frac{268}{303 - 268}\right]\left[\frac{393 - 303}{393}\right] = \mathbf{1.7535}$$

$$\text{Actual COP} = 0.8 \times (COP)_{ideal} = 0.8 \times 1.7535$$

$$= \mathbf{1.4028}$$

$$\text{Actual COP} = \frac{\text{Refrigerating load}}{\text{Actual heat supplied}}$$

$$\text{Actual heat supplied} = \frac{60 \times 3.516}{1.4028} = 150.38 \text{ kJ/sec}$$

$$\text{Heat supplied per kg} = x \times h_{fg} = 0.95 \times 2200$$

$$= \mathbf{2090 \text{ kJ/kg}}$$

$$\text{Mass of steam needed} = \frac{150.38 \times 3600}{2090}$$

$$= \mathbf{259.02 \text{ kg/hrs}}$$

5.15 Lithium - Bromide Absorption

AU : May-16

- This refrigeration system is used to chill water.
- In this system water is used as refrigerant and Li - Br salt used as absorbent.
- Because of low vapour pressure Li - Br solution has a strong affinity for water vapour.
- It consists of absorber, generator, heat exchanger, condenser and evaporator.
- In the evaporator, refrigerant water evaporates by absorbing the latent heat from water which is to be chilled.
- The water vapours are drawn in absorber.
- In the absorber weak solution of Li - Br absorbs the water vapour and gets converted into strong solution.
- In the absorber cooling coil is placed, to increase the affinity for water vapour.
- The strong solution of Li - Br is pumped to a generator through heat exhanger.
- The function of heat exchanger is to exchange the heat from weak solution to strong solution.
- In the generator strong solution is heated by means of heating coil to release the water vapour.
- As the water vapours are releases, the strong solution is converted into weak solution.

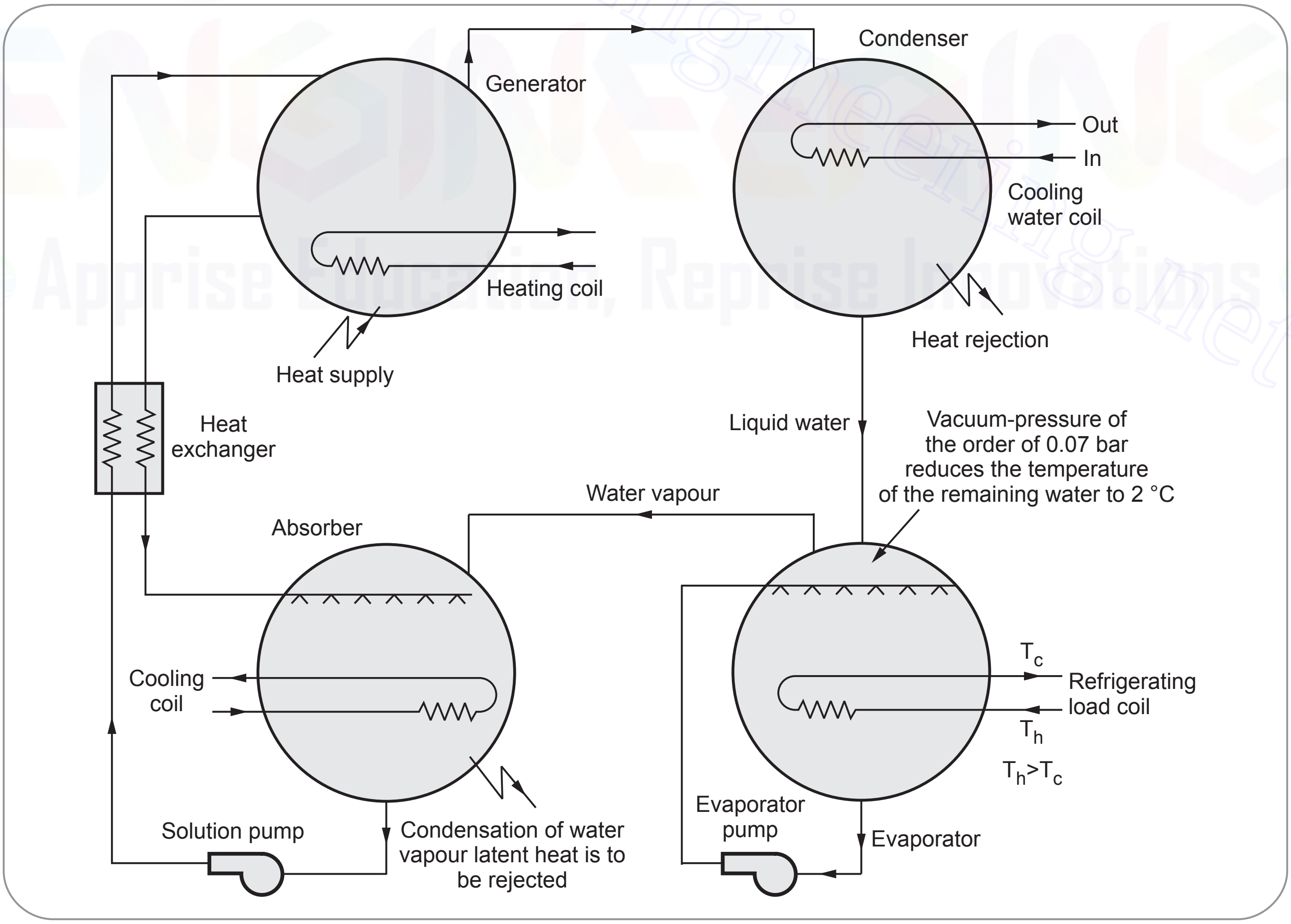


Fig. 5.49 Li - Br - Absorption refrigeration system

- This weak solution is passed to the absorber through heat exchanger.
- The water vapours which are generated in the generator are passed to condenser, where it is condensed by condensing water supplied externally.
- This condensed liquid refrigerant finally passed to the evaporator where evaporation will takes place.

5.16 Practical Li - Br Absorption System OR Two Shell type Li - Br Absorption System

- It consists of following major components.

i) Generator ii) Heat exchanger

iii) Absorber iv) Solution Pump

v) Evaporator pump vi) Condenser

vii) Evaporator

- The weak Li - Br solution transfered to generator through heat exchanger with the help of solution pump.
- In the heat exchanger, weak solution from the absorber is heated by strong solution flowing from the generator.
- It reduces heat requirement in the generator and cooling load in absorber.
- The water vapour refrigerant is condensed in condenser by circulation of water.
- The condensed water vapour refrigerant flows down towards evaporator. During which the refrigerants pressure is reduced upto evaporator pressure by pressure reducing valve.
- In the evaporator cooled water absorbs the heat from hot water.
- During this process hot water gives away the heat and converted into chilled water.

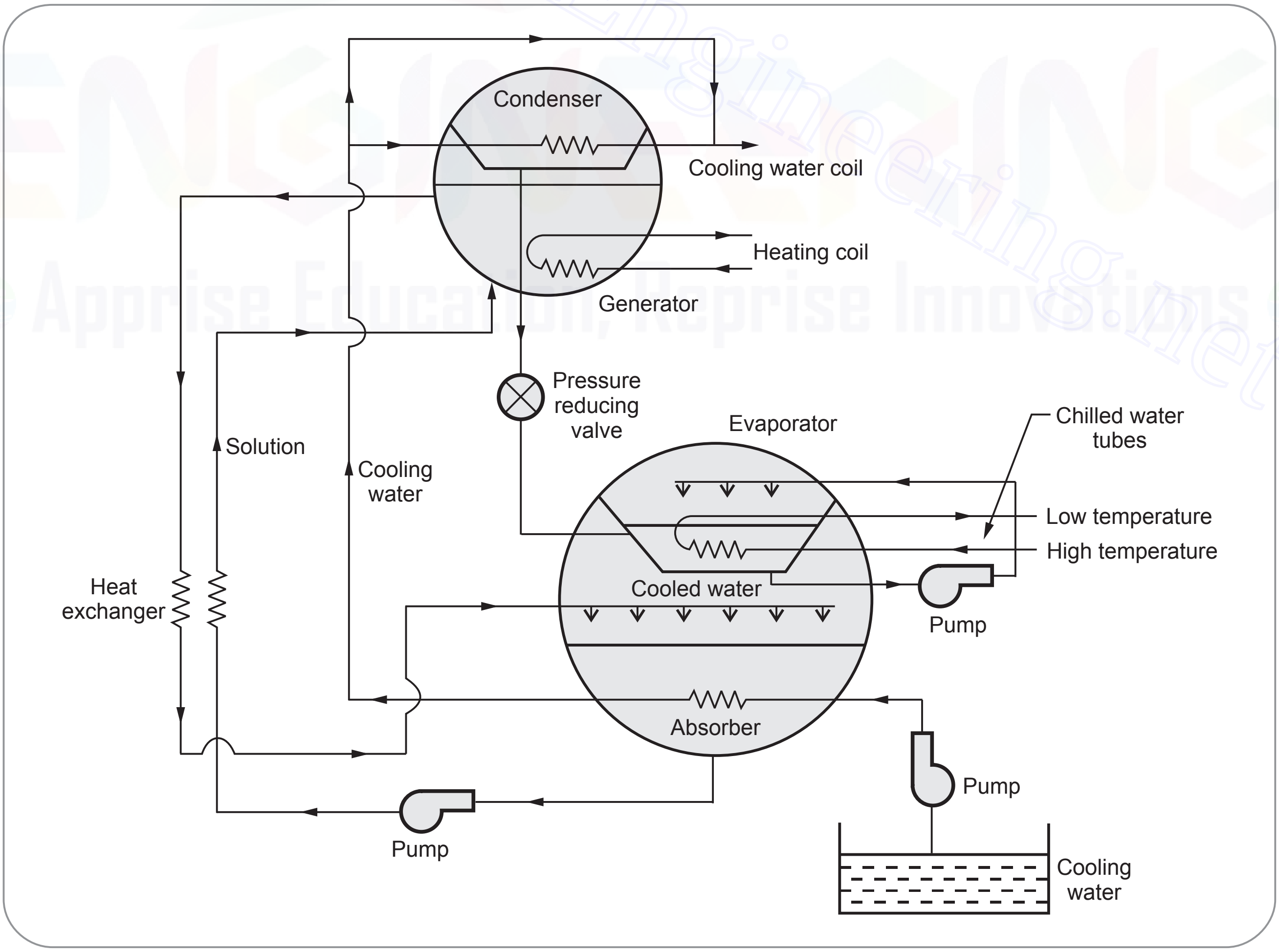


Fig. 5.50 Practical VAS (Li - Br)

5.16.1 Advantages and Disadvantages

Advantages

- Operation and maintenance cost is very low.
- The pressure inside the system is below atmospheric.
- These plants have been built up to 1 lakh TR capacity.
- The work energy requirements are negligible.

Disadvantages

- Lithium - Bromide is corrosive.
- All joints must be made leakproof to prevent the leakages.
- Once the system stops working, the salt solution may solidify and it causes replacement of pipe.

5.17 Three Fluid System (Electrolux Refrigerator)

- In this system, three fluids are used hence it is called as three fluid system.
- The three fluids used in this system are ammonia (NH_3), Hydrogen (H_2) and Water (H_2O).
- The main purpose of this system is to eliminate the pump hence system will become noiseless.
- **Ammonia** (NH_3) used as **refrigerant** because it possesses most desirable properties.
- The **hydrogen** (H_2) is the lightest gas used to **increase the rate of evaporation** of liquid ammonia passing through evaporator.
- The **water** is used as **solvent** because it has the ability to absorb ammonia.
- The strong ammonia solution from absorber enters into the generator through heat exchanger.
- In the generator ammonia solution is heated by external source.
- During heating process, ammonia vapours are formed, and these ammonia vapours are removed from solution and passed to condenser.
- The rectifier is fitted between the generator and condenser to remove the water vapour. If these water vapours are not removed it will enter into the evaporator and causes freezing and choking of machine.

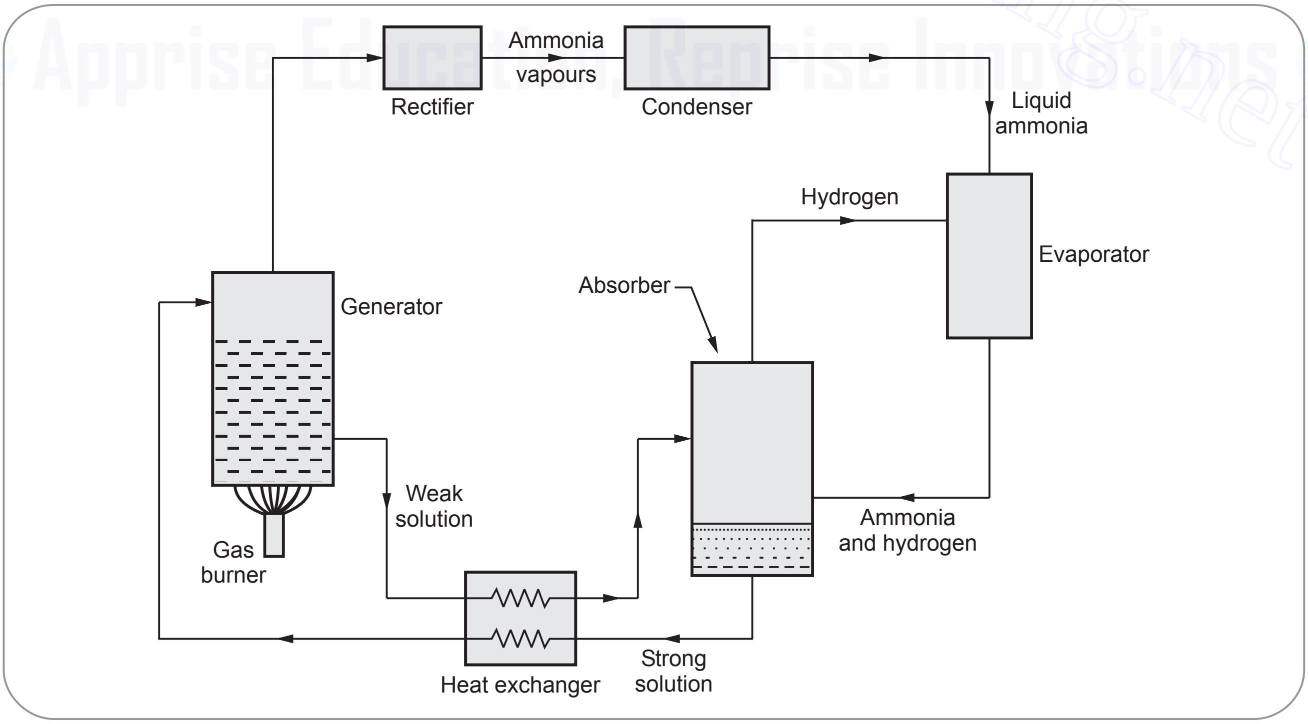


Fig. 5.51 Three fluid system

- The hot weak solution which is in the generator is passed to absorber through heat exchanger.
- In the heat exchanger heat exchange will takes place between weak solution and strong solution.
- Ammonial vapours in condenser gets condensed by using external cooling source.
- The liquid refrigerant from the condenser flows into the evaporator where it meets hydrogen (H_2) gas.
- Finally the mixture of ammonia vapour and hydrogen is passed to absorber, where ammonia (NH_3) is absorbed in water and hydrogen flows back to evaporator.
- COP of the system can be find out,

$$COP = \frac{\text{Heat absorbed in evaporator}}{\text{Heat supplied to generator}}$$

5.18 Comparison of VCC and VAS

Sr.No.	Vapour Compression System	Vapour Absorption System
1.	This system is provided with compressor.	Compressor is replaced by absorber, generator, pump and pressure reducing valve.
2.	More wear and tear due to more no. of parts in compressors.	No wear and tear.
3.	Due to presence of compressor system is noisy.	System is quite.
4.	Cooling effect is instant.	System takes some time to produce cooling effect.
5.	Poor performance at partial load.	Better performance at partial load.
6.	It requires high grade energy.	It can be used with low grade energy.
7.	More chances of refrigerant leakage.	Less chances of refrigerant leakage.
8.	COP of the system is high.	COP of system is low.
9.	More maintenance is required	Less maintenance is required.
10.	Charging of refrigerant is simple.	Charging of refrigerant is difficult.

5.19 Thermoelectric Refrigeration

- Thermoelectric cooling uses the Peltier effect to create a heat flux between the junctions of two different types of materials.
- Thermoelectricity is based upon following basic principles :

1. Seebeck effect 2. Peltier effect

3. Thomson effect 4. Joule effect

5. Fourier effect

SeeBeck Effect :

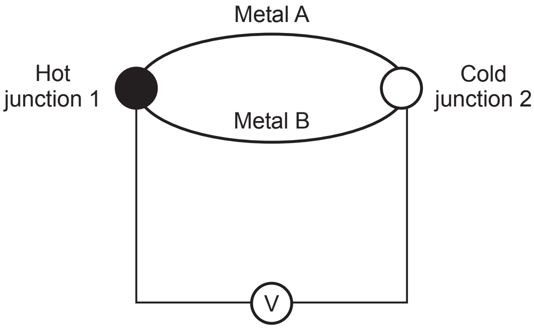


Fig. 5.52

- Thermoelectric power supply generators are based on the Seebeck effect.
- It is based on voltage generation along a conductor subjected to a gradient of temperature.
- An electromotive force is produced because of the temperature gradient is applied to a conductor.
- The generated voltage difference is proportional to the temperature difference across the thermoelectric module between the two junctions i.e. the hot and the cold junction. $\Delta V \propto \Delta T$.

Peltier Effect

- The Peltier effect plays the vital role in all thermoelectric cooling applications.

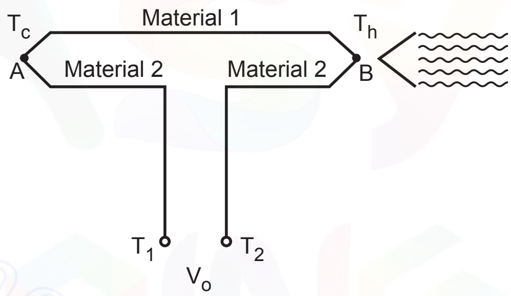


Fig. 5.53

- It is responsible for removal of heat and absorption of heat.
- It states that when an electric current flows across two dissimilar conductors, the junction of the conductors will either absorb or emit heat depending on the flow of the electric current.
- The heat absorbed or released at the junction is proportional to the input electric current.
- The constant of proportionality is called the Peltier coefficient.

Thomson Effect

- It states when an electric current is passed through a conductor having a temperature gradient over its length, heat will be either absorbed by or expelled from the conductor.
- The Thompson effect governs the cooling and the heating of a material carrying a current and subjected to a temperature gradient.

$$\frac{dQ}{dX} = \tau l \left[\frac{dT}{dX}\right]$$

Joule Effect :

- When electrical Current I flows through a conductor of resistance R, there is dissipation of electrical energy.
- This is well known joule effect.
- The energy dissipated is given by

$$Q = I^2 R$$

Fourier Effect :

- If the ends of any element are maintained at different temperatures, the heat transfer from the hot end to the cold end is related by

$$Q = U(T_h - T_c)$$

5.20 Basic Psychrometry

- The **psychrometry** is the field of engineering science concerned with the study of behaviour of gas and vapour mixtures. The word psychometric is derived from the Greek words 'psuchron' means "cold" and 'metron' means "*way of measurement*".
- From the subject point of view, psychometry is the study of various thermodynamic properties of air and water vapour mixture. It is used extensively to express and analyze the characteristics of various air conditioning processes.
- Understanding of properties of air and water vapour mixture and the various processes involving air is fundamental of air conditioning design.
- The moist air or air is a mixture of various gases, water vapour (moisture) and a number of pollutants. The air in absence of water vapour is called as dry air.
- The ordinary atmospheric air is always contains some amount of water vapour so the pure dry air doesn't really exist.
- The dry air is composed of various gases, mainly nitrogen (78 %) and oxygen (21 %). The remaining 1 % of the gases includes carbon dioxide and very small quantities of inert gases like hydrogen, helium, neon and argon.
- Dry air is used as the basis for defining properties of air since its composition is approximately remains same under any sets of conditions.
- However, the amount of moisture in air keep changing from one location to other and depending on the atmospheric conditions at a particular location. The places near to the sea areas contain more moisture while the desert areas contain less moisture.
- Similarly, during the rainy season, the moisture content of the air is high whereas during summer and winter seasons, it is low.
- It is important to note that, the water vapour exists in the superheated condition, but when it is cooled or heated there is change in its phases, hence it absorbs or liberates sensible heat as well as the latent heat due to changes in its phases. This is what makes the whole process of air conditioning highly complicated.
- Cooling of water vapour results in its condensation, while its heating leads to superheating.
- The following Table 5.2 represents properties of air constituents :

Gas	Ratio compared to dry air (%)		Molecular mass	Chemical symbol	Boiling point (°C)
	By volume	By weight	- M - (kg/kmol)		
Oxygen	20.95	23.20	32.00	O_2	– 196
Nitrogen	78.09	75.47	28.02	N_2	– 183
Carbon dioxide	0.03	0.046	44.01	CO_2	– 78.5
Hydrogen	0.00005	~ 0	2.02	H_2	– 252.87
Argon	0.933	1.28	39.94	Ar	– 186
Neon	0.0018	0.0012	20.18	Ne	– 246
Helium	0.0005	0.00007	4.00	He	– 269

Table 5.2 : Properties of air constituents

5.21 Psychrometric Terms

- There are number of psychrometric terms, some of them are as follows :

(1) Dry air	(2) Moist air
(3) Saturated air	(4) Degree of saturation
(5) Specific humidity	(6) Absolute humidity
(7) Relative humidity	(8) Dry bulb temperature
(9) Wet bulb temperature	(10) Wet bulb depression
(11) Dew point temperature	
(12) Dew point depression	

1) **Dry air :** It is a mixture of number of gases such as oxygen, nitrogen, carbon dioxide, hydrogen, argon, helium, etc.

2) **Moist air :** It is a mixture of dry air and water vapour. The amount of water vapour present in the air depends upon the absolute pressure and temperature of the mixture.

3) **Saturated air :** It is a mixture of dry air and water vapour, when the air has diffused the maximum amount of water vapour into it.

4) **Degree of saturation :** It is the ratio of actual mass of water vapour in a unit mass of dry air to the mass of water vapour in the same mass of dry air when it is saturated at the same temperature.

5) **Specific Humidity (w) [Moisture content] :** It is the ratio of mass of water vapour to mass of dry air in a particular volume of moist air (dry air plus the water vapour). **Humidity** or **humidity ratio** or **Specific humidity** is related to mixing ratio on mass basis. It is expressed in grams per kg of dry air (g/kg of dry air).

6) **Absolute Humidity (AH) :** Absolute Humidity is the mass of water vapour present in unit volume of moist air at a given temperature and pressure. It is measured in grams per cubic metre of air (g/m^3).

7) **Relative Humidity (ϕ) :**

- Relative Humidity (RH) is the most commonly used psychrometric unit. The "relative" in relative humidity expresses the relation between the amount of water vapour present and the maximum amount that is physically possible at that temperature.
- In other words, relative humidity (expressed in percentage) is the partial water vapour pressure in relation to it's saturation pressure.

8) **Dry Bulb Temperature (DBT) :** The Dry Bulb Temperature refers basically to the ambient air temperature as measured by a standard thermometer with a dry sensing bulb. It is called as Dry Bulb because the air temperature is indicated by a thermometer not affected by the moisture of the air. It is denoted by t_d or t_{db}.

9) **Wet Bulb Temperature (WBT) :** Temperature of air as measured by a thermometer using a sensing bulb covered by a wet cloth. The adiabatic evaporation of water from the thermometer and the cooling effect is indicated by a WBT. As the thermometer is moved through the air, water will evaporate from the cloth at a rate determined by the relative humidity of the surrounding air. The wet bulb temperature is always lower than the dry bulb temperature but at 100 % relative humidity, it will be identical as there would no moisture evaporation from the wet cloth. It is denoted by t_w or t_{wb}.

10) **Wet bulb depression :** The wet-bulb depression is the difference between the dry-bulb temperature and the wet-bulb temperature. It indicates relative humidity of the air.

11) **Dew Point Temperature (DPT) :**

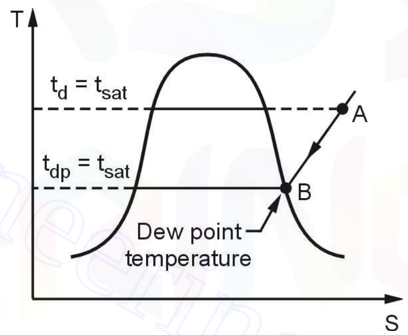


Fig. 5.54 : Dew point temperature

- The mass of water vapour that air can hold is dependent on the air temperature (and pressure). When the air is cooled, the amount of water it can hold is reduced and the relative humidity subsequently rises.
- If air is cooled to the point where it becomes 100 % saturated, then any further cooling will cause moisture to deposit out of the air in the form of condensation or dew.
- The temperature at which this saturation occurs is the 'dew point' of the air. Above this temperature the moisture will stay in the air.
- Thus, the dew point temperature can be defined as the temperature at which condensation of moisture begins when the air is cooled. The dew point is always lower than (or equal to) the air temperature.

- If unsatured air containing superheated water vapour is cooled at constant pressure, the partial pressure remains constant until the water vapour reaches the saturated state as shown in Fig. 8.1 (Point B).
- At this point B dew will be formed, hence the corresponding temperature is called as dew point temperature. It is denoted by $\mathbf{t_{dp}}$.

12) Dew point depression : It is the difference between the dry bulb temperature and dew point temperature of air.

In addition to psychrometric terms it is necessary to revise the following terms.

13) Specific Volume (v) : The specific volume is the volume of unit mass of dry air at a given temperature normally expressed as m^3/kg and is also shown in the psychrometric chart. Specific volume is the inverse of density.

14) Sensible Heat (SH) : It can be defined as heat which when applied to (or removed from) a substance, causes only a change in temperature. Sensible heat is measured in joules per kilogram (J/kg).

15) Latent Heat (LH) : It can be defined as heat which when applied to (or removed from) a substance, produces a change in state without any change to temperature. Latent heat is measured in joules per kilogram (J/kg).

16) Total heat : Total heat is the sum of sensible and latent heat.

5.21.1 Psychrometer

- To measure the different psychrometric properties psychrometer is used.
- There are different types of psychrometer but the **sling psychrometer** is commonly used.
- It consists of a dry bulb thermometer and wet bulb thermometer mounted side by side in a protective case. Refer Fig. 5.55.
- The protective case is attached to a handle by a swivel connection so that it can be easily rotated as per the requirement.

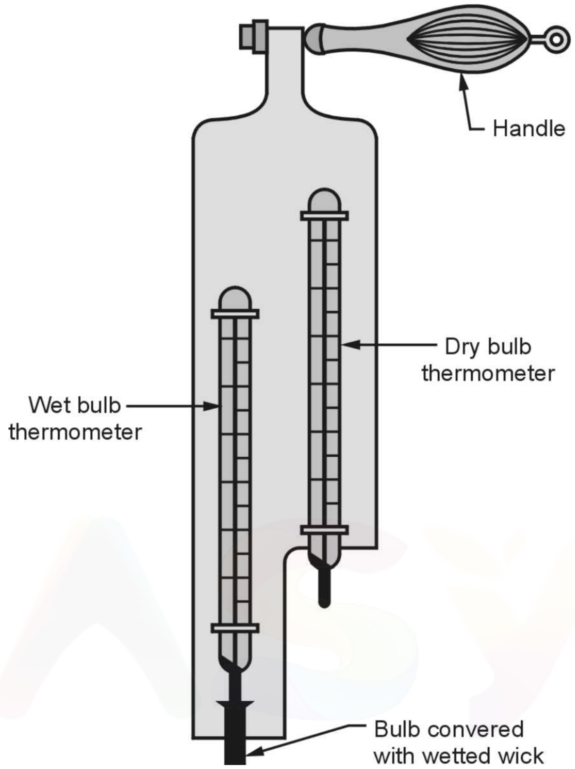


Fig. 5.55 : Sling psychrometer

- The sling psychrometer is rotated in the air for almost 1 minute and the readings of both the thermometers are taken.
- The dry bulb thermometer is directly exposed to air and measures the actual temperature of air.
- But, the bulb of wet bulb thermometer is covered by a wick (throroughly wetted by using distilled water).
- Now the temperature measured by this wick covered bulb is the temperature of liquid water in the wick which is known as **wet bulb temperature**.

5.22 Psychrometric Relations

- In previous section, we have studied psychrometric terms used in air conditioning. These terms have some co-relation between one another. Some of the important co-relations are discussed in this section as given below :

5.22.1 Dalton's Law of Partial Pressure

- It is used to evaluate the pressure of individual constituent of mixture of gases.
- This law states that, "The total pressure exerted by the mixture of air and water vapour is equal to the sum of the pressures, which each constituent would exert, if it is occupied the same space by itself."
- In simple words, the total pressure exerted by air and water vapour mixture is equal to the barometric pressure. Thus, in case of moist air the barometric pressure is given by,

$$p_b = p_a + p_v$$

where, p_b = Barometric pressure exerted by moist air

p_b = Partial pressure exerted by dry air

p_v = Partial pressure exerted by water vapour

- According to Dr. Carrier, the partial pressure of water vapour can be determined as follows :

$$p_v = p_w - \frac{[p_f - (p_{vs})_{wb}](t_{db} - t_{wb})}{1527.4 - 1.3 t_{wb}}$$

where, p_w = Saturation pressure of water vapour corresponding to wet bulb temperature (from steam table)

p_b = Barometric pressure of moist air

t_{db} = Dry bulb temperature (°C)

t_{wb} = Wet bulb temperature (°C)

5.22.2 Specific Humidity or Humidity (w)

- It is defined as, the mass of water vapour per unit mass of dry air in the moist air. It is denoted by 'w'.
- Mathematically specific humidity is given by,

$$\text{Specific humidity (w)} = \frac{\text{Mass of water vapour}}{\text{Mass of dry air}}$$

$$w = \frac{m_v}{m_a} \qquad \ldots(5.8)$$

Air is assumed to be perfect ideal gas, therefore using ideal gas equation, we have,

$$m_a = \frac{p_a v_a}{R_a T_a} \qquad \ldots(5.9)$$

and

$$m_v = \frac{p_v v_v}{R_v T_v} \qquad \ldots(5.10)$$

where, p_a = Partial pressure of dry air

p_v = Partial pressure of water vapour

v_a = Volume of dry air

v_v = Volume of water vapour

T_a = Temperature of dry air

T_v = Temperature of water vapour

R_a = Characteristic gas constant of dry air

R_v = Characteristic gas constant of water vapour

From equation (5.9) and (5.10),

$$w = \left(\frac{p_v v_a}{R_v T_a}\right) \Big/ \left(\frac{p_a v_v}{R_a T_v}\right)$$

$$= \frac{p_v v_a}{R_v T_a} \times \frac{R_a T_v}{p_a v_v}$$

$$w = \frac{R_a}{R_v}\frac{p_v}{p_a} \quad \ldots (\because T_a = T_v = T_d \text{ and } v_a = v_v)$$

But, $$R_a = \frac{R_o}{m_a} = \frac{8.3143}{28.97} = 0.287 \text{ kJ/kg K}$$

and $$R_v = \frac{R_o}{m_v} = \frac{8.3143}{18} = 0.462 \text{ kJ/kg K}$$

Thus, $$w = \frac{0.287}{0.462} \times \frac{p_v}{p_a} = 0.622 \frac{p_v}{p_a}$$

$$\mathbf{w = 0.622\left(\frac{p_v}{p_b - p_v}\right)}$$

$$(\because p_b = p_a + p_v) \qquad \ldots(5.11)$$

5.22.3 Relative Humidity (RH) (ϕ)

- According to definition of relative humidity,

$$RH(\phi) = \frac{\text{Mass of water vapour associated with unit mass of dry hair}}{\text{Mass of water vapour associated with unit mass of dry saturated air}}$$

$$\phi = \frac{m_v}{m_{vs}}$$

$$= \left(\frac{p_v v_v}{R_v T_v}\right) \Big/ \left(\frac{p_{vs} \cdot v_{vs}}{R_{vs} \cdot T_{vs}}\right)$$

But, $v_v = v_{vs}$, $T_v = T_{vs}$ and $R_v = R_{vs}$

$$\therefore \qquad \phi = \frac{p_v}{p_{vs}}$$

Also,

$$\phi = \frac{\mathbf{v}_{\mathbf{vs}}}{\mathbf{v}_{\mathbf{v}}} \qquad \ldots(5.12)$$

- Thus, the relative humidity can also be defined as the ratio of partial pressure of water vapour in an unsaturated air at a given temperature to the partial pressure of saturated air at the same temperature. It is generally expressed in percentage.
- When p_v is equal to p_{vs}, RH becomes unity and the air becomes fully saturated and is considered to have 100 % RH. Relative humidity indicates the closeness of air to its saturated condition. It decides the rate of evaporation.

As,

$$RH(\phi) = \frac{p_v}{p_{vs}}$$

But, we know,

$$w = 0.622 \frac{p_v}{p_a} \quad \text{or} \quad p_v = \frac{w}{0.622} p_a$$

$$\phi = \frac{p_v}{p_{vs}} = \frac{w}{0.622} \frac{p_a}{P_{vs}}$$

5.22.4 Degree of Saturation (μ)

- It is the ratio of actual mass of water vapour in a unit mass of dry air to the mass of water vapour in the same mass of dry air when it is saturated at the same temperature.
- In other words, it is also defined as the ratio of actual specific humidity to the specific humidity of saturated air at the same dry bulb temperature.
- Degree of saturation is the another property of air that gives relation of air to its saturation state. It is generally, denoted as μ and mathematically written as,

$$\mu = \frac{w}{w_s}$$

where, w and w_s are specific humidities of air and saturated air respectively.

- From the equations of specific humidity above equation can be modified as,

$$\mu = \frac{0.622\left(\dfrac{p_v}{p_b - p_v}\right)}{0.622\left(\dfrac{p_{vs}}{p_b - p_{vs}}\right)}$$

$$\mu = \frac{p_v}{p_{vs}}\left(\frac{p_b - p_{vs}}{p_b - p_v}\right) = \phi\left(\frac{p_b - p_{vs}}{p_b - p_v}\right)$$

$$\phi = \mu\left(\frac{p_b - p_v}{p_b - p_{vs}}\right) = \mu\left(\frac{1 - \dfrac{p_v}{p_b}}{1 - \dfrac{p_{vs}}{p_b}}\right) \quad \ldots (5.13)$$

5.22.5 Enthalpy of Moist Air

- The total heat (enthalpy) of moist air is mathematically equal to the enthalpy of dry air plus the enthalpy of water vapour associated with dry air.

Mathematically, $h = h_a + w \cdot h_v$

Enthalpy of 1 kg of dry air is,

$$h_a = C_{pa}\, t_{db}$$

where, C_{pa} = Specific heat of dry air = 1.005 kJ/kg K

t_{db} = Dry bulb temperature

and enthalpy of water vapour associated with 1 kg of dry air.

$$h_v = w\, h_s$$

where, w = Mass of water vapour in 1 kg of dry air

h_s = Enthalpy of water vapour per kg of dry air at dew point temperature (t_{dp})

- If the moist air is superheated, then the enthalpy of water vapour is,

$$h_v = w\, C_{ps}(t_{db} - t_{dp})$$

Where, C_{ps} = Specific heat of superheated water vapour (generally taken as 1.88 kJ/kg K)

$t_{db} - t_{dp}$ = Degree of superheat of water vapour

Now, the total enthalpy of superheated water vapour is given by,

$$h = (C_{pa} + w\, C_{ps})\, t_{db} + w\,[h_{fgdp} + 2.3\, t_{dp}] \quad \ldots (5.14)$$

- The term $(C_{pa} + w\, C_{ps})$ is called **humid specific heat (C_{pm})**. It is the specific heat or heat capacity of moist air i.e. (1 + w) kg/kg of dry air. The general value of humid specific heat in air conditioning range is taken as 1.022 kJ/kg K.

Enthalpy of moist air is given by, $h = 1.022\, t_{db} + w\,(h_{fgdp} + 2.3\, t_{dp})$ kJ/kg

where, h_{fgdp} = Latent heat of vapourisation of water corresponding to dew point temperature (from steam table) = 2500 kJ/kg.

- Better approximation may be obtained by following equation,

$$\mathbf{h = 1.005\, t_{db} + w(2500 + 1.88\, t_d)\, kJ / kg} \quad \dots (5.15)$$

5.23 Solved Examples

Ex. 5.43 : *For a sample of air having DBT of 22 °C, RH 30 % at barometric pressure of 0.76 m of Hg determine the following parameters by using psychrometric chart :*
i) Vapour pressure ii) Humidity ratio iii) Vapour density iv) Enthalpy.

Sol. : Given data :

DBT (t_d) = 22 °C, RH ϕ = 30 % = 0.3,

p_b = 0.76 m of Hg

To find : i) p_v ii) w iii) ρ iv) h

- Mark the initial condition of air i.e. 22 °C DBT and 30 % RH on a psychrometric chart as shown in Fig. 5.56. (Say point A).
- Now, from point A draw a horizontal line which meets the vapour pressure line at B and humidity ratio line at C.

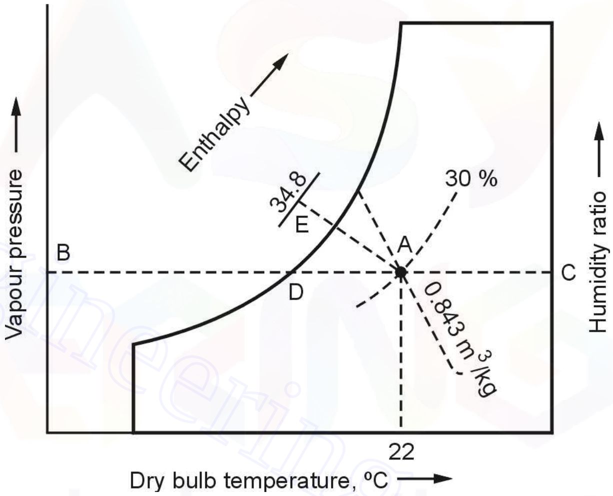


Fig. 5.56

By measurement on psychrometric chart, we get,

(i) Vapour pressure at point B,

$$p_v = 5.95 \text{ mm of Hg} = 5.95 \times 10^{-3} \text{ m of Hg}$$

$$\therefore \quad p_v = 5.95 \times 10^{-3} \times 13.6 \times 9810 = 793.8252 \text{ Pa} \quad \dots (\because p = \gamma h)$$

$$\mathbf{p_v = 793.8252 \times 10^{-5}\ bar = 0.007938\ bar} \quad \dots \textbf{Ans.}$$

(ii) Humidity ratio at point C,

$$w = 5 \text{ g/kg of dry air} = \mathbf{5 \times 10^{-3}\ kg/kg\ of\ dry\ air} \quad \dots \textbf{Ans.}$$

(iii) Specific volume at point A,

$$v_a = 0.843 \text{ m}^3/\text{kg of dry air}$$

(iv) Vapour density,

$$\rho_v = \frac{w}{v_a} = \frac{5 \times 10^{-3}}{0.843} = \mathbf{5.9311 \times 10^{-3}\ kg / m^3} \quad \dots \textbf{Ans.}$$

Also draw a line from point A to the WBT line meeting the enthalpy line at point E.

(v) Enthalpy from the chart is,

$$h = \mathbf{34.8\ kJ/kg\ of\ dry\ air} \quad \text{... Ans.}$$

Ex. 5.44 : *Atmospheric air at 1.0132 bar has a DBT of 32 °C and a WBT of 26 °C. Compute.*
i) Partial pressure of water vapour ii) Specific humidity
iii) Dew point temperature iv) Relative humidity
v) Degree of saturation vi) Density of air in the mixture
vii) Density of vapour in the mixture and
viii) Enthalpy of the mixture.

Sol. : Given data :

$$p_b = 1.0132 \text{ bar},\ t_{db} = 32\ °C,\ t_{wb} = 26\ °C$$

To find : p_v, w, t_{dp}, ϕ, μ, ρ_v, h

Step 1 : Calculate partial pressure of water vapour.

Partial pressure of water vapour is,

$$p_v = p_w - \frac{(p_b - p_w)(t_{db} - t_{wb})}{1544 - 1.44\, t_{wb}}$$

From psychrometric chart,

$$\text{At } t_{db} = 32\ °C,\ p_{vs} = 0.0479 \text{ bar}$$

$$\text{At } t_{wb} = 26\ °C,\ p_w = 0.0338 \text{ bar}$$

$$\therefore \quad p_v = 0.0338 - \frac{(1.0132 - 0.0338)(32 - 26)}{1544 - 1.44\,(26)}$$

$$\therefore \quad \mathbf{p_v = 0.02989\ bar} \quad \text{... Ans.}$$

Step 2 : Calculate specific humidity and dew point temperature.

As, $$w = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622 \times 0.02989}{1.0132 - 0.02989}$$

$$\therefore \quad \mathbf{w = 0.01890\ kg/kg\ of\ dry\ air.} \quad \text{... Ans.}$$

From steam table,

$$\text{At } p_v = 0.02989 \text{ bar},\ \mathbf{t_{dp} = 23\ °C} \quad \text{... Ans.}$$

Step 3 : Calculate relative humidity and degree of saturation

Relative humidity is given by,

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.02989}{0.0479} = 0.6240$$

$$\therefore \quad \phi = \mathbf{62.40\ \%} \quad \text{... Ans.}$$

Now, $$(w_{sat}) = \frac{0.622\, p_{vs}}{p_b - p_{vs}} = \frac{0.622 \times 0.0479}{1.0132 - 0.0479}$$

$$\therefore \quad w_{sat} = 0.03086 \text{ kg/kg of dry air}$$

Now, degree of saturation is given by,

$$\therefore \quad \mu = \frac{w}{w_{sat}} = \frac{0.01890}{0.03086} = \mathbf{0.6124} \quad \text{... Ans.}$$

Step 4 : Calculate density of air and density of vapour in the mixture.

We know that,

$$\rho_v = \frac{w\,(p_b - p_v)}{R \cdot t_{db}}$$

$$= \frac{0.01890\,(1.0132 - 0.02989)\,10^5}{287 \times (32 + 273)}$$

$$\therefore \quad \rho_v = \mathbf{0.0237\ kg/m^3\ of\ dry\ air} \quad \text{... Ans.}$$

$$\text{Specific volume of dry air} = \frac{R \cdot t_{db}}{p_b - p_v}$$

$$\therefore \quad v_a = \frac{287 \times (32 + 273)}{(1.0132 - 0.02989)\,10^5}$$

$$= 0.8902 \text{ m}^3/\text{kg of dry air}$$

$$\text{Now, } \rho_a = \frac{1}{v_a} = \frac{1}{0.8902}$$

$$= 1.123 \text{ kg/m}^3 \text{ of dry air}$$

Step 5 : Calculate the enthalpy

Enthalpy is given by,

$$h = 1.005\, t_{db} + w\,(2500 + 1.88\, t_{db})$$

$$\therefore \quad h = 1.005\,(32) + 0.01890\,(2500 + 1.88\,(32))$$

$$\therefore \quad \mathbf{h = 80.547\ kJ/kg} \quad \text{... Ans.}$$

Ex. 5.45 : *A sling thermometer reads 40 °C DBT and 28 °C WBT. Find the following :*
i) Specific humidity ii) Relative humidity
ii) Dew point temperature iv) Vapour density.

Sol. : Given : $t_{db} = 40\ °C,\ t_{wb} = 28\ °C$

To find : i) w ii) ϕ iii) t_{dp} iv) ρ_v

Step 1 : Calculate specific humidity and relative humidity

From steam table,

$$\text{at } t_{db} = 40\ ^\circ C,\quad p_{vs} = 0.0737 \text{ bar}$$

$$\text{at } t_{wb} = 28\ ^\circ C,\ p_w = 0.03778 \text{ bar}$$

By Carrier's equation,

$$p_v = p_w - \left[\frac{(p_b - p_w)(t_{db} - t_{wb})}{1544 - 1.44\, t_{wb}}\right]$$

$$\therefore \quad p_v = 0.03778 - \left[\frac{(1.0132 - 0.03778)(40 - 28)}{1544 - 1.44 \times 28}\right]$$

$$= 0.03 \text{ bar}$$

Specific humidity is given by,

$$w = 0.622\left[\frac{p_v}{p_b - p_v}\right] = 0.622\left(\frac{0.03}{1.0132 - 0.03}\right)$$

$$\therefore \quad w = \mathbf{0.0189\ kg/kg\ of\ dry\ air} \qquad \text{... Ans.}$$

Relative humidity is given by,

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.03}{0.0737} = 0.407 = \mathbf{40.7\ \%} \qquad \text{... Ans.}$$

Step 2 : Calculate dew point temperature and vapour density

From steam table,

$$\text{at } p_v = 0.03 \text{ bar},\ \mathbf{t_{dp} = 26\ ^\circ C} \qquad \text{... Ans.}$$

$$\text{Specific volume of dry air} = v_a = \frac{R \cdot t_{db}}{p_a} = \frac{R \cdot t_{db}}{p_b - p_v}$$

$$\therefore \quad v_a = \frac{287 \times (40 + 273)}{(1.0132 - 0.03) \times 10^5}$$

$$= 0.9136\ m^3/kg$$

We know that,

$$\rho_v = \frac{w}{v_a} = \frac{0.0189}{0.9136} = \mathbf{0.0206\ kg/m^3}$$

... **Ans.**

Ex. 5.46 : *Calculate : i) Relative humidity ; ii) Humidity ratio ; iii) Dew point temperature ; iv) Density and v) Enthalpy of atmospheric air when the DBT is 35 ºC, WBT = 23 ºC and the barometer reads 750 mm of Hg.*

Sol. : Given :

$$t_{db} = 35\ ^\circ C,\ t_{wb} = 23\ ^\circ C,\ P_b = 750 \text{ mm of Hg}$$

To find : i) ϕ ii) w iii) t_{dp} iv) ρ_v v) h

Step 1 : Calculate dew point temperature

From steam table, saturation pressure at $t_{wb} = 23\ ^\circ C$ is,

$$p_w = 0.028 \text{ bar}$$

We know that,

$$p_b = 750 \text{ mm of Hg} = 0.75 \times 13.6 \times 9810$$

$$= 100.062 \times 10^3\ N/m^2$$

$$\therefore \quad p_b = 1.0006 \text{ bar}$$

Partial pressure of water vapour is given by,

$$p_v = p_w - \frac{(p_b - p_w)(t_{db} - t_{wb})}{1544 - 1.44\, t_{wb}}$$

$$\therefore \quad p_v = 0.028 - \frac{(1.0006 - 0.028)(35 - 23)}{1544 - 1.44 \times 23}$$

$$= 0.0202 \text{ bar}$$

For partial pressure of water vapour we can find saturation temperature from steam table. This saturation temperature is dew point temperature.

$$\therefore \quad \mathbf{t_{dp} = 17\ ^\circ C} \qquad \text{... Ans.}$$

Step 2 : Calculate relative humidity and specific humidity

We can find saturation pressure of vapour corresponding to dry bulb temperature from steam table.

$$\therefore \quad p_{vs} = 0.0562 \text{ bar}$$

Relative humidity is given by,

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.0202}{0.0562} = 0.3594$$

$$\therefore \quad \phi = \mathbf{35.94\ \%} \qquad \text{... Ans.}$$

Specific humidity is given by,

$$w = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622 \times 0.0202}{1.0006 - 0.0202}$$

$\therefore$ **w = 0.0128 kg/kg of dry air** ... **Ans.**

Step 3 : Calculate vapour density and enthalpy

Vapour density is given by,

$$\rho_v = \frac{w\,(p_b - p_v)}{R \cdot t_{db}}$$

$$= \frac{0.0128\,(1.0006 - 0.0202) \times 10^5}{287 \times (273 + 35)}$$

$\therefore$ ρ_v = **0.01419 kg/m³ of dry air** ... **Ans.**

Enthalpy is given by,

$$h = 1.005\, t_{db} + w\,(2500 + 1.88\, t_{db})$$

$$\therefore \quad h = 1.005 \times 35 + 0.0128\,(2500 + 1.88 \times 35)$$

$\therefore$ **h = 68.0172 kJ/kg of dry air** ... **Ans.**

Ex. 5.47 : *Moist air at 35 °C has a dew point of 15 °C. Calculate its relative humidity, specific humidity and enthalpy.*

Sol. : Given data :

t_{db} = 35 °C = 308 K, t_{dp} = 15 °C = 288 K

To find : i) ϕ ii) w iii) h

Step 1 : Calculate relative humidity and specific humidity

From Psychrometric chart,

At t_{db} = 35 °C, p_{vs} = 0.0563 bar

At t_{dp} = 15 °C, p_v = 0.017 bar

We know that,

$$\text{Relative humidity, } \phi = \frac{p_v}{p_{vs}} = \frac{0.017}{0.0563} = 0.3019$$

$\therefore$ ϕ = **30.19 %** ... **Ans.**

Specific humidity is given by,

$$w = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622 \times 0.017}{1.01325 - 0.017}$$

$\therefore$ **w = 0.01061 kg/kg of air** ... **Ans.**

Step 2 : Calculate the enthalpy of moist air

Enthalpy of moist air is given by,

$$h = 1.005\, t_{db} + w\,(2500 + 1.88\, t_{db})$$

$$\therefore \quad h = (1.005 \times 35) + 0.01061\,(2500 + 1.88 \times 35)$$

$\therefore$ **h = 62.39 kJ/kg** ... **Ans.**

Ex. 5.48 : *Moist air at 20 °C, 1.01325 bar has relative humidity of 80 %, without using psychrometric chart determine degree of saturation, dew point temperature, specific volume and density of air.*

Sol. : Given :

t_{db} = 20 °C, p_b = 1.0132 bar, ϕ = 80 % = 0.8

To find : i) μ ii) t_{dp} iii) v_a iv) ρ_a

Step 1 : Calculate dew point temperature

From steam table,

at 20 °C, p_{vs} = 0.02337 bar

$$\text{Relative humidity } \phi = \frac{p_v}{p_{vs}}$$

$$\therefore \quad 0.8 = \frac{p_v}{0.02337} \qquad \therefore p_v = 0.01869 \text{ bar}$$

From steam table, at p_v = 0.01869 bar, we get dew point temperature as,

t_{dp} = **16 °C** ... **Ans.**

Step 2 : Calculate degree of saturation and mass of air

Degree of saturation is given by,

$$\mu = \phi\left(\frac{p_b - p_{vs}}{p_b - p_v}\right)$$

$$= 0.8\left(\frac{1.01325 - 0.02337}{1.01325 - 0.01869}\right) = \mathbf{9.6443} \quad \text{... Ans.}$$

We know that, $p_b = p_a + p_v$

$$\therefore \quad p_a = p_b - p_v = 1.01325 - 0.01869$$

$$= 0.9945 \text{ bar}$$

Now,

$$\text{Specific volume} = v_a = \frac{RT}{p_a} = \frac{287 \times 293}{0.9945 \times 10^5}$$

$$= 0.8455 \, m^3/kg$$

$$\text{Specific humidity} = w = 0.622 \left(\frac{p_v}{p_b - p_v} \right)$$

$$= 0.622 \left(\frac{0.01869}{1.0132 - 0.01869} \right)$$

$$w = 0.0118 \text{ kg/kg of dry air}$$

Now, $$\rho_v = \frac{w}{v_a} = \frac{0.0118}{0.8455}$$

$$= \mathbf{0.0139 \, kg/m^3} \text{ of dry air} \quad \textbf{... Ans.}$$

Ex. 5.49 : *Atmospheric air at 101.325 kPa has 30° DBT and 15 °C DPT. Without using psychrometric chart calculate partial pressure of air and vapour, Specific humidity, Relative humidity, vapour density and enthalpy of moist air.*

Sol. : Given :

p_b = 101.325 kPa = 1.0132 bar, t_{db} = 30 °C,

t_{dp} = 15 °C

To find : i) p_a and p_v ii) w iii) ϕ iv) ρ_v v) h

Step 1 : Calculate partial pressure of air and water vapour

From steam table,

at t_{db} = 30 °C, p_{vs} = 0.04242 bar

at t_{dp} = 15 °C, $\mathbf{p_v}$ = **0.01704 bar ... Ans.**

We know that, $p_b = p_a + p_v$

$\therefore$ $\mathbf{p_a} = p_b - p_v$ = 1.0132 – 0.01704 = **0.99616 bar**

... Ans.

Step 2 : Calculate specific humidity and relative humidity

$$\text{Specific humidity} = w = 0.622 \frac{p_v}{p_a} = 0.622 \times \frac{0.01704}{0.99616}$$

$\therefore$ **w = 0.01063 kg/kg of dry air ... Ans.**

$$\text{Relative humidity} = \phi = \frac{p_v}{p_{vs}} = \frac{0.01704}{0.04242} = 0.4016$$

$\therefore$ ϕ = **40.16 % ... Ans.**

Step 3 : Calculate enthalpy and vapour density

Enthalpy of moist air is given by,

$$h = 1.005 \, t_{db} + w \, (2500 + 1.88 \, t_{db})$$

$$\therefore \quad h = 1.005 \times 30 + 0.01063 \, (2500 + 1.88 \times 30)$$

$\therefore$ **h = 57.3245 kJ/kg of dry air ... Ans.**

$$\text{Specific volume of air} = v_a = \frac{R \cdot t_{db}}{p_a} = \frac{287 \times 303}{0.9961 \times 10^5}$$

$$= 0.873 \, m^3/kg$$

$$\text{Vapour density} = \rho_v = \frac{w}{v_a} = \frac{0.01063}{0.873}$$

$$= \mathbf{0.0121 \, kg/m^3} \quad \textbf{... Ans.}$$

Ex. 5.50 : *A room measures 5 m × 5 m × 3 m contains atmospheric air at 100 kPa, DBT = 30 °C and relative humidity = 30 %. Find the mass of dry air and the mass of associated water vapour in the room. Solve the problem without using psychrometric chart.*

Sol. : Given :

Volume = 5 × 5 × 3 = 75 m^3, p_b = 100 kPa = 1 bar,

ϕ = 30 % = 0.3, t_{db} = 30 °C

To find : i) m_a ii) m_v

Step 1 : Calculate mass of air

From steam table,

At DBT = 30 °C, p_{vs} = 0.04242 bar = 4.242 kPa

$$\text{We know that, } \phi = \frac{p_v}{p_{vs}} \qquad \therefore \; 0.3 = \frac{p_v}{4.242}$$

$$\therefore \quad p_v = 1.2726 \text{ kPa}$$

For moist air barometric pressure is given by,

$$p_b = p_a + p_v \quad \therefore \quad p_a = p_b - p_v$$

$$\therefore \qquad p_a = 100 - 1.2726 = 98.7274 \text{ kPa}$$

$$= 98.7274 \times 10^3 \text{ Pa}$$

By using ideal gas equation,

$$m_a = \frac{p_a \, v}{R_a \, T} = \frac{98.7274 \times 10^3 \times 75}{287 \times 303}$$

$$\therefore \qquad \mathbf{m_a = 85.1479 \text{ kg}} \qquad \textbf{... Ans.}$$

Step 2 : Calculate mass of vapours

Characteristic gas constant for ideal vapour

$= R_v = 461.8$

By using ideal gas equation

$$m_v = \frac{p_v \, v}{R_v \, T} = \frac{1.2726 \times 10^3 \times 75}{461.8 \times 303}$$

$$\mathbf{m_v = 0.6821 \text{ kg}} \qquad \textbf{... Ans.}$$

5.24 Psychrometric Chart

- The psychrometric chart is a plot of various thermodynamic properties of moist air. It is very useful to find out the various properties of air required in air condition and eliminate tedious calculations.
- The psychrometric chart gives the interrelationships between dry air, moisture content and enthalpy of air.
- For a proper understanding of air conditioning, it is essential to get familiar with the psychrometric chart.
- It is suitable to design charts at a constant barometric pressure because barometric pressure does not change significantly over much of the occupied surface of the earth.
- There is a slight variation in the charts prepared by different air-conditioning manufactures but basically they all are same.
- The psychrometric chart is generally drawn at standard atmospheric pressure of 0.76 m of Hg or 1.01325 bar.
- In a psychrometric chart, DBT is taken as abscissa (ºC) and moisture content i.e specific humidity as ordinate (g/kg of dry air).
- Now the saturation curve is drawn by plotting the various saturation points at corresponding DBT.
- The saturation curve shows 100 % relative humidity at various DBT. It also represents the wet bulb temperature dew point temperature.

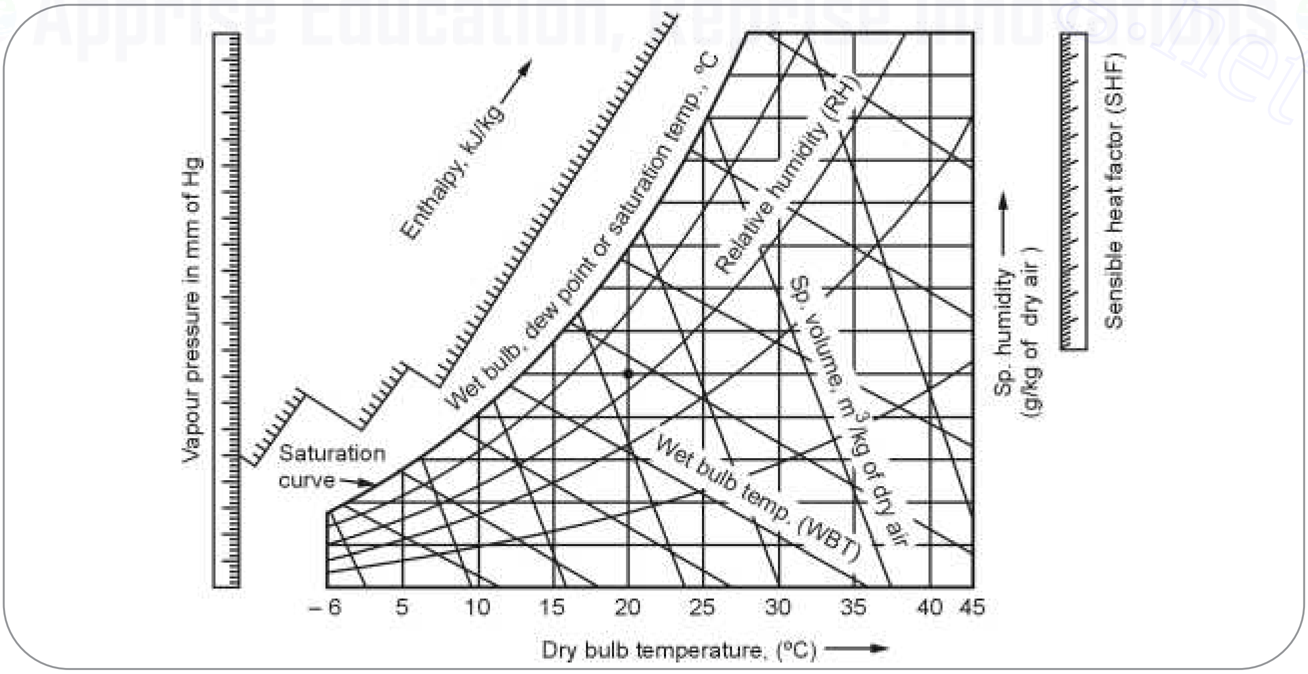


Fig. 5.57 : Psychrometric chart

- The psychrometric chart has a number of details, some of the important lines are as follows :

1) Dry bulb temperature lines
2) Specific humidity lines
3) Dew point temperature lines
4) Wet bulb temperature lines
5) Enthalpy or total heat lines
6) Specific volume lines
7) Relative humidity lines.

(1) Dry bulb temperature lines :

- These lines are vertical i.e. parallel to the ordinate and uniformly spaced as shown in Fig. 5.58.
- The temperature range of these lines on psychrometric chart is from – 6 ºC to 45 ºC.
- These lines drawn with difference of every 5 ºC and up to the saturation curve.

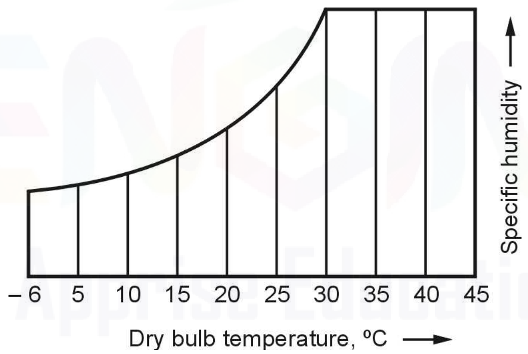


Fig. 5.58 : Dry bulb temperature lines

(2) Specific humidity or moisture content lines :

- These lines are horizontal i.e. parallel to the abscissa and are also uniformly spaced as shown in Fig. 5.59.

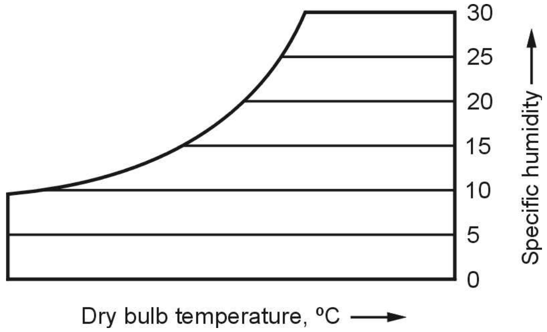


Fig. 5.59 : Specific humidity lines

- The moisture content range of these lines on psychrometric chart is from 0 to 30 g / kg of dry air (or from 0 to 0.030 kg / kg of dry air).
- The moisture content lines are drawn with a difference of every 1 g (0.001 kg) and up to the saturation curve.

(3) Dew point temperature lines :

- These lines are horizontal i.e. parallel to the abscissa and non-uniformly spaced as shown in Fig. 5.60.

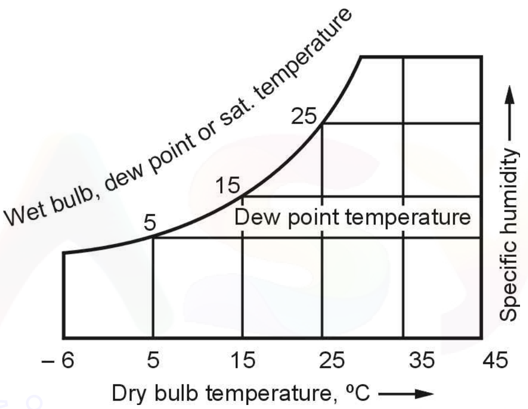


Fig. 5.60 : Dew point temperature lines

- At any point on the saturation curve, the dry bulb temperature and dew point temperature are same.
- The values of DPT are generally given along the saturation curve of the chart.

(4) Wet bulb temperature lines :

- These lines are inclined straight lines and are not uniformly spaced as shown in Fig. 5.61.
- At any point on the saturation curve, the dry bulb and wet bulb temperatures are equal.

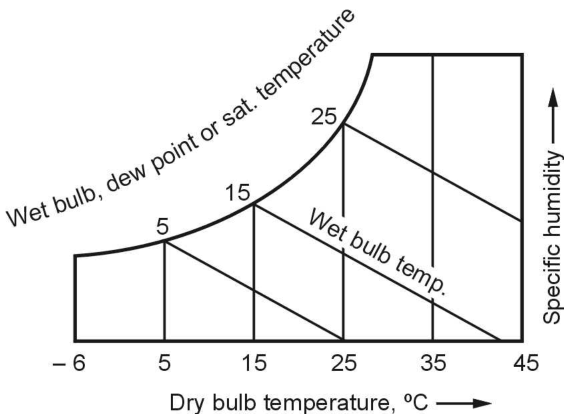


Fig. 5.61 : Wet bulb temperature lines

- The values of WBT are generally given along the saturation curve of the chart.

(5) Specific enthalpy lines :

- These lines are inclined straight lines and uniformly spaced as shown in Fig. 5.62.

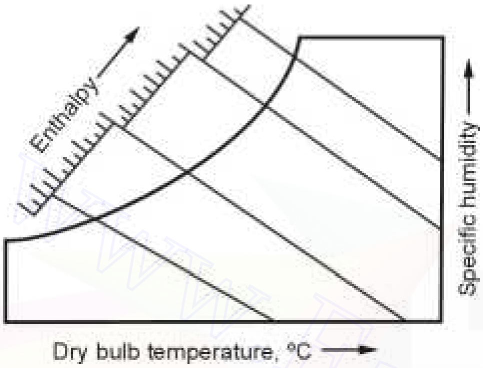


Fig. 5.62 : Enthalpy lines

- These lines are parallel to the wet bulb temperature lines and are drawn up to the saturation curve. Some lines coincide with the WBT lines.
- The values of total enthalpy are given on a scale above the saturation curve.
- The specific enthalpy will increase with DBT and moisture content.

(6) Specific volume lines :

- These lines are obliquely inclined straight lines and uniformly spaced as shown in Fig. 5.63.

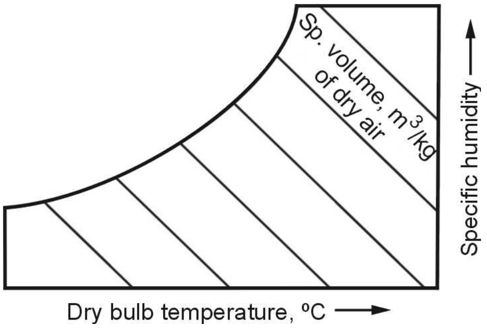


Fig. 5.63 : Specific column lines

- These lines are drawn up to the saturation curve. The values of volume lines are generally given at the base of the chart.

(7) Relative humidity lines :

- These lines are curved lines and follow the saturation curve. Generally, these lines are drawn with values of relative humidity 10 %, 20 %, 30 % etc. and up to 100 %.
- The saturation curve represents 100 % relative humidity line. The values of relative humidity lines are generally given along the lines themselves.

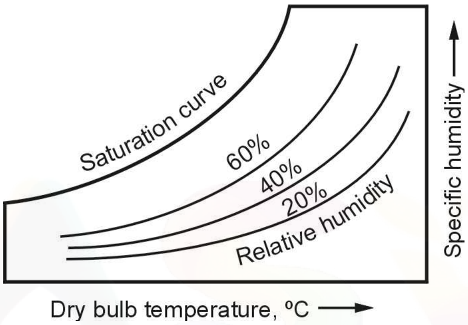


Fig. 5.64 : Relative humidity lines

5.25 Psychrometric Processes **AU : May-16**

- After developing the relationships between all the different properties of moist air, the next step in the design and analysis of air conditioning system is to study the various processes being performed on air.
- All these processes can be plotted easily on a psychrometric chart for quick visualization of changes in important properties such as temperature, moisture content, relative humidity, enthalpy, etc. In this section the various air conditioning processes are treated individually at first and later combined to form complete cycles.
- The important process that air undergoes in a typical air conditioning system are :

(1) Sensible cooling
(2) Sensible heating
(3) Humidification and dehumidification
(4) Cooling and dehumidification
(5) Cooling with adiabatic humidification
(6) Heating and humidification
(7) Heating and dehumidification
(8) Adiabatic mixing of two streams

5.25.1 Sensible Cooling

- The cooling of air, without change in its moisture content (i.e. specific humidity) is known as sensible cooling.
- When air at temperature t_{d1} passes over a cooling coil whose temperature (t_{d3}) is below the DBT of entering air t_{d1} and above the DPT of entering air (t_{dp1}) then moisture content is unaffected. So this process is called as sensible cooling as only sensible heat is removed from the air.
- In this process, the dew point temperature and latent heat content of air remains constant. Therefore, the process is indicated as a horizontal line from right to left, based on the end conditions of air, as shown in Fig. 5.65.
- Fig. 5.65 (a) shows the cooling process on psychrometric chart as horizontal line 1-2. The point 3 represents the surface temperature of the cooling coil.
- If the cooling coil is perfect (i.e. 100 % effective) then the leaving temperature of air (t_{d2}) will be equal to the surface temperature of coil (t_{d3}).
- But in actual practice, the leaving air temperature will be higher than the cooling coil temperature as it requires contact of air with coil surface for enough period of time.
- During the sensible cooling process, the specific humidity of air remains constant (i.e. $w_1 = w_2$). The DBT of air reduces from t_{d1} to t_{d2} and RH of air increases from ϕ_1 to ϕ_2 as shown in Fig. 5.65 (b).
- The heat rejected by air is obtained from the psychrometric chart by enthalpy difference $(h_1 - h_2)$ as shown in Fig. 5.65 (b).
- The total heat rejected during sensible cooling is equal to the change in sensible heat and is given by,

Heat rejected, $q = h_1 - h_2$

$$= C_{pa}(t_{d1} - t_{d2}) + w\,C_{pv}(t_{d1} - t_{d2}) \quad \ldots (\because\ h = C_p \cdot \Delta T)$$

$$q = (C_{pa} + wC_{pv})\ (t_{d1} - t_{d2})$$

$$= C_{pm}(t_{d1} - t_{d2})$$

- The term $(C_{pa} + C_{pv})$ is called **humid specific heat** **(C_{pm})** and its value is taken as 1.022 kJ/kg K.

$\therefore$ Heat rejected, $q = 1.022\ (t_{d1} - t_{d2})$ kJ/kg

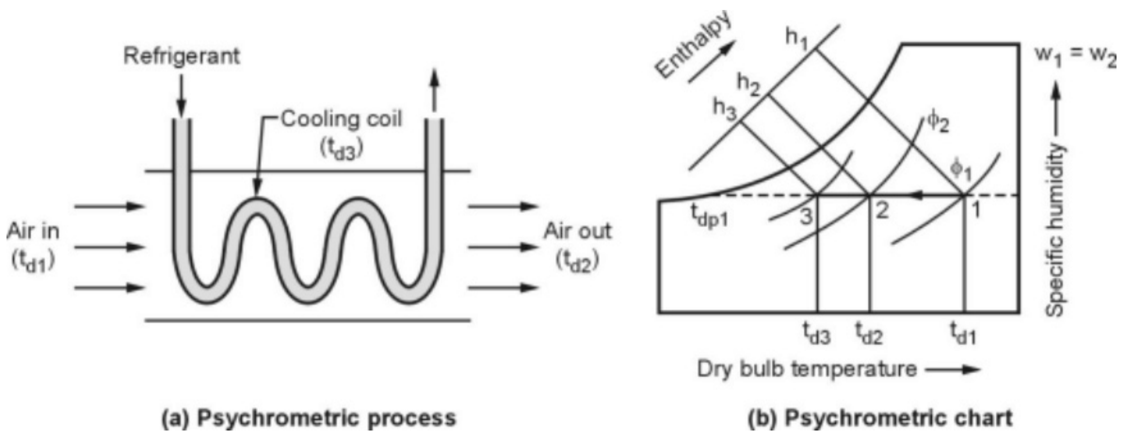
- The cooling capacity of coil(sensible heat per minute) is,

$$Q = m_a C_{pm}(t_{d1} - t_{d2}) \text{ kJ/min}$$

where, $\dot{m}_a$ = Mass flow rate of air in kg/min.

- For air conditioning applications generally flow rate is mentioned in cmm i.e (m^3/min). Therefore, the sensible heat transfer per min is given as,

$$SH = V \cdot r \cdot C_{pm}(\Delta t)\ ,\ \text{kJ/min}$$



(a) Psychrometric process (b) Psychrometric chart

Fig. 5.65 : Sensible cooling

where, V = Flow rate of air in cmm i.e. m^3/min

ρ = Density of moist air = 1.2 kg/m^3

C_{pm} = Humid specific heat = 1.022 kJ/kg K and

Δt = Temperature difference between entering and leaving air condition

Putting the values of ρ and C_{pm} in above equation we get,

$$SH = v \times 1.2 \times 1.022 \times (\Delta t)$$

$$= 1.2264\ V \times (\Delta t)\ kJ/min = \frac{1.2264\, v \times \Delta t}{60}$$

$\therefore$ **SH = 0.0204 V × t, kW**

Note : The sensible cooling can be possible only upto the dew point temperature (t_{dp}) as shown Fig. 5.65 (b). The further cooling will result in condensation of moisture.

Numericals on Sensible Cooling

Ex. 5.51 (Sensible cooling) : *In a summer air conditioner, an atmospheric air enters at 30 ºC DBT and 18 ºC WBT. The air leaves the air conditioner at 20 ºC DBT without changing its moisture content. Find the following :*
(i) Initial enthalpy and specific humidity of air. (ii) Final relative humidity of air and its WBT.
(iii) Sensible heat removed per kg of air.

Sol. : Refer Fig. 5.66.

Given data :

t_{db1} = 30 ºC, t_{wb1} = 18 ºC, t_{db2} = 20 ºC

(i) Initial enthalpy (h_1) and specific humidity :

- Locate point 1 at the intersection of 30 ºC DBT and 18 ºC WBT lines.

From psychrometric chart at point 1,

h_1 = **50 kJ/kg of air**

and w_1 = **0.008 kg/kg of dry air** ... **Ans.**

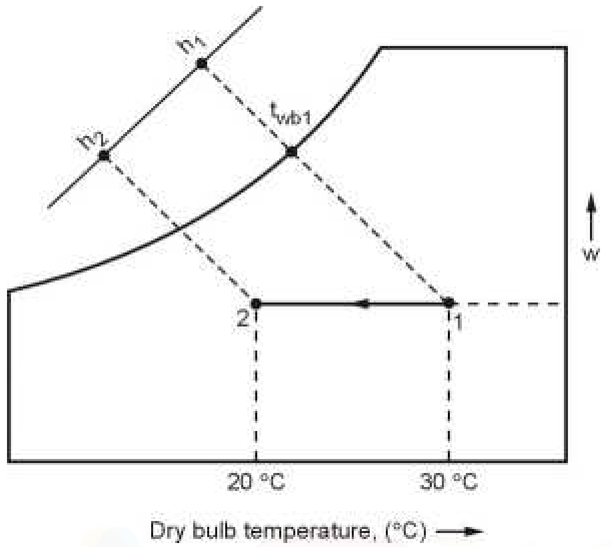


Fig. 5.66

(ii) Final relative humidity ϕ_2 and t_{wb2} :

- As the air is cooled without changing its moisture content ($w_1 = w_2$) upto 20 ºC WBT, draw a horizontal line from point 1 that cuts the 20 ºC vertical DBT line at point 2.
- From psychrometric chart,

We get, ϕ_2 = **55 %** and t_{wb2} = **14.5 ºC**

Also, h_2 = **42 kJ/kg** ... **Ans.**

(iii) Sensible heat removed per kg of air (Q) :

$$Q = (h_1 - h_2) = (50 - 42)$$

$\therefore$ **Q = 8 kJ/kg of air** ... **Ans.**

5.25.2 Sensible Heating

- The heating of air, without any change in its moisture content (i.e. specific humidity), is known as sensible heating.
- When air at temperature (t_{d1}) passes over a heating coil, whose surface temperature (t_{d3}) is above DBT of entering air (t_{d1}), heating of air takes place keeping moisture constant.
- Since no moisture is added or removed from the air, the process is shown by horizontal line 1-2 from left to right as shown in Fig. 5.67 an psychrometric chart.

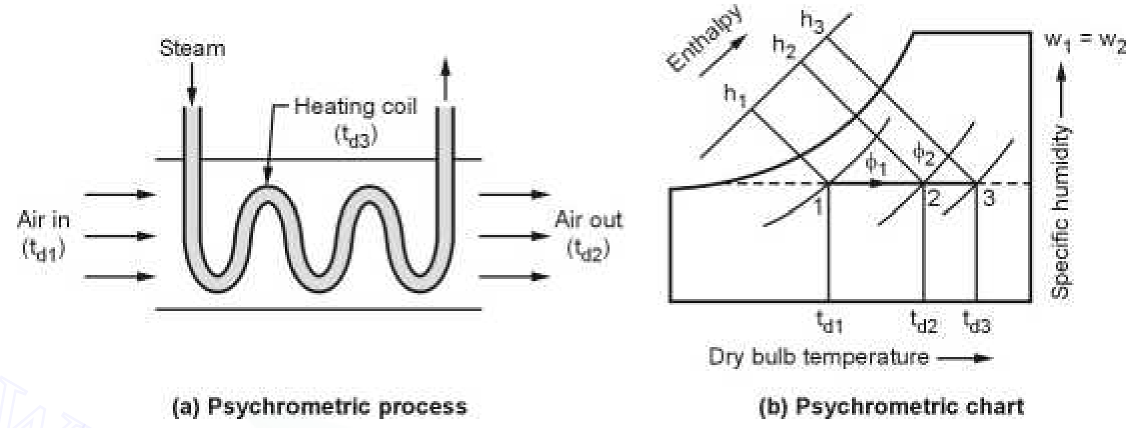


Fig. 5.67 : Sensible heating

The point 3 represents the surface temperature of heating coil.

- During sensible heating, the specific humidity remains constant $(w_1 = w_2)$. The DBT of entering air increases form t_{d1} to t_{d2} and RH reduces from ϕ_1 to ϕ_2 as shown in Fig. 5.67 (b).
- Generally, sensible heating is carried over the air by passing the steam or hot water through the heating coil.
- The heat absorbed by the air during sensible heating is obtained from the psychrometric chart by the enthalpy difference $(h_2 - h_1)$ as shown in Fig. 5.67 (b).
- The total heat transfer during this process is equal to the change in sensible heat and is given as,

Heat added, $q = h_2 - h_1$

$$= C_{pa}(t_{d2} - t_{d1}) + w\,C_{pv}(t_{d2} + t_{d1})$$

$$q = (C_{pa} + w\,C_{pv})(t_{d2} - t_{d1})$$

$$= C_{pm}(t_{d2} - t_{d1})$$

- The term $(C_{pa} + w\,C_{pv})$ is called **humid specific heat (C_{pm})** and its value is taken as 1.22 kJ/kg K.

$\therefore$ Heat added SH $= 1.022\,(t_{d2} - t_{d1})$, kJ/kg

and the heating capacity of coil is given by,

$$Q = \dot{m}_a C_{pm}(t_{d2} - t_{d1}) \text{ kJ / min}$$

where, $\dot{m}_a$ = Mass flow rate of air in kg/min

For air conditioning purpose it given as,

Heating capacity of coil,

$$Q = 0.0204 V \times t, \text{ kW}$$

where v = Volume flow rate of air in cmm (m^3/min)

Numericals on Sensibile Heating

Ex. 5.52 (Sensible heating) : *Moist air of mass flow rate 200 m^3/min at 15 ºC DBT and 75 % RH is heated until its temperature reaches to 25 ºC. Find the following :*

(i) RH of heated air (ii) Wet bulb temperature of heated air (iii) Heat added to air.

Sol. : Refer Fig. 5.68.

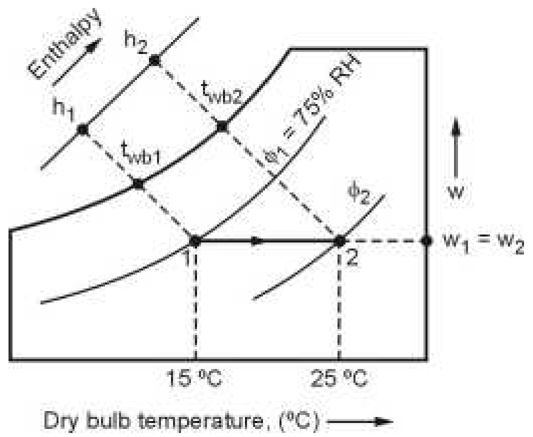


Fig. 5.68

Given data : V = 200 m^3/min t_{db1} = 15 ºC,

$\phi_1 = 75\ \%,\ t_{db2} = 25\ ^\circ C$

- Locate point 1 on psychrometric chart at the intersection of 15 ºC DBT and 75 % RH lines. Through point 1 draw a constant specific humidity line ($w_1 = w_2$) to cut 25 ºC DBT line and get point 2.
- Read the following values from the psychrometric chart :

h_1 = 354.4 kJ/kg and h_2 = 45.2 kJ/kg,

V_1 = 0.825 m^3/kg of dry air

(i) RH of heated air (from chart at point 2) :

$$\phi_2 = \mathbf{41\ \%}$$

(ii) WBT of heated air (from chart at point 2) :

$$t_{wb} = \mathbf{16.1\ ^\circ C}$$

(iii) Heat added to air :

$$\dot{m} = \frac{V}{V_1} = \frac{200}{0.825} = 242.42 \text{ kg/min}$$

$$Q = \dot{m}(h_2 - h_1) = \frac{242.42}{60}(45.2 - 35.4)$$

$\therefore$ **Q = 39.59 kJ/kg of air** **... Ans.**

5.25.3 Bypass Factor for Heating and Cooling Coil

- In sensible heating and cooling, we have discussed that, temperature of air leaving the apparatus (t_{d2}) is less than surface temperature of coil (t_{d3}) in a heating case and more than t_{d3} in cooling case.
- This is due to the fact that, not all air comes in contact with surface of coil and contacted air will not remain in touch with surface for sufficient time. To understand this phenomenon the concept of by pass factor is very important in air conditioning design.
- Let us consider the air at temperature t_{d1} passes over coil having surface temperature of t_{d3} as shown in Fig. 5.69.
- If the mass of air is 1 kg, then some quantity of this air say 'x' kg just by-passes the coil untouched and remaining (1–x) kg comes in contact with the coil, this phenomenon of by-passing the air is measured in terms of a by-pass factor.
- The by-pass factor depends upon the following factors :
  - (i) Coil surface area
  - (ii) The number of tubes
  - (iii) The number of fins provided in a unit length
  - (iv) The number of rows in a coil
  - (v) Pitch of the fins
  - (vi) Velocity of air

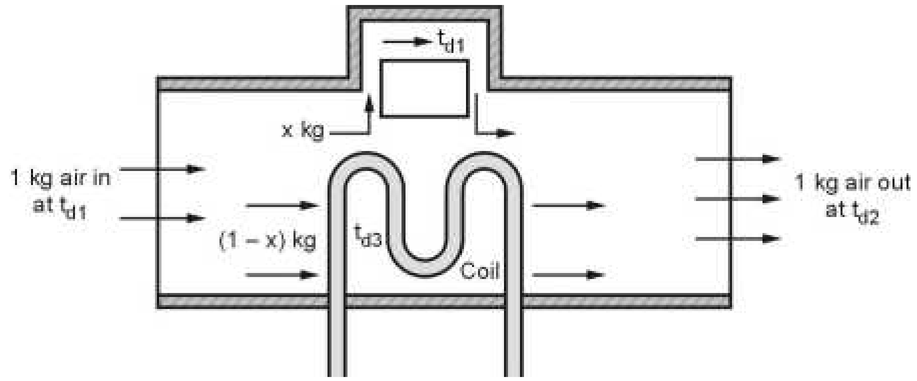


Fig. 5.69 : By pass factor

- Applying the energy balance, we can write,

$$x\, C_{pm} t_{d1} + (1-x)\, C_{pm} t_{d3} = 1 \times C_{pm} t_{d2}$$

where, C_{pm} = Humid specific heat (kJ/kg K)

$$\therefore\ x\, t_{d1} + (1-x)\, t_{d3} = t_{d2} \text{ or } x\, t_{d1} + t_{d3} - x\, t_{d3} = t_{d2}$$

$$\therefore\ x\,(t_{d3} - t_{d1}) = t_{d3} - t_{d2}$$

$$\therefore\ \mathbf{x} = \frac{\mathbf{t_{d3} - t_{d2}}}{\mathbf{t_{d3} - t_{d1}}}$$

where, x is called the **by-pass factor of the coil** and is generally written as BPF.

For heating coil, the by-pass factor is given by,

$$BPF = \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}}$$

Similarly, for cooling coil $BPF = \dfrac{t_{d2} - t_{d3}}{t_{d1} - t_{d3}}$

- The term (1 – x) i.e. (1 – BPF) is called as **contact factor or efficiency of coil.**

∴ Efficiency of heating coil is,

$$\eta_H = 1 - BPF = 1 - \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}} = \frac{t_{d2} - t_{d1}}{t_{d3} - t_{d1}}$$

and efficiency of cooling coil is,

$$\eta_c = 1 - BPF = 1 - \frac{t_{d2} - t_{d3}}{t_{d1} - t_{d3}} = \frac{t_{d1} - t_{d2}}{t_{d1} - t_{d3}}$$

Numericals on BPF

Ex. 5.53 (Sensible cooling, BPF) : *Moist air enters the cooling coil at 40 °C DBT and 20 °C WBT. Finally, the air is sensibly cooled to 26 °C DBT. Plot the process on psychrometric chart and determine*
(i) Final WBT of air.
(ii) The total heat transferred in kW, if air is flowing at the rate of 100 m³/min.
If the cooling coil surface temperature is 22 °C, find the bypass factor of coil.

Sol. : Refer Fig. 5.70.

Given data :

t_{db1} = 40 °C, t_{wb1} = 20 °C, t_{db2} = 26 °C

$$v_a = 100\ m^3/min,\ ADP = 22\ °C$$

- Locate point 1 at the intersection of 40 °C DBT and 20 °C WBT lines.
- Through point 1 draw a horizontal line that cuts the saturation curve at point 2.
- Line 1-2 represents sensible cooling process as shown in Fig. 5.70.

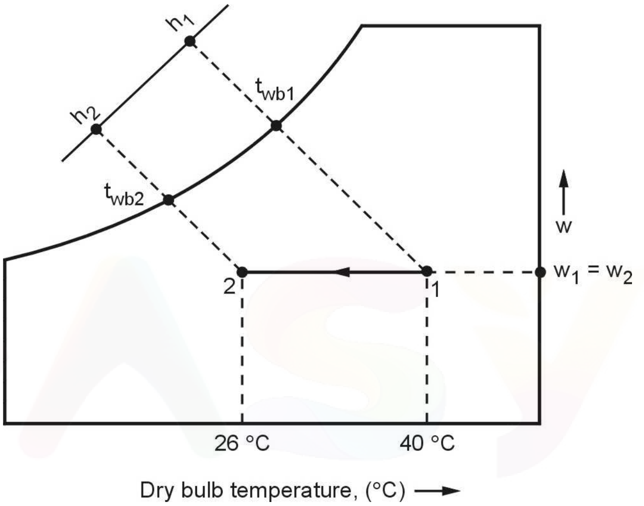


Fig. 5.70

- Locate point 3 at the intersection of 26 °C DBT line and line 1-2.

(i) WBT of air : From psychrometric chart, at point '3' we get

$$t_{wb3} = 15.5\ °C,\ h_3 = 41.5\ kJ/kg$$

(ii) Total heat transferred (Q) :

Mass flow rate of air is,

$$m_a = \frac{v}{v_1}$$

From psychrometric chart, at point 7

$$v_1 = 0.896\ m^3/kg \text{ dry air},\ h_1 = 57.5\ kJ/kg$$

$$\therefore\ m_a = \frac{100}{0.896 \times 60} = 1.8601\ kg/s$$

Thus $Q = m_a (h_1 - h_3) = 1.8601(57.5 - 41.5)$

$$\therefore\ Q = \mathbf{29.7616\ kW}$$

(iii) By pass factor of coil is

$$\mathbf{BPF} = \frac{T_3 - T_2}{T_1 - T_2} = \frac{26-22}{40-22} = \mathbf{0.22} \quad \textbf{... Ans.}$$

Ex. 5.54 (Sensible heating) : *In a heating application, moist air enters a steam heating coil at 15 °C DBT and 10 °C WBT at the flow rate of 5 kg/sec. The temperature of heating coil is 40 °C, while its bypass factor is 0.4. Determine DBT, WBT, RH of the air leaving the coil. Also determine the capacity of heating coil.*

Sol. : Refer Fig. 5.71.

Given data : t_{db1} = 15 °C ; t_{wb1} = 10 °C, t_{db3} = 40 °C, BPF = 0.4

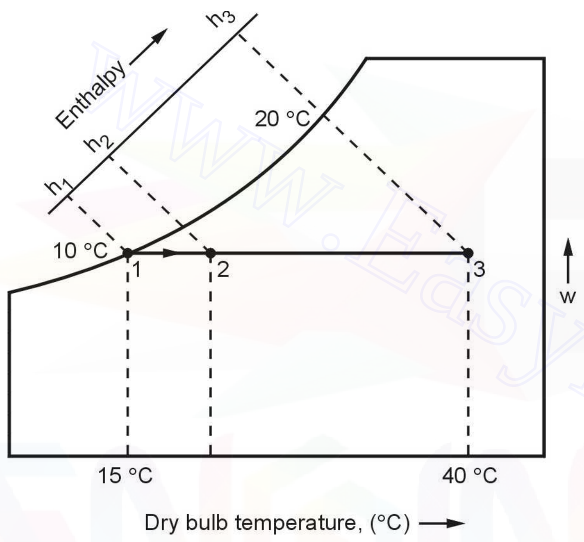


Fig. 5.71

- Locate point 1 at the intersection of 15 °C DBT and 10 °C WBT lines as shown an psychrometric chart in Fig. 5.71.
- Through point 1 draw a horizontal lines to intersect the vertical line at the DBT of heating coil at 40 °C to get point 2. Then, line 1-2 represents sensible heating process.
- Point 3 lies on the line 1-2. Let tdb3 be the temperature of air leaving the heating coil.

(i) DBT of air leaving the coil (t_{db3}) :

We know that,

$$BPF = \frac{t_{db3} - t_{db2}}{t_{db3} - t_{db1}}$$

$$0.4 = \frac{40 - t_{db2}}{40 - 15}$$

$$\therefore \quad \mathbf{t_{db2} = 30\ °C} \quad \text{... Ans.}$$

(ii) WBT of air leaving the coil (t_{wb2}) and relative humidity (ϕ_2) :

- Locate point 3 an vertical line of DBT = 30 °C, that cuts the line 1-2 at point 3.
- From the psychrometric chart, the outlet state at point 3 is :

$$\mathbf{t_{wb2} = 16\ °C \text{ and } \phi_2 = 21\ \%}$$

(iii) Capacity of heating coil (Q) :

From psychrometric chart,

$$h_1 = 29.5 \text{ kJ/kg} \quad h_3 = 45 \text{ kJ/kg}$$

Capacity of heating coil

$$(Q) = \dot{m}(h_3 - h_1) = 50\,(45 - 29.5)$$

$$= \mathbf{775\ kW} \quad \text{... Ans.}$$

5.25.4 Humidification and Dehumidification

- **Humidification** is the process of addition of water vapour in the air at constant dry bulb temperature. Similarly, removal of water vapour from the air, without change in its dry bulb temperature is known as **dehumidification.**
- During perfect humidification process, the dry bulb temperature of air is remain constant, therefore it is represented on psychrometric chart as a vertical line from bottom to top as shown in Fig. 5.72 (a). It is to be noted that in humidification, the RH of air increases from ϕ_1 to ϕ_2 and specific humidity also increases from w_1 to w_2.
- Similarly, ideal dehumidification process is represented on psychrometric chart as a vertical line from top to bottom as shown in Fig. 5.72 (b). During dehumidification process, the RH and specific humidity decreases from ϕ_1 to ϕ_2 and w_1 to w_2 respectively.
- Since the DBT of air remains constant during humidification and dehumidification process, sensible heat also remains constant.
- Therefore, the total heat transfer during these processes is the latent heat which causes the change in

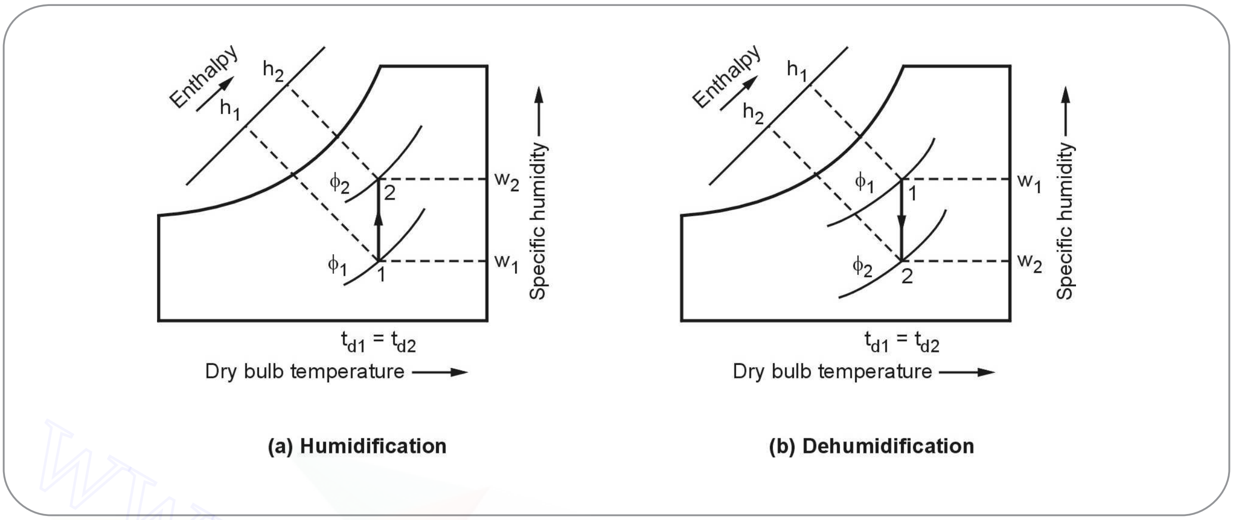


Fig. 5.72 : Humidification and dehumidification

enthalpy per kg of dry air due to increased moisture content equal to $(w_2 - w_1)$ kg per kg of dry air.

- Thus, the latent heat transfer during humidification and dehumidification is given by,

$$LH = h_2 - h_1 \quad \ldots \text{ for humidification}$$

$$= h_{fg}(w_2 - w_1)$$

where, h_{fg} = Latent heat of vapourisation at DBT (t_{d1})

- Similarly, for dehumidification the above equation may be written as,

$$LH = (h_1 - h_2) = h_{fg}(w_1 - w_2)$$

In air conditioning practice, the latent heat load per minute is given as :

$$LH = \dot{m}_a(\Delta h) = \dot{m}_a \cdot h_{fg}(\Delta w) \quad (\because m_a = V \cdot \rho)$$

$$LH = V \cdot \rho\, h_{fg}(\Delta w)$$

where, ρ = Density of moist air = 1.2 kg/m^3

h_{fg} = Latent heat of vapourisation of water = 2500 kJ/kg

(Δw) = Specific humidity difference between inlet and exit air condition

Putting these values in above equation,

$$LH = V \times 1.2 \times 2500 \times (\Delta w)$$

$$= 3000\, V \times (\Delta w) \text{ kJ/min}$$

$$\therefore \quad LH = \frac{3000\, V \times \Delta w}{60}$$

$$= 50\, V \times \Delta w \text{ kJ/s or kW}$$

5.25.5 Sensible Heat Factor (SHF)

- The heat added during a psychrometric process may be divided into sensible heat (SH) and latent heat (LH).
- Sensible Heat Factor (SHF) or Sensible Heat Ratio (SHR) is the ratio of sensible heat to the total heat. It is given as,

$$SHF = \frac{\text{Sensible heat}}{\text{Total heat}} = \frac{SH}{SH + LH}$$

- SHF is shown on the right side of psychrometric chart.

5.25.6 Cooling and Dehumidification

- In summer air conditioning system, the outside air is cooled and dehumidified to obtain desired comfort conditions.
- During cooling of coil, the moisture is removed from the air only when air is cooled to a temperature below its dew point temperature.
- Therefore, for dehumidification the effective surface temperature of cooling coil i.e. apparatus dew point (ADP) (t_{d4}) must be less than the dew point temperature of entering air (t_{dp1}).

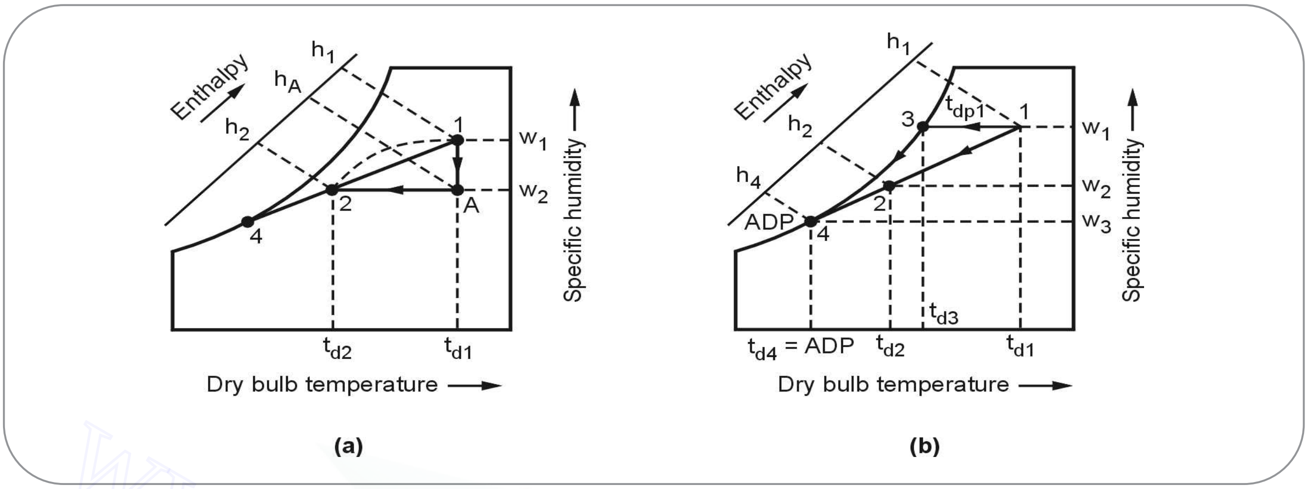


Fig. 5.73 : Cooling and dehumidification

- The cooling and dehumidification process is represented by process 1-2 on psychrometric chart as shown in Fig. 5.73 (a).
- Let, t_{d1} = DBT of air entering the coil

  t_{dp1} = DPT of air entering the coil = t_{d3}

  t_{d2} = DBT of leaving air

  t_{d4} = Effective surface temperature of coil
- In ideal conditions, the DBT of air leaving the coil (t_{d4}) should be same as that of surface temperature of coil (ADP), but in actual practice it is not possible due to inefficiency of cooling coil.
- Therefore, actual condition of air leaving the cooling coil is indicated by the point '2' an straight line joining the points 1 and ADP.
- In this case BPF is given by,

$$\text{BPF} = \frac{t_{d2} - t_{d4}}{t_{d1} - t_{d4}} = \frac{w_2 - w_4}{w_1 - w_4} = \frac{h_2 - h_4}{h_1 - h_4}$$

- For calculation of psychrometric properties, only end points are considered.
- Thus, the cooling and dehumidification process as shown by a line 1-2 is divided as 1-A (dehumidification) and A-2 (cooling).

The total heat removed from the air during cooling and dehumidification process is,

$$q = h_1 - h_2 = (h_1 - h_A) + (h_A - h_2)$$

$$= \text{LH} + \text{SH}$$

where,

LH = Latent heat removed due to condensation of moisture

SH = Sensible heat removed

- The total heat absorbed by the coil is sum of SH and LH and is given by enthalpy difference $(h_1 - h_2)$ as shown in Fig. 5.73 (a). The proportion of sensible heat to the total heat is expressed in sensible heat factor.
- The SHF of process gives the slope of line representing the process an psychrometric chart as shown in Fig. 5.73 (a) by line 1-ADP.

Thus, from Fig. 5.73 (a),

$$\text{SHF} = \frac{\text{SH}}{\text{SH} + \text{LH}} = \frac{h_A - h_2}{(h_A - h_2) + (h_1 - h_A)}$$

$$= \frac{h_A - h_2}{h_1 - h_2}$$

Numericals on Cooling and Dehumidification

Ex. 5.55 (Cooling and dehumidification) : *In a cooling application, air at 32 °C DBT and 20 °C WBT is passed through a cooling coil maintained at 5 °C. The heat removed by the cooling coil from air is 14 kW and air flow rate is 42.5 m^3/min*

Determine : (i) DBT and WBT of the air leaving the coil. (ii) Coil by-pass factor.

Sol. : Refer Fig. 5.74.

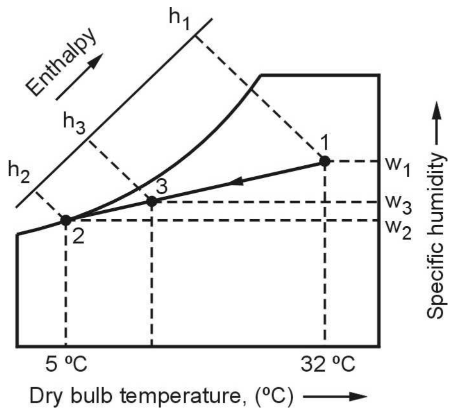


Fig. 5.74

Given data : t_{db1} = 32 ºC, t_{wb1} = 20 ºC, t_{db2} = 5 ºC, V = 42.5 m^3/min

Heat removed by the cooling = 14 kW

- Locate the point 1 at the intersection of 32 ºC DBT and 20 ºC WBT lines.
- Locate the point 2 at the intersection of 5 ºC DBT and the saturation curve.

Join point 1 and 2.

Line 1-2 represents cooling and dehumidification process.

From psychrometric chart, corresponding to points 1 and 3

We have, w_1 = 0.0098 kg/kg of dry air and

h_1 = 57.6 kJ/kg

v_1 = 0.875 m^3/kg ,

w_2 = 0.0054 kg/kg of dry air

h_2 = 18.7 kJ/kg

Mass of air passed through the coil is given by,

$$\dot{m}_a = \frac{V}{v_1} = \frac{42.5}{0.875} = 48.57 \text{ kg/min}$$

Heat removed per kg of air by the cooling coil is,

$$Q = \dot{m}_a \cdot (h_1 - h_3)$$

$$\therefore \quad 14 \times 60 = 48.57 \times (57.6 - h_3)$$

$$\therefore \quad h_3 = 40.31 \text{ kJ/kg}$$

(i) DBT and WBT of the air leaving the coil

The equation for the condition line can be written as :

$$\frac{h_1 - h_3}{h_1 - h_2} = \frac{w_1 - w_3}{w_1 - w_2}$$

$$\therefore \quad \frac{57.6 - 40.31}{57.6 - 18.7} = \frac{0.0098 - w_3}{0.0098 - 0.0054}$$

$$\therefore \quad w_3 = 0.00785 \text{ kg}$$

- Corresponding to h_3 = 40.31 kJ/kg and w_3 = 0.00785, locate the point 3 on the psychrometric chart and final :

$$\mathbf{t_{db3} = 20.3\ ºC \qquad t_{wb3} = 14.4\ ºC} \quad \text{... Ans.}$$

(ii) Coil by-pass factor

$$BPF = \frac{t_{db3} - t_{db2}}{t_{db1} - t_{db2}} = \frac{20.3 - 5}{32 - 5}$$

$$\mathbf{BPF = 0.56} \quad \text{... Ans.}$$

Ex. 5.56 (Cooling and dehumidification) : *The moist air at 30 ºC DBT and 75 % RH enters a refrigeration coil at the rate of 120 m^3/min. The coil dew point temp. is 14 ºC and the by-pass factor of the coil is 0.1. Determine :*

(i) The temp. of air leaving the cooling coil

(ii) The capacity of the cooling coil in TR and in kW

(iii) The amount of water vapour removed per min.

(iv) The sensible heat factor for the process.

Sol. : Refer Fig. 5.75.

Given data : t_{db1} = 30 ºC, ϕ_1 = 75 %, V = 120 m^3/min , ADP = 14 ºC, BPF = 0.1

- Locate the initial condition of air (point 1) at the intersection of 30 ºC DBT and 75 % RH lines on psychrometric chart as shown in Fig. 5.75.

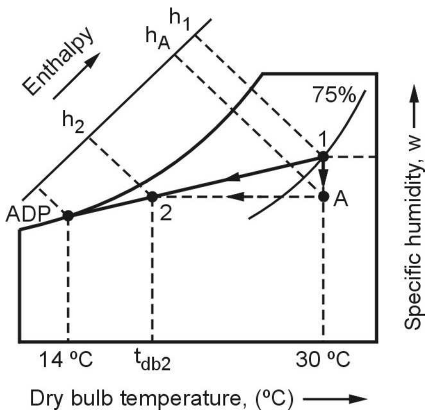


Fig. 5.75

From psychrometric chart, we have

$$t_{dp1} = 25.2\ ^\circ C$$

Since the dew point temperature (~ADP) is less than dew point temperature of entering air, therefore it is a process of cooling and dehumidification.

(i) Temperature of air leaving the cooling coil, (t_{db2})

We know that, BPF = $\frac{t_{db2} - ADP}{t_{d1} - ADP}$ $\therefore$ $0.1 = \frac{t_{db2} - 14}{30 - 14}$

$\therefore$ t_{db2} = **15.6 °C** ... **Ans.**

(ii) Capacity of cooling coil,

The air coming out of the coil is indicated by point 2 on the line joining 1 and ADP. as shown in Fig. 5.75. The line 1-2 represents the cooling and dehumidification process, which is assumed to have followed the path 1-A (i.e. dehumidification) and A-2 (i.e. cooling). Now from psychrometric chart, we get

Specific humidity of entering air at point 1,

$$w_1 = 0.0202 \text{ kg/kg of dry air}$$

specific humidity of leaving air at point 2,

$$w_2 = 0.011 \text{ kg/kg of dry air}$$

Specific volume of entering air at point 1,

$$v_1 = 0.884 \text{ m}^3\text{/kg of dry air}$$

Enthalpy at point 1, h_1 = 82 kJ/kg

Enthalpy at point A, h_A = 58 kJ/kg

Enthalpy at point 2, h_2 = 43.5 kJ/kg

We know that, mass flowing through the cooling coil,

$$\dot{m}_a = \frac{V}{v_1} = \frac{120}{0.884} = 135.75 \text{ kg/min}$$

$\therefore$ Capacity of the cooling coil in kW = $\dot{m}_a\ (h_1 - h_2)$

$$= \frac{135.75}{60}(82 - 43.5)$$

= **87.106 kW** ... **Ans.**

$\therefore$ Capacity of the cooling coil in TR = **87.106/3.516**

= **24.77 TR** ... **Ans.**

(iii) Amount of water vapour removed

$$= \dot{m}_a\ (w_1 - w_2)$$

$$= 135.75 \times (0.0202 - 0.011)$$

= **1.25 kg/min** ... **Ans.**

(iv) Sensible heat factor of process is,

$$SHF = \frac{h_A - h_2}{h_1 - h_2} = \frac{58 - 43.5}{82 - 43.5} = \mathbf{0.377}$$

Ex. 5.57 (Cooling and dehumidification, SHF line) : *In air conditioning application, the atom spheric air enters a dehumidifying coil at initial condition of 30 °C DBT and 20° WBT respectively. It is desired that, the air leaving the coil have 17 °C DBT and 14 °C WBT, Determine :*

(i) The effective surface temperature of coil
(ii) The by-pass factor of coil
(iii) The sensible heat factor of coil.
(iv) The sensible heat removed per kg of air
(v) The latent heat removed per kg of air
(vi) The mass of water vapour condensed per kg of air.

Sol. : Refer Fig. 5.76.

Given data : t_{db1} = 30 °C, t_{wb1} = 20 °C, t_{db2} = 17 °C, t_{wb2} = 14 °C

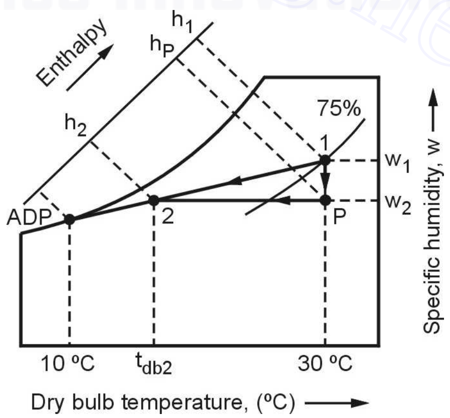


Fig. 5.76

- Locate the entering condition of air i.e. point 1 act the intersection of 30 °C DBT and 20 °C WBT lines.
- Similarly, locate the leaving condition of air i.e. point 2 at the intersection of 17 °C DBT and 14 °C WBT lines.

- Join the points 1 and 2. The line 1-2 represent sensible heat factor line to provide simultaneous cooling and dehumidification of air as shown in Fig. 5.76 an psychrometic chart.

(i) The effective surface temp. of coil (ADP) :

- Extend the process line 1-2 till it cuts the saturation curve at point 3 as shown in Fig. 5.76. The intersection point '3' gives the required surface temperature of coil. From psychrometic chart,

$$ADP = t_{db3} = 10\ ºC$$

(ii) The by-pass factor of coil (BPF) :

We know that, $BPF = \dfrac{t_{db2} - t_{db3}}{t_{db1} - t_{db3}} = \dfrac{17-10}{30-10}$

∴ **BPF = 0.35** **... Ans.**

(iii) The Sensible Heat Factor of Coil (SHF) :

- Locate a reference point an psychrometic chart as shown in Fig. 5.76 (at 25º C DBT and 50 % RH.)
- Through a reference point 'O', draw a line parallel to process line 1-2 that will cut the SHF scale at point 'A' as shown in Fig. 5.76. The intersection point of line OA and SHF scale gives the required sensible heat factor of coil.
- From SHF scale of psychrometic chart, we get

SHF = 0.73 **... Ans.**

(iv) Sensible heat removed per kg of air (SH) :

- The cooling and dehumidification process as shown in Fig. 5.76 is assumed to have followed the path 1-P (i.e. dehumidification) and P-2 (i.e. cooling).

From psychrometic chart,

$h_1 = 57.60$ kJ/kg, $h_2 = 39.5$ kJ/kg
and $h_p = 52.50$ kJ/kg

We know that, sensible heat removed is,

$$Q_s = SH = \dot{m}_a\ (h_p - h_2)$$

$$= 1\ (52.50 - 39.5)$$

$= \mathbf{13\ kJ/kg\ of\ dry\ air}$ **... Ans.**

(v) Latent heat removed per kg of air (LH) :

It is given by, $LH = Q_1 = \dot{m}_a\ (h_1 - h_p)$

$$= 1\ (57.60 - 52.50)$$

$= \mathbf{5.1\ kJ/kg\ of\ dry\ air}$ **... Ans.**

Thus total heat removed = SH + LH = 13 + 5.1

$= \mathbf{18.1\ kJ/kg}$ **... Ans.**

(vi) The mass of water vapour condensed per kg of air (Δw) :

From psychrometic chart,

$w_1 = 0.01063$ kg/kg of dry air

and $w_2 = 0.0088$ kg/kg of dry air

∴ The mass of locater vapour condensed is,

$$= \dot{m}_a\ (w_2 - w_1)$$

$$= 1\ (0.01063 - 0.0088)$$

$= \mathbf{0.00183\ kg/kg\ of\ dry\ air}$ **... Ans.**

Ex. 5.58 (Cooling and dehumidification) :

40 m³/min of air is at 31 ºC DBT and 19 ºC WBT is passed over the cooling coil whose surface temperature is 5 ºC. The cooling capacity is 4 TR under the given conditions of air. Determine DBT and WBT of the air leaving the cooling coil. Also calculate by-pass factor of cooling coil.

Sol. : Refer Fig. 5.77.

Given data : $v_a = 40\ m^3/min$, $t_{db1} = 31$ ºC, $t_{wb1} = 19$ ºC, ADP = $t_{d3} = 5$ ºC, $Q_C = 4$ TR

- Locate the entering air condition i.e. point 1 at the intersection of 31 ºC DBT and 19 ºC WBT lines.

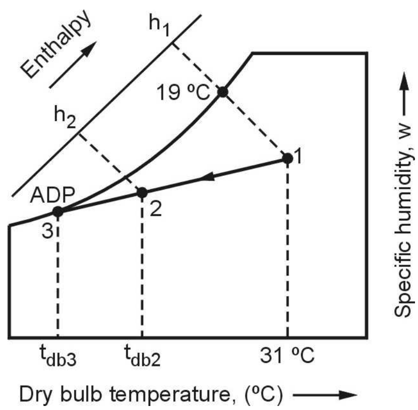


Fig. 5.77

- Locate the surface temperature of cooling coil i.e. point '3' on saturation surve at 5 ºC DBT as shown in Fig. 5.77 on psychrometric chart.
- Join the points 1 and 3. The line 1-3 represents cooling and dehumidification process.
- Let, point '2' be the condition of air leaving the cooling coil.

(i) DBT and WBT of leaving air (t_{db2}, t_{wb2}) :

From psychrometric chart, we get,

$$v_1 = 0.874 \text{ m}^3/\text{kg}, \ h_1 = 54 \text{ kJ/kg of dry air}$$

We know that, Mass flow rate of air is given by,

$$\dot{m}_a = \frac{v_a}{v_1} = \frac{40}{0.874 \times 60} = 0.7628 \text{ kg/sec.}$$

Thus, capacity of cooling coil is

$$Q = \dot{m}_a (h_1 - h_2)$$

$$\therefore \quad 4 \times 3.516 = 0.7628 \, (54 - h_2)$$

$$\therefore \quad h_2 = 35.56 \text{ kJ/kg of dry air}$$

- Now, plot point '2' on line 1-3 such that, the enthalpy h_2 = 35.56 kJ/kg.

From psychrometric chart,

DBT of leaving air is, $\mathbf{t_{db2} = 17.5}$ **ºC. ... Ans.**

and WBT of leaving air is, $\mathbf{t_{wb2} = 12.6}$ **ºC. ... Ans.**

(ii) By-pass factor of cooling coil

It is given by, $BPF = \dfrac{t_{db2} - t_{db3}}{t_{db1} - t_{db3}} = \dfrac{17.5 - 5}{31 - 5}$

$\therefore$ **BPF = 0.48 ... Ans.**

5.25.7 Cooling with Adiabatic Humidification / Evaporative Cooling

- When the air is passed through an insulated chamber, having sprays of water maintained at a temperature (t_w) higher than the dew point temperature of entering air (t_{dp1}), but lower than its dry bulb temperature (t_{db1}) of entering air or equal to the wet bulb temperature of the entering air (t_{wb1}), then the air is said to be cooled and humidified.
- Since no heat is supplied or rejected from the spray water (water is circulated again and again) this process is called as **adiabatic cooling process.**
- This process is shown by line 1-3 or the psychrometric chart and follows the path along the constant wet bulb temperature line or constant enthalpy line. Refer Fig. 5.78 (a) and (b).
- During the process, the DBT of air decreases from t_{d1} to t_{d2} and RH increases from ϕ_1 to ϕ_2.

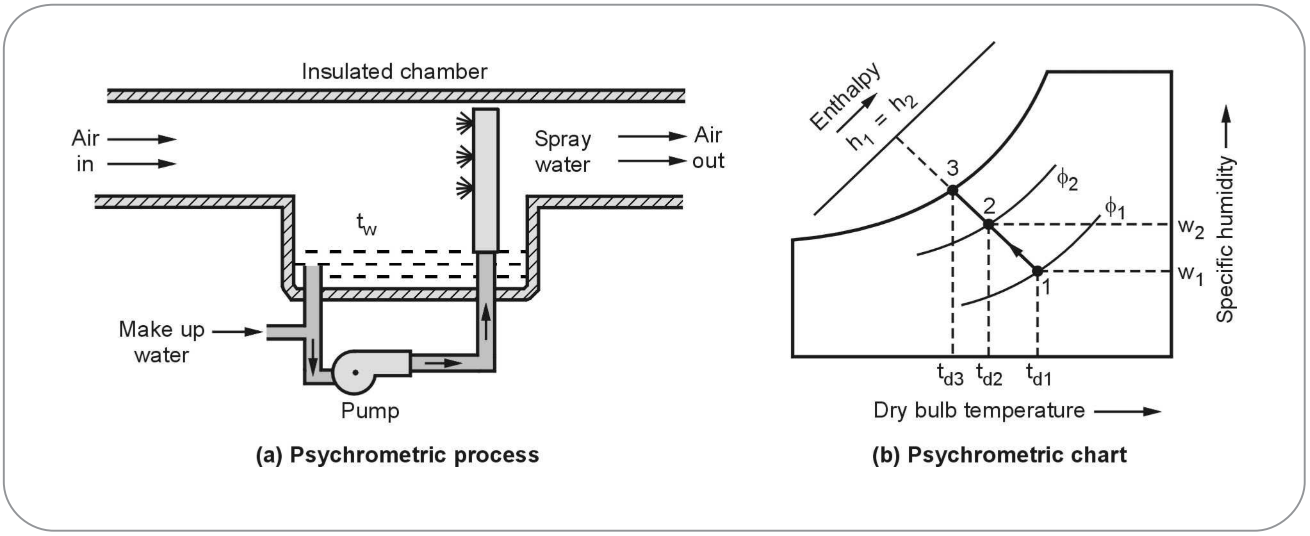


Fig. 5.78 : Cooling with adiabatic humidification

- When humidification is perfect (ideal condition), the final condition of air would be point 3. But practiclly, perfect humidification is never achieved. Therefore, the final condition of air at the outlet is indicated by point 2 and the line 1-3 as shown in Fig. 5.78 (b).
- The performance of spray chamber is measured by the term effectiveness humidifying efficiency as,

$$\eta_H = \frac{\text{Actual drop in DBT}}{\text{Ideal drop in DBT}}$$

or
$$\eta_H = \frac{t_{d1} - t_{d2}}{t_{d1} - t_{d3}} = \frac{w_2 - w_1}{w_3 - w_1}$$

Numericals on Cooling with - Humidification

Ex. 5.59 (Cooling humidification) :

On a particular day, the atmospheric air conditions are recorded as 30 ºC DBT and 40 % RH. Determine the dew point and wet bulb temperature of air. If this air cooled in the air washer using recirculated spray water and having a humidifying efficiency of 90 %, what is the dry bulb temperature and dew point temperature of air leaving the air washer ?

Sol. : Refer Fig. 5.79.

Given data : $t_{db1} = 30$ ºC, $\phi_1 = 40$ %, $\eta_H = 0.9$

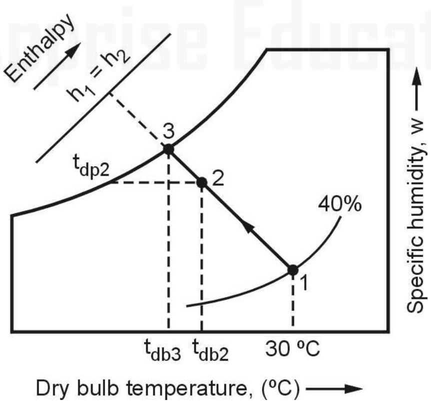


Fig. 5.79

(i) Dew point temperature and wet bulb temperature (t_{dp1}, t_{wb1}) :

- Locate initial condition of air i.e. point 1 at the intersection of 30 ºC DBT and 40 % . RH as shown in Fig. 5.79 on psychrometric chart.

From psychrometric chart, we get,

$$t_{dp1} = \mathbf{15\ ºC}$$

and $t_{wb1} = \mathbf{t_{d3}} = 19.8$ ºC

(ii) Dry bulb temperature and dew point temperature of air leaving air leaving air washer (t_{db2}, t_{dp2}) :

Let t_{dp2} be the DBT of air leaving the air washer.

In an ideal case i.e when the humidification is perfect the final condition of air will be at point '3' as shown in Fig. 5.79 (i.e. at temperature t_{db3}, 100 % RH and humidifying efficiency as 100 %).

However, in given case air washer is only 90 % efficient.

Let, the condition of air leaving the air washer is indicated by point 2 at temperature of t_{db2} as shown in figure an line 1-3.

Humidifying efficiency of air washer is,

$$\eta_H = \frac{t_{dp1} - t_{dp2}}{t_{dp1} - t_{dp3}} = \frac{30 - t_{dp2}}{30 - 19.8} = 0.9$$

$\therefore$ $\mathbf{t_{dp2} = 20.82\ ºC}$ **... Ans.**

- Now, plot point '2' on the constant wet bulb temperature line 1-3 such that, $t_{dp2} = 20.82$ ºC. From psychrometric chart.

The dew point temperature at point 2 is,

$\mathbf{t_{dp2} = 19.4\ ºC}$ **... Ans.**

Ex. 5.60 (Cooling with adiabatic humidification) :

For a hall to be air conditioned,
Outdoor conditions : DBT = 40 ºC,WBT = 20 ºC, Required conditions : DBT = 20 ºC, RH = 60 %. Seating capacity of hall = 1500, Amount of outdoor air supplied = 0.3 m³/min person. If required conditions are achieved first by adiabatic humidification and then by cooling, estimate : (i) Capacity of cooling coil in TR
(ii) Capacity of humidifier in kg/hr.

Sol. : Refer Fig. 5.80.

Given data :

Outside conditions (1) = 40 ºC DBT and 20 ºC WBT

Required conditions (3) = 20 ºC DBT and 60 % RH

No. of persons = 1500, V_a = 0.3 m^3/min. person.

- Locate outside conditions of air i.e. point '1' at the intersection of 40 °C DBT and 20 °C WBT lines.
- Similarly, locate required conditions of air i.e. point '3' at the intersection of 20 °C DBT and 60 % RH lines.
- According to given condition first adiabatic humidification is carried over air and then cooling process.
- Therefore, draw constant enthalpy line through point '1' and draw horizontal line through 3. The intersection of these lines gives the point '2' which represents the end of adiabatic humidification and begining of cooling process as shown in Fig. 5.80 an psychrometric chart.

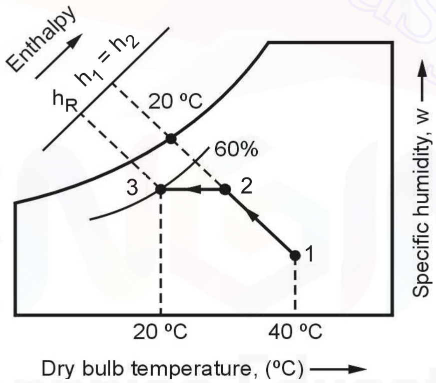


Fig. 5.80

- From psychrometric chart,

At point '1' :

Enthalpy h_1 = 57.4 kJ/kg of dry air

Specific humidity w_1 = 0.0066 kg/kg of dry air

Specific volume v_1 = 0.896 m^3/kg of dry air

At point '2' :

Enthalpy $h_2 = h_1 = 57.4$ kJ/kg of dry air

Specific humidity w_2 = 0.0088 kg/kg of dry air

At point '3' :

Enthalpy h_3 = 42.6 kJ/kg of dry air

Total air supplied, V = Seating capacity × Air flow/min/person

$$= 1500 \times 0.3 = 450 \text{ m}^3/\text{min}$$

(i) Cooling capacity of coil $= \dfrac{\text{Air supplied}}{\text{Specific volume} \times 60} \times (h_2 - h_3)$

$$= \frac{450}{0.896 \times 60}(57.4 - 42.6) = 123.88 \text{ kW}$$

$$= \frac{123.88}{3.516} = \mathbf{35.23\ TR}$$ **... Ans.**

(ii) Capacity of humidifier in kg/hr = $\dot{m}_a (w_2 - w_1)$

$$= \frac{V}{V_o}(w_2 - W_1)$$

$$= \frac{450}{0.896} \times (0.0088 - 0.0066)$$

$$= 1.1049 \text{ kg/min}$$

$$= \mathbf{66.29\ kg/hr}$$ **... Ans.**

Ex. 5.61 (Cooling with adiabatic humidification) :

200 m^3 of air per min is passed through adiabatic humidifier. The condition of air at inlet is 40 °C DBT and 15 % relative humidity. The outlet condition is 23 °C DBT and specific humidity of 0.12 kg/kg of dry air.
Find DPT and amount of water vapour added.

Sol. : Refer Fig. 5.81.

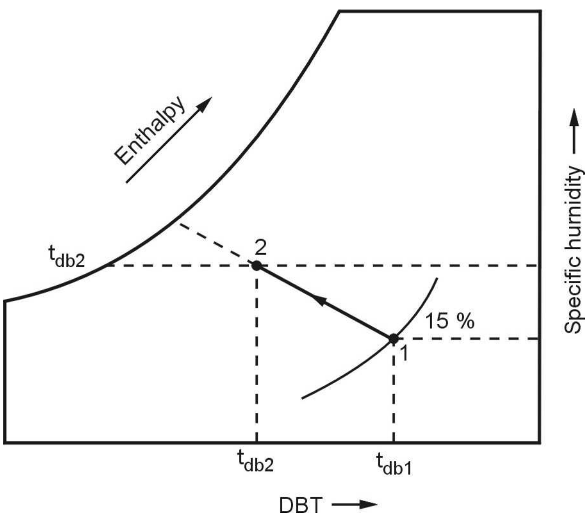


Fig. 5.81

Given data : $V = 200\ m^3/min$, $t_{db1} = 40\ °C$,
$\phi = 15\ \%$, $t_{db2} = 23\ °C$, $w_2 = 0.12$ kg/kg of dry air

- Mark inlet condition of air at 40 ºC DBT and 15 % RH lines. This is point 1 as shown in Fig. 5.81.
- Outlet condition is marked on psychrometric chart at 23 ºC DBT. Specific humidity of 0.012 kg/kg of dry air. This is point 2 as shown in Fig. 5.81.

(i) Dew point temperature

Take a horizontal line from point 2 on psychrometric chart upto saturation curve from that point dew point temperature is given by,

$$t_{dp2} = \mathbf{18.8\ °C} \quad \text{... Ans.}$$

(ii) Amount of water vapour added to air per minute

From psychrometric chart,

At point 1 $\quad v_1 = 0.895\ m^3/kg$ of dry air

$w_1 = 0.0066$ kg/kg of dry air

Mass of air supplied

$$\dot{m}_a = \frac{V}{V_1} = \frac{200}{0.88} = 227.27 \text{ kg/min}$$

∴ Mass of water vapour added

$$= \dot{m}_a (w_2 - w_1)$$

$$= 227.27\ (0.012 - 0.0066)$$

$$= \mathbf{1.22\ kg/min} \quad \text{... Ans.}$$

5.25.8 Heating and Humidification

- It is the reverse process of cooling and dehumidification.
- It is generally used in winter air conditioning to warm and humidify the air as the outside air is at lower temperature with lower moisture content.
- To achieve heating and humidfication process, the temperature of water to be sprayed in the air stream is kept at higher temperature than the DBT of entering air so that heat of vapourization of water will be transferred to air to make it hot.
- In this way, the unsaturated air reaches the condition of saturation and the heat of vapourizisation of water will be transferred to air to make it hot.
- The heat of vapourization of water is absorbed from the spray water itself so that the spray water gets cooled.
- The process of heating and humidification is represented by line 1-2 on psychrometric chart as shown in Fig. 5.82.
- During this process, the specific humidity, DBT, WBT, DPT and enthalpy of air increases whereas RH of air may increase or decrease. The air enters at state 1 and leaves at state 2 as shown in Fig. 5.82.

The mass balance for the water spray is,

$$(m_{w1} - m_{w2}) = m_a\ (w_2 - w_1)$$

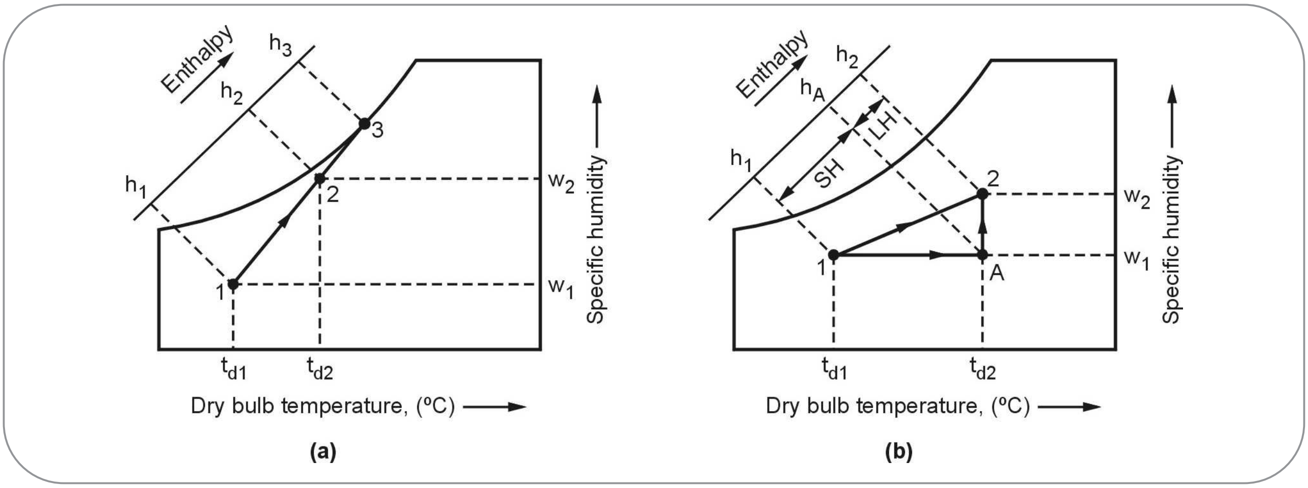


Fig. 5.82 : Heating and humidification

where,

m_{w1} and m_{w2} = Mass flow rates of water entering and leaving the humidifier in kg

m_a = Mass of entering dry air

w_1 and w_2 = Specific humidity of entering and leaving air respectively.

The enthalpy balance is,

$$m_{w1} - h_{fw1} - m_{w2}\, h_{fw2} = m_a\,(h_2 - h_1)$$

Where,

h_{fw1} and h_{fw2} = Enthalpy of spray water entering and leaving the humidifier respectively.

The heating and humidification process shown by line 1-2 is assumed to have followed the path 1-A (i.e. heating) and A-2 (i.e. humidification) as shown in Fig. 5.82 (b). Thus, the total heat added to the air during heating and humidification is,

$$q = h_2 - h_1 = (h_2 - h_A) + (h_A - h_1)$$

$$\therefore \quad q = SH + LH$$

where, $SH = h_A - h_1$ = Sensible heat added

$LH = h_2 - h_A$ = Latent heat of vapourization of increased moisture content $(w_2 - w_1)$

We know that, sensible heat factor is,

$$SHF = \frac{SH}{\text{Total heat}} = \frac{SH}{SH + LH}$$

$$SHF = \frac{h_A - h_1}{h_2 - h_1}$$

Numericals on Heating with Humidification

Ex. 5.62 (Heating and humidification) :

The air of 19 ºC DBT and 13 ºC WBT is heated in a furnace to get the humidified air of specific humidity 0.019 kg/kg of dry air and 35 ºC DBT. Find heat gain and moisture gain alongwith SHF.

Sol. : Refer Fig. 5.83.

Given data :

t_{db1} = 19 ºC, t_{wb1} = 13 ºC, t_{db2} = 35 ºC,

w_2 = 0.019 kg/kg of dry air

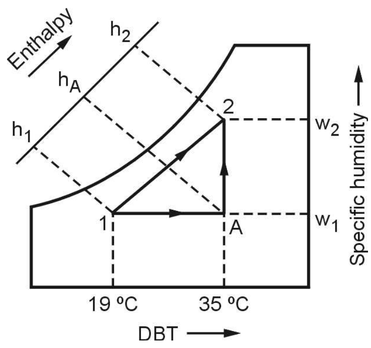


Fig. 5.83

- In psychrometric chart mark point 1 at 19 ºC DBT and 13 ºC WBT.
- Also mark point 2 as 35 ºC DBT and 0.012 kg/kg of dry air specific humidity. Join 1 - 2.
- Draw vertical line through 2 and horizontal line from 1. Mark intersection as A.
- From psychrometric chart,

at point 2, h_2 = 84 kJ/kg

at point A, h_A = 53 kJ/kg

at point 1, h_1 = 37 kJ/kg

(i) Heat added to the air,

$$= h_2 - h_1 = (84 - 37)$$

$$= \mathbf{47\ kJ/kg} \qquad \text{... Ans.}$$

(ii) Moisture added to the air

at point 1 w_1 = 0.007 kg/kg of dry air

at point 2 w_2 = 0.012 kg/kg of dry air

$\therefore$ Moisture added to the air,

$$= w_2 - w_1 = 0.012 - 0.007$$

$$= \mathbf{0.005\ kg/kg\ of\ dry\ air} \qquad \text{... Ans.}$$

(iii) Sensible heat factor

$$\mathbf{SHF} = \frac{h_A - h_1}{h_2 - h_1} = \frac{53 - 37}{84 - 37} = \mathbf{0.34} \qquad \text{... Ans.}$$

Ex. 5.63 (Heating and humidification) :

Cold air at DBT 10 ºC and 60 % RH enters into a room heater at the rate of 100 m^3/min .

It is heated at DBT of 25 ºC and moisture content increased to 0.011 kg/kg of dry air. Find sensible and latent heat added, SHF and capacity of heating coil.

Sol. : Refer Fig. 5.84.

Given data : t_{db1} = 10 ºC, ϕ_1 = 60 %,

V = 100 m^3/min, t_{db2} = 25 ºC,

w_1 = 0.011 kg/kg of dry air

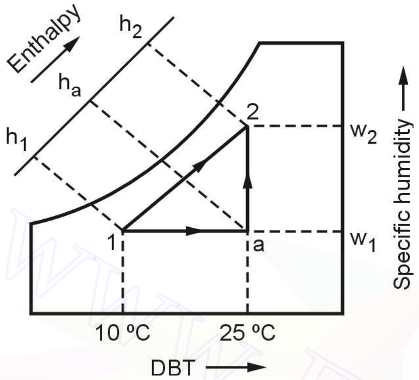


Fig. 5.84

- Mark point 1 on psychrometric chart at 10 ºC DBT and 60 % RH.
- Mark point 2 on psychrometric chart at 25 ºC DBT and 0.011 kg/kg of dry air join 1 - 2.
- Draw vertical line from 2 and horizontal line from 1. Mark the intersection as 'a'.
- From psychrometric chart,

at point 1, h_1 = 21.5 kJ/kg, v_1 = 0.81 m^3/kg of dry air

w_1 = 0.0046 kg/kg of dry air

at point 2, h_2 = 53 kJ/kg

at point a, h_a = 36.5 kJ/kg

Now, mass of air-supplied,

$$\dot{m}_a = \frac{V}{v_1} = \frac{100}{0.81} = 123.46 \text{ kg/min}$$

(i) Sensible heat added

$$SH = \dot{m}_a (h_a - h_1) = 123.46\ (36.5 - 21.5)$$

SH = 1851.9 kJ/min **... Ans.**

(ii) Latent heat added

$$LH = \dot{m}_a (h_2 - h_a) = 123.46\ (53 - 36.5)$$

∴ **LH = 2037.09 kJ/min** **... Ans.**

(iii) Sensible heat factor

$$\textbf{SHF} = \frac{SH}{SH + LH} = \frac{1851.9}{1851.9 + 2037.09} = \textbf{0.47}$$ **... Ans.**

(iv) Capacity of heating coil

$$C = \dot{m}_a (h_2 - h_1) = \frac{123.46}{60} = (53 - 21.5)$$

∴ **C = 64.8165 kW = 18.4347 TR** **... Ans.**

5.25.9 Heating and Dehumidification / Chemical Dehumidification

- In some applications of air conditioning, the moisture requirement is very low which is not possible to achieve by a cooling coil due to restriction of its temperature. In such cases, moisture from air is absorbed by a chemical in its path.
- In this process, the outside air is passed over chemicals which have strong affinity for water vapours.
- As air comes in contact with these chemicals, the water vapour gets condensed out of the air by giving its latent heat. This latent heat of condensation is used to heat the air and hence increases its dry bulb temperature.
- In ideal case, the latent heat released by condensation of water vapour is taken up by the air as sensible heat raising its temperature. Therefore, this process is called adiabatic dehumidification and is represented by constant enthalpy line 1-2 as shown on psychrometric chart in the Fig. 5.85.

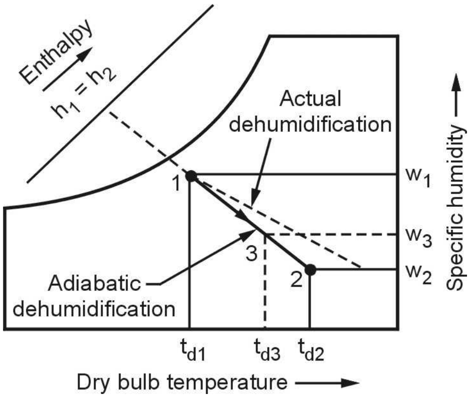


Fig. 5.85 : Heating and dehumidification

- Due to condensation, the specific humidity decreases and the heat of condensation supplies sensible heat for heating the air. This increases its DBT.

- The efficiency of dehumidifier or effectiveness is given as,

$$\eta_H = \frac{t_{d3} - t_{d1}}{t_{d2} - t_{d1}}$$

- Heating and dehumidification process is mainly used in industrial air conditioning and can also be used for some comfort air conditioning requiring either a low RH or low dew point temperature in the room.
- The two types of chemicals are commonly used for dehumidification viz **absorbents** and **adsorbents.**
- The absorbents are substance which can take up moisture from air and changes chemically and physically. These includes water solutions or brines of calcium chloride, lithium chloride, lithium bromide and enthylene glycol.
- The adsorbents are the solid substance which can take up moisture from air and do not change it chemically or physically. These includes silica gel and activated alumina.

Numericals on Heating with Dehumidification

Ex.5.64 (Adiabatic chemical dehumidification) : *Saturated air at 21 ºC is passed through a drier so that its final relative humidity is 20 %. The drier uses silica gel absorbent. The air is then passed through a cooler until its final temperature is 21 ºC without change in specific humidity. Determine :*

(i) The temperature of air at the end of drying process.
(ii) The heat rejected during at the end of cooling process.
(iii) The relative humidity at the end of cooling process.
(iv) The dew point temperature at the end of drying process.
(v) The moisture removed during the drying process.

Sol. : Refer Fig. 5.86.

Given data : $t_{d1} = t_{d3} = 21$ ºC, $\phi_2 = 20$ %

(i) Temperature of air at the end of drying process :

- Locate the initial condition of air i.e. point '1' on the saturation curve at 21 ºC DBT as shown in Fig. 5.86.
- Since the process is a chemical dehumidification process, it follows a path along constant enthalpy line

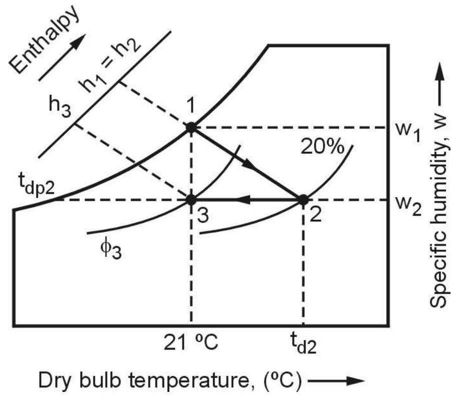


Fig. 5.86

as shown by the line 1-2 in Fig. 8.33. Plot the point 2 at the RH of 20 % on line 1-2.

From psychrometric chart, the temperature at the end of drying process at point '2',

$$t_{d2} = \mathbf{38.5\ ºC} \qquad \textbf{... Ans.}$$

(ii) Heat rejected during the cooling process (Q_r) :

Line 2-3 on psychrometric chart represents cooling process. From psychrometric chart,

Enthalpy of air at point '2' is $h_2 = 61$ kJ/kg of dry air

and enthalpy of air at point '3' is $h_3 = 43$ kJ/kg of dry air

Heat rejected during cooling process $= h_2 - h_3 = 61 - 43$

$$\therefore \quad Q_r = \mathbf{18\ kJ/kg\ of\ dry\ air} \qquad \textbf{... Ans.}$$

(iii) Relative humidity at end of cooling process (ϕ_3) :

From the psychrometry chart, we get

Relative humidity end of cooling process (at point 3)

$$\phi_3 = \mathbf{55\ \%} \qquad \textbf{... Ans.}$$

(iv) Dew point temperature at the end of drying process (t_{dp2}) :

From chart, $t_{dp2} = \mathbf{11.6\ ºC}$ **... Ans.**

(v) Moisture removed during the drying process, (Δw) :

From psychrometric chart,

Moisture in air before drying process at point 1 is,

$$w_1 = 0.0157 \text{ kg/kg of dry air.}$$

Moisture in air after the drying process at point 2 is

$$w_2 = 0.0084 \text{ kg/kg of dry air.}$$

$\therefore$ Moisture removed during drying process

$$= w_1 - w_2 = 0.0157 - 0.0084$$

$$= \mathbf{0.0073 \text{ kg/kg of dry air}} \quad \text{... Ans.}$$

5.25.10 Adiabatic Mixing of Two Air Streams

- Consider the adiabatic mixing of different quantities of air in two different states at constant pressure.
- Let, the two air streams 1 and 2 are mixed then the final condition of the air mixture depends upon the masses involved, enthalpy and specific humidity of each of the constituent which enter the mixture.
- Let

m_1, h_1 and w_1 = Mass flow rate, enthalpy and specific humidity of entering air at '1'.

m_2, h_2 and w_2 = Corresponding values of entering air at '2'.

m_3, h_3 and w_3 = Corresponding values of mixture leaving at 3. Refer Fig. 5.87.

- Neglecting losses during the air mixing process, the mass balance for the mixing process,

$$m_1 + m_2 = m_3 \quad \text{... (i)}$$

The energy balance is given by,

$$m_1 h_1 + m_2 h_2 = m_3 h_3 \quad \text{... (ii)}$$

and mass balance for the water vapour is,

$$m_1 w_1 + m_2 w_2 = m_3 w_3 \quad \text{... (iii)}$$

Substituting the value of m_3 from equation (i),

$$m_1 h_1 + m_2 h_2 = (m_1 + m_2) h_3$$

$$m_1 h_1 - m_1 h_3 = m_2 h_3 - m_2 h_2$$

$$m_1 (h_1 - h_3) = m_2 (h_3 - h_2)$$

$$\therefore \quad \frac{m_1}{m_2} = \frac{h_3 - h_2}{h_1 - h_3} \quad \text{... (iv)}$$

Similarly, substituting the value of m_3 from equation (i) in equation (iii), we have,

$$\frac{m_1}{m_2} = \frac{w_3 - w_2}{w_1 - w_3}$$

$$\therefore \quad \frac{m_1}{m_2} = \frac{h_3 - h_2}{h_1 - h_3} = \frac{w_3 - w_2}{w_1 - w_3}$$

- The adiabatic process is represented on psychrometric chart as shown in Fig. 5.87 (b).
- The final condition of mixture point 3 lies on the straight line 1-2. The point '3' divides the line 1-2 in the inverse ratio of the mixing masses. By calculating the value of w_3 from equation (iv), the point '3' can be plotted on the line 1-2.

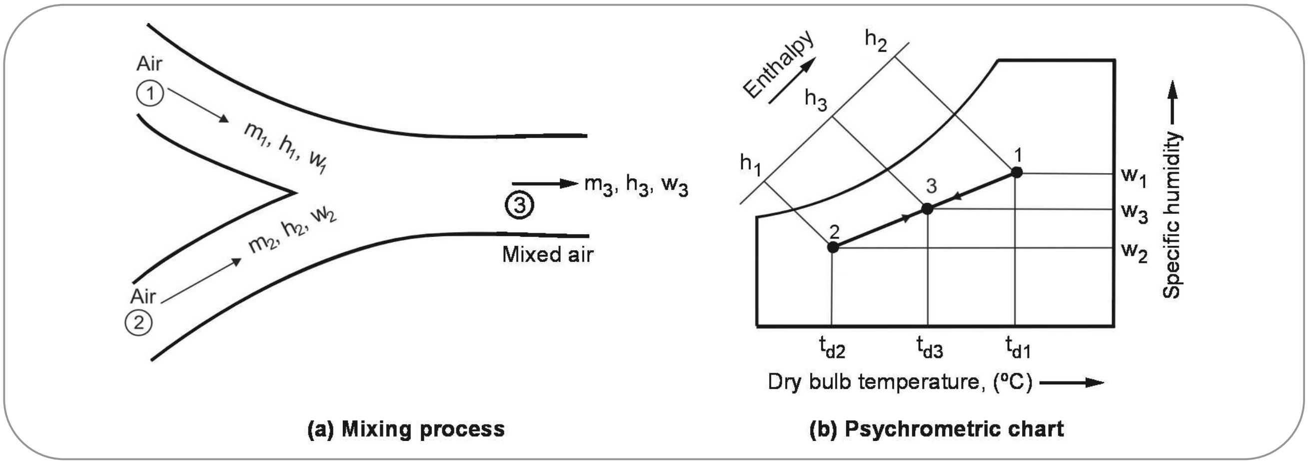


Fig. 5.87 : Adiabatic mixing of two air streams

Note : When warm and high humidity air is mixed with cold air, the resulting mixture will be a fog. In such cases, point 3 on the psychrometric chart will lie to the left or above the saturation curve. The fog can be cleared by heating the fog, mixing the fog with warmer unsaturated air or separate the water droplets from the air.

Numericals on Mixing of Air Streams

Ex. 5.65 (Mixing of two air streams) :

Two air streams are mixed steadily and adiabatically. The first air stream enters at 32 °C DBT and 40 % RH while second enters at 12 °C and 90 % RH. The flow rates of the two streams are 20 m^3/min. and 25 m^3/min. respectively. Determine the specific humidity, relative humidity and flow rate after mixing.

Sol. : Refer Fig. 5.88.

Given data :

First air stream condition '1' = 32 °C DBT and 40 % RH .

Second air stream condition '2' = 12 °C DBT and 90 % RH

Flow rate of first air stream = V_1 = 20 m^3/min.

Flow rate of second air stream = V_2 = 25 m^3/min.

- Locate 1st air stream condition i.e. point '1' at the intersection of 32 °C DBT line and 40 % RH lines.
- Similarly, locate 2nd air stream condition i.e. point '2' at the intersection of 12 °C DBT line and 90 % RH lines as shown in Fig. 5.88 on psychrometric chart. Join line 1-2.
- From psychrometric chart, we get,

$$v_1 = 0.881 \text{ m}^3\text{/kg of dry air.}$$

and $$v_2 = 0.818 \text{ m}^3\text{/kg of dry air.}$$

Mass flow rate of 1st stream is given by,

$$\dot{m}_1 = \frac{V_1}{v_1} = \frac{20}{0.881} = 22.7 \text{ kg/min.}$$

Similarly, mass flow rate of 2nd stream is,

$$\dot{m}_2 = \frac{V_2}{v_2} = \frac{25}{0.818} = 30.56 \text{ kg/min.}$$

According to mass and energy balance, we can write,

$$(\dot{m}_1 + \dot{m}_2)\, t_{d3} = \dot{m}_1 \cdot t_{d1} + \dot{m}_2 \cdot t_{d2}$$

$$\therefore \quad t_{d3} = \frac{22.7 \times 32 + 30.56 \times 12}{22.7 + 30.56} = 20.5 \text{ °C}$$

- Now, plot point '3' (i.e. mixing state) on line 1-2 such that t_{d3} = 20.5 °C.
- From psychromatric chart, v_3 = 0.844 m^3/kg

(i) Specific humidity of mixture at point '3' is

$$\mathbf{w_3 = 0.0096 \text{ kg/kg of dry air}} \quad \text{... Ans.}$$

(ii) RH of Mixture is,

$$\phi_3 = \mathbf{64\ \%} \quad \text{... Ans.}$$

(iii) Flow rate of mixture is,

$$V_3 = m_3 \times v_3$$

$$V_3 = (22.7 + 30.56) \times 0.844 = \mathbf{44.95 \text{ m}^3\text{/min}} \quad \text{... Ans.}$$

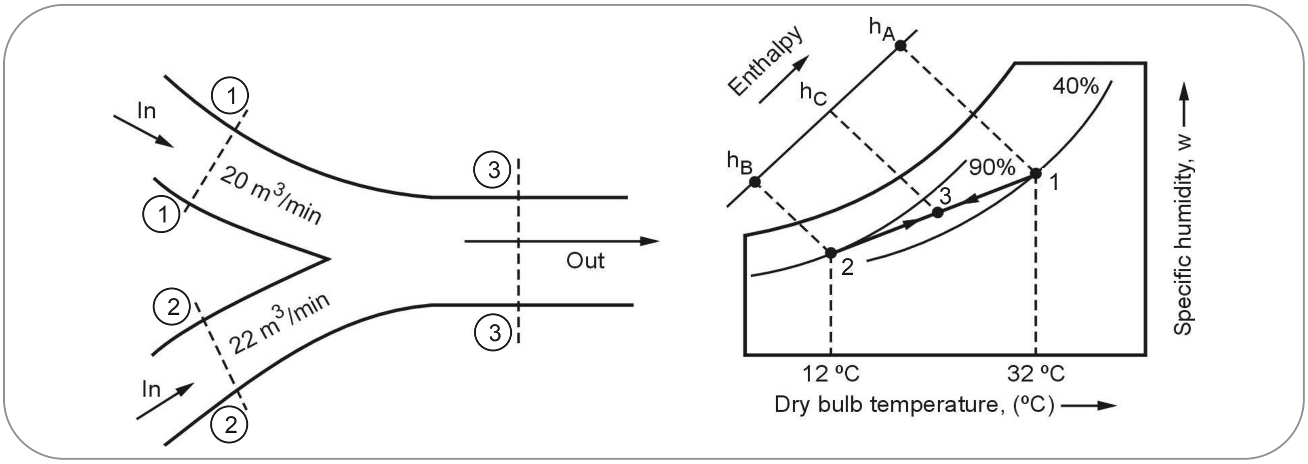


Fig. 5.88

Ex. 5.66 (Mixing of two air streams) : *An air stream of 24 °C DBT and 22 °C WBT is mixed with fresh air of 32 °C DBT and 70 % RH. The volume flow rates of streams is 200 m^3/min and 850 m^3/min respectively. Determine specific humidity, enthalpy and DPT of mixture.*

Sol. : Refer Fig. 5.89.

Given data : t_{db2} = 24 °C, t_{wb2} = 22 °C,

V_2 = 850 m^3/min, t_{db1} = 32 °C, ϕ_1 = 70 %,

V_1 = 200 m^3/min

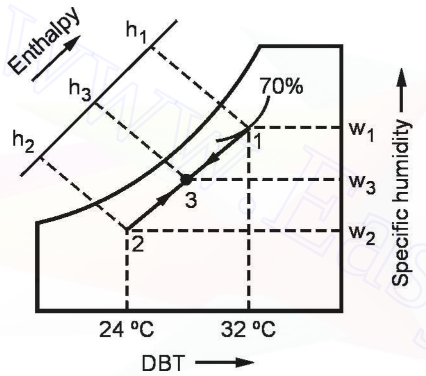


Fig. 5.89

- On psychrometric chart plot point 1 at 70 % RH and 32 °C DBT. Plot point 2 at 24 °C DBT and 22 °C WBT. Join 1-2.
- From psychrometric chart,

At point 1, h_1 = 90 kJ/kg,

w_1 = 0.028 kg/kg of dry air

v_1 = 0.897 kg/kg of dry air

At point 2, h_2 = 51 kJ/kg,

w_2 = 0.0104 kg/kg of dry air,

v_2 = 0.855 kg/kg of dry air

- Mass of fresh air at point 1,

$$\dot{m}_1 = \frac{200}{0.897} = 222.96 \text{ kg/min}$$

Mass of recirculated at point 2.

$$\dot{m}_2 = \frac{V_2}{v_2} = \frac{850}{0.855} = 994.15 \text{ kg/min}$$

Now, $$\frac{m_1}{m_2} = \frac{h_3 - h_2}{h_1 - h_3} \quad \therefore \quad \frac{222.96}{994.15} = \frac{h_3 - 51}{90 - h_3}$$

$$222.96\,(90 - h_3) = 994.15\,(h_3 - 51)$$

$$\mathbf{h_3 = 58.14 \text{ kJ/kg}} \qquad \textbf{... Ans.}$$

- Mark point 3 on line 1 – 2 at h_3 = 58.14 kJ/kg

∴ From psychrometric chart,

$$\mathbf{w_3 = 0.0126 \text{ kg/kg of dry air}} \qquad \textbf{... Ans.}$$

$$\mathbf{t_{dp3} = 17.5 \text{ °C}} \qquad \textbf{... Ans.}$$

5.26 Numericals form University Question Paper

Ex. 5.67 *The sling psychrometer reads 40 °C DBT and 28 °C WBT calculate following i) Specific humidity ii) Relative humidity iii) Vapor density iv) DPT v) Enthalpy of mixture. Assume atm. pressure is 1.03 bar*

Sol. : Given data : 40 °C DBT, 28 °C WBT

i) Specific humidity

From steam table,

at t_{db} = 40 °C p_{vs} = 0.0737 bar

t_{wb} = 28 °C p_w = 0.03778 bar

By carrier equation

$$p_v = p_w - \left[\frac{(p_b - p_w)(t_{db} - t_{wb})}{1544 - 1.44\, t_{wb}}\right]$$

$$= 0.03778 - \left[\frac{(1.0132 - 0.03778)(40 - 28)}{1544 - 1.44 \times 28}\right]$$

$$= 0.03 \text{ bar}$$

Specific humidity

$$\omega = 0.622\left[\frac{p_v}{p_b - p_v}\right] = 0.622\left[\frac{0.03}{1.03 - 0.03}\right]$$

$$= 0.01866 \text{ kg/kg of dry air}$$

ii) Relative humidity

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.03}{0.0737} = 0.407 = 40 \%$$

iii) Vapor density

Specific volume of dry air = $V_a = \dfrac{R \times t_{db}}{p_a} = \dfrac{R \times t_{db}}{p_b - p_v}$

$$V_a = \frac{287 \times (40 + 273)}{(1.03 - 0.03) \times 10^5} = 0.9136 \text{ m}^3/\text{kg}$$

$$\rho_v = \frac{\omega}{V_a} = \frac{0.089}{0.9136} = 0.0206 \text{ kg/m}^3$$

iv) DPT

From steam table,

at $p_v = 0.03$ bar, $t_{dp} = 26$ °C

v) Enthalpy

$$h = 1.005\, t_{db} + \omega(2500 + 1.88\, t_{db})$$

$$= 1.005 \times 40 + 0.0189(2500 + 1.88 \times 40)$$

$$= 88.87 \text{ kJ/kg}$$

Ex. 5.68 : *The sling psychrometer reads 35 °C DBT and 25 °C WBT calculate following i) Specific humidity ii) Relative humidity iii) DPT iv) Enthalpy of mixture. Assume atm. pressure 1.01 bar*

Sol. :

Given data : 35 °C DBT, 25 °C WBT

From steam table,

$t_{db} = 35$ °C $\quad p_{vs} = 0.05622$ bar

$t_{wb} = 25$ °C $\quad p_w = 0.03166$ bar

By carrier equation

$$p_v = p_w - \left[\frac{(p_b - p_w)(t_{db} - t_{wb})}{1544 - 1.44\, t_{wb}}\right]$$

$$p_v = 0.03166 - \left[\frac{(1.01 - 0.03166)(35 - 25)}{1544 - 1.44(25)}\right]$$

$$= 0.025 \text{ bar}$$

$$\omega = 0.622\left[\frac{p_v}{p_b - p_v}\right] = 0.622\left[\frac{0.025}{1.01 - 0.025}\right]$$

$$= 0.0157 \text{ kg/kg of dry air}$$

ii) Relative humidity

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.025}{0.05622} = 0.44 = 44\ \%$$

iii) DPT

From steam table,

at $p_v = 0.025$ bar, $t_{dp} = 21$ °C

v) Enthalpy

$$h = 1.005\, t_{db} + \omega(2500 + 1.88\, t_{db})$$

$$= 1.005 \times 35 + 0.0157(2500 + 1.88 \times 35)$$

$$= 75.45 \text{ kJ/kg}$$

Ex. 5.69 *A mixture of dry air and water vapour is at a temperature of 22 °C under a total pressure of 730 mm of Hg. The dew temperature is 15 °C. Find i) Partial pressure of water vapour ii) Relative humidity iii) Specific humidity iv) Enthalpy v) Specific volume of air*

Sol. :

Given data : $t_{db} = 22$ °C, $t_{dp} = 15$ °C

$$p_t = 1.01325 \times \frac{730}{760} = 0.9732 \text{ bar}$$

i) Partial pressure of water vapour

$$p_v = 0.017 \text{ bar}$$

... From steam table at $t_{dp} = 15$ °C

ii) Relative humidity

From steam table

at $t_{db} = 22$ °C $\quad p_{vs} = 0.0264$ bar

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.017}{0.0264} = 0.644 = 64\ \%$$

iii) Specific humidity

$$\omega = \frac{0.622\, p_v}{p_t - p_v} = \frac{0.622 \times 0.017}{0.9732 - 0.017}$$

$$= 0.011 \text{ kg/kg of dry air}$$

iv) Enthalpy

$$h = 1.005\, t_{db} + \omega(2500 + 1.88\, t_{db})$$

$$= 1.005 \times 22 + 0.011(2500 + 1.88 \times 22)$$

$$= 50.06 \text{ kJ/kg}$$

v) Specific volume

$$V = V_a = \frac{R \times T_{db}}{(p_t - p_v)} = \frac{287 \times (22 + 273)}{(0.9732 - 0.017) \times 10^5}$$

$$= 0.8856 \text{ m}^3/\text{kg of dry air}$$

Ex. 5.70 : *The barometer for air reads 750 mm of Hg. The DBT and WBT measured using sling psychrometer is 33 °C and 23 °C calculate i) Vapour pressure ii) Relative humidity iii) Humidity ratio iv) DPT v) Specific enthalpy vi) Wet bulb depression vii) Dew point depression.*

Sol. :

Given data : $t_{db} = 33$ °C, $T_{wb} = 23$ °C

$$p_t = 1.01325 \times \frac{750}{760} = 0.99 \text{ bar}$$

From steam table,

$$t_{db} = 33 \text{ °C} \qquad p_{vs} = 0.05029 \text{ bar}$$

$$t_{wb} = 23 \text{ °C} \qquad p_w = 0.02808 \text{ bar}$$

i) Vapour pressure

$$p_v = p_w - \left[\frac{(p_b - p_w)(t_{db} - t_{wb})}{1544 - 1.44\, t_{wb}}\right]$$

$$= 0.02808 - \left[\frac{(1.013 - 0.02808)(35 - 23)}{1544 - 1.44 \times 23}\right]$$

$$= 0.02156 \text{ bar}$$

ii) $$\phi = \frac{p_v}{p_{vs}} = \frac{0.02156}{0.05029} = 0.4287 = 42\ \%$$

iii) $$\omega = \frac{0.622 \times p_v}{p_t - p_v} = \frac{0.622 \times 0.02151}{0.99 - 0.02151}$$

$$= 0.0138 \text{ kg/kg of dry air}$$

iii) DPT

From steam table,

at $p_v = 0.0215$ bar, $t_{dp} = 19$ °C

iv) Specific enthalpy

$$h = 1.005 \times t_{db} + \omega \times (2500 + 1.88 \times t_{db})$$

$$= 1.005 \times 33 + 0.0138(2500 + 1.88 \times 33)$$

$$= 68.52 \text{ kJ/kg}$$

v) Wet bulb depression

$$WBD = T_{db} - T_{wb} = 33 - 23 = 10 \text{ °C}$$

vi) Dew point depression

$$DPD = T_{db} - T_{dp}$$

$$= 33 - 19 = 14 \text{ °C}$$

Ex. 5.71 *40 m^3 of air at 35 °C DBT and 50 % R.H. is cooled to 25 °C DBT maintaining its specific humidity constant. Determine :*

i) Relative humidity (R.H.) of cooled air;

ii) Heat removed from air.

Sol. :

Given data : $V_1 = 40$ m^3

$t_{db1} = 35$ °C and 50 % RH, $t_{db2} = 25$ °C

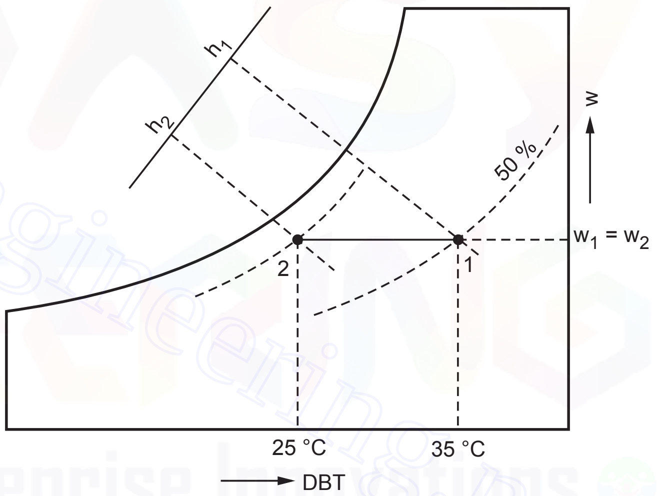


Fig. 5.90

To find :

i) ϕ of cooled air ii) $Q_{removed}$ iii) R.H. of cooled air

from psychrometric chart

$$(\phi)_2 = 88\ \%$$

To find heat removed

$$Q = m_a(h_1 - h_2)$$

$$m = \frac{V}{V_{S1}} = \frac{40}{0.89} = 44.94 \text{ kg/kg of dry air}$$

$$Q = 44.94(81 - 70) \ldots (h_1 \text{ and } h_2 \text{ from chart})$$

$$= 494 \text{ kg}$$

5.27 Air Conditioning System AU : May-16

- An air conditioning is defined as the process of conditioning the air for the simultaneous control of indoor air within the acceptable limits.
- Temperature control can be performed by refrigeration systems, but the simultaneous control of temperature, relative humidity, air movement and purity of air in the space can be performed by air conditioning systems.
- The air conditioning composed of components and equipment arranged in sequence to condition the air to provide comfort conditions. Most air conditioning systems perform the following functions :
  - Condition the supply air i.e. heat or cool, humidify or dehumidify, clean and purify and reduce any objectionable noise produced by the equipment.
  - Distribute the conditioned air, containing sufficient outdoor air, to the conditioned space.
  - Control and maintain the indoor environmental parameters such as temperature, humidity, cleanliness, air movement, sound level and pressure differential between the conditioned space and surroundings within predetermined limits.
- Air conditioning systems can be classified according to their applications as follows :

(a) Comfort air conditioning systems and
(b) Industrial air conditioning systems.

(a) Comfort Air Conditioning Systems :

- Comfort air conditioning systems provide occupants with a comfortable and healthy indoor environment to carry out their activities. The various sectors of the economy using comfort air conditioning systems are as follows :
  - The commercial sector includes office buildings, supermarkets, department stores, shopping centers, restaurants and others. Many high-rise office buildings use complicated air conditioning systems to satisfy multiple tenant requirements.
  - The institutional sector includes schools, colleges, universities, libraries, museums, indoor stadiums, cinemas, theaters, concert halls and recreation centers.
  - The residential and lodging sector consists of hotels, motels, apartment houses and private homes.
  - The health care sector encompasses hospitals, nursing homes and convalescent care facilities. Special air filters are generally used in hospitals to remove bacteria and particulates of sub-micrometer size from areas such as operating rooms, nurseries and intensive care units.
  - The transportation sector includes aircraft, automobiles, railroad cars, buses and cruising ships.

(b) Industrial Air Conditioning Systems :

- Industrial air conditioning systems provide needed indoor environmental control for manufacturing, product storage or other research and development processes. The following areas are examples of industrial air conditioning systems :
  - In textile mills, natural fibers and manufactured fibers are hygroscopic. Proper control of humidity increases the strength of the yarn and fabric during processing.
  - Many electronic products require clean rooms for manufacturing like integrated circuits, since their quality is adversely affected by airborne particles.
  - Precision manufacturers always need precise temperature control during production of precision instruments, tools and equipment.
  - Pharmaceutical products require temperature, humidity and air cleanliness control.

5.27.1 Winter Air Conditioning

- Fig. 5.91 shows the arrangement of winter air conditioning system. In this system, the air is heated which is accompanied by humidification.

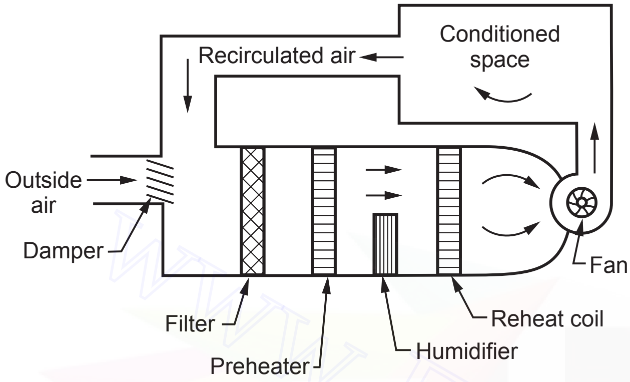


Fig. 5.91 : Winter air conditioning system

- The outside air passes through a damper and mixes up with the recirculated air.
- The mixed air flows through a filter to remove dirt, dust and other impurities.
- Now, to prevent the freezing of water and to control the evaporation of water in the humidifier, the air passes through a preheat coil.
- After passing through preheat coil, now the air is made to pass through a reheat coil to bring the air to required DBT.
- Now, the conditioned air is supplied to the conditioned space by a fan and a part of the used air is exhausted to the atmosphere by using exhaust fans.
- The remaining part of the used air called as recirculated air again conditioned.

5.27.2 Summer Air Conditioning

- The schematic arrangement of a summer air conditioning system is shown in Fig. 5.92.
- It is the most important type of air conditioning in which the air is cooled and dehumidified.
- The outside air flows through the damper and mixes with recirculated air (obtained from the conditioned space).

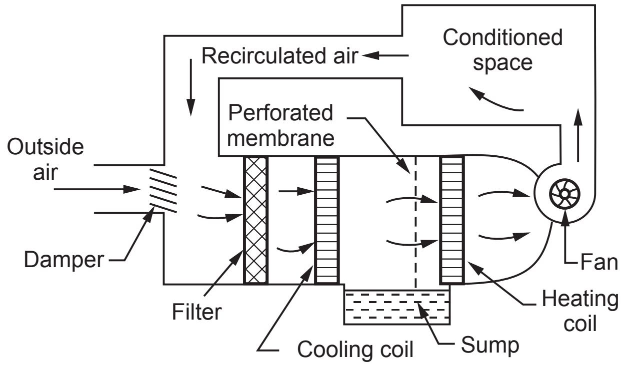


Fig. 5.92 : Summer air conditioner

- The mixed air is then passed through a filter in order to remove dust, dirt and other impurities.
- This air then passes through a cooling coil. This coil has a temperature below the required dry bulb temperature of the air in the conditioned space.
- The cooled air passes through a perforated membrane and loses its moisture in the condensate form which is collected in a sump.
- Then the air is made to pass through a heating coil which heats up the air slightly. This is done to bring the air to the designed dry bulb temperature and relative humidity.
- These conditioned air is supplied to the conditioned space by a fan.
- A part of the used air is exhausted to the atmosphere by the exhaust fans or ventilators from the conditioned space.
- The remaining part of the used air (recirculated air) is again conditioned. The outside air is sucked and made to mix with recirculated air in order to make up for the loss of conditioned air through exhaust fans or ventilators from conditioned space.

5.27.3 Year Round Air Conditioning

- The year - round air conditioning system should have equipment for both summer and winter air conditioning.
- The schematic arrangement of year - round air conditioning system is shown in Fig. 5.93.

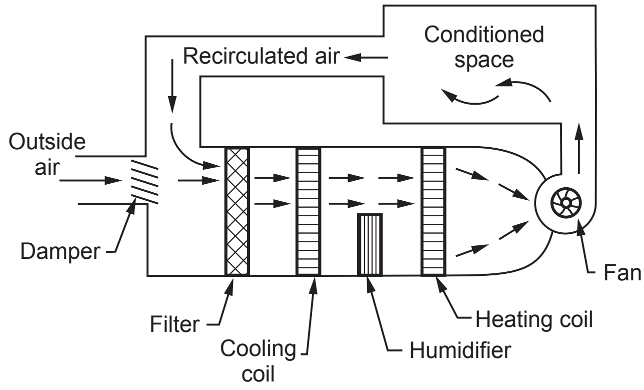


Fig. 5.93 : Year - round air conditioner

- The outside air flows through the damper and mixes up with the recirculated air.
- The mixed air passes through the filter to remove dirt, dust and other impurities.
- In summer air conditioning, the cooling coil operates to cool the air to the desired value. The dehumidification is obtained by operating the cooling coil at a temperature lower than the dew point temperature.
- In winter, the cooling coil is made inoperative and the heating coil operates to heat the air.

5.28 Introduction to Cooling Load

- For the designing of any air conditioning system it is necessary to understand heat sources and their nature.
- The total amount of heat which is to be removed from the space by the use of air conditioning equipment is known as cooling load.
- The cooling load estimation plays the vital role to determine the size of air conditioning equipment.
- There are two types of loads

  i) Sensible heat load and ii) Latent heat load.

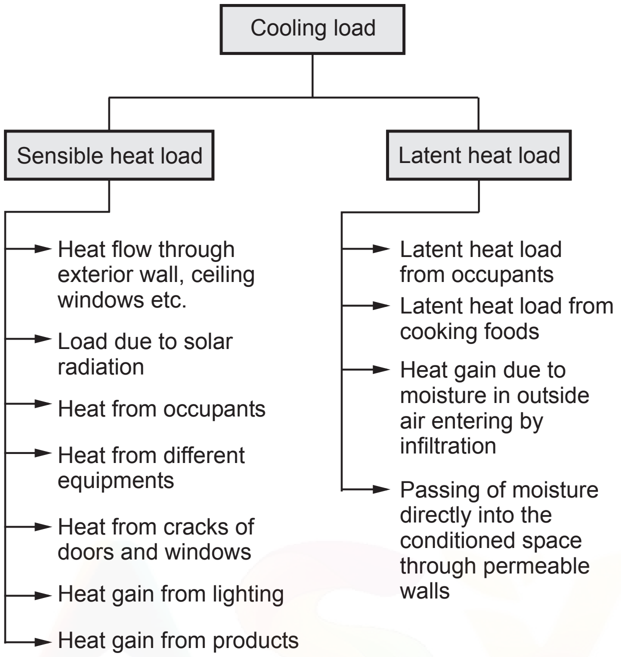


Fig. 5.94

5.28.1 Heat Flow Due to Conduction

- Consider a plane wall which is made up of single material.
- The heat passing through wall is first received by the wall surface which is exposed to atmosphere.
- It then flows through a interior portion of the wall.
- If steady state of heat flow is considered then heat flow is given by equation.

$$Q = UA(T_o - T_i)$$

Where, T_o - Outside air temperature

T_i - Conditioned air temperature

A - Area through which heat is flowing

U - Overall heat transfer coefficient

- The overall heat transfer coefficient is given by

$$\frac{1}{U} = \frac{1}{h_o} + \frac{x_1}{K_1} + \frac{x_2}{K_2} + \ldots \frac{1}{h_i}$$

$$= \frac{1}{h_o} + \sum_{1}^{n} \left(\frac{x}{K}\right) + \frac{1}{h_i}$$

h_o - Heat transfer coefficient of outer surface.

h_i - Heat transfer coefficient on inner surface.

- When the wall is made up of number of materials is known as composite wall.

5.28.2 Solar Radiation Through Fenestration

- Fenestration refers to any transparent apertures in a building, such as glass doors, windows, skylights etc.
- The fenestration or glazed surfaces contribute a major part of cooling load of a building.
- Because of its transparent nature it transmit solar radiation into the building.
- This heat transfer through transparent surfaces is different from heat transfer through opaque surfaces.
- When solar radiation is incident on an opaque building wall, a part of it is absorbed while the remaining part is reflected back.
- Only a fraction of the radiations are absorbed by the opaque surfaces is transferred to the interiors of the building.
- However, in case of transparent surfaces, a major portion of the solar radiation is transmitted directly to the interiors of the building, while the remaining small fraction is absorbed and reflected back.
- The energy transfer due to fenestration depends on the characteristics of the surface and its orientation, weather and solar radiation conditions.
- The amount of solar radiation passing through a transparent surface can be written as :

$$Q_{sg} = A(\tau \cdot I_t + N \cdot \alpha \cdot I_t) \tag{5.16}$$

Where : A = Area of the surface exposed to radiation

I_t = Total radiation incident on the surface

τ = Transmittivity of glass for direct, diffuse and reflected radiations.

α = Absorptivity of glass for direct, diffuse and reflected radiations.

Assuming the transitivity and absorptivity of the surface same for direct, diffuse and reflected components of solar radiation.

5.28.3 Load Due to Occupants

The internal cooling load due to occupants consists of both sensible and latent heat components.

- The rate at which the sensible and latent heat transfer take place depends mainly on the population and activity level of the occupants.
- Since a portion of the heat transferred by the occupants is in the form of radiation.
- The sensible heat transfer to the conditioned space due to the occupants is given by the equation :

$$Q_{s,\,occupants} = (\text{Number of people}) \cdot (\text{Sensible heat gain / person}) \cdot \text{CLF}$$

- The value of Cooling Load Factor (CLF) for occupants depends on the hours after the entry of the occupants into the conditioned space, the total hours spent in the conditioned space and type of the building.

Activity	Total heat gain, W	Sensible heat gain fraction
Sleeping	70	0.75
Seated, quiet	100	0.60
Standing	150	0.50
Walking @ 3.5 kmph	305	0.35
Office work	150	0.55
Teaching	175	0.50
Industrial work	300 to 600	0.35

- Table shows typical values of total heat gain from the occupants and also the sensible heat gain fraction as a function of activity in an air conditioned space.

Ex. 5.72 : *100 m^3 of air per minute at 15 °C DBT 80% RH is heated until its temperature in 22 °C. Calculate heat added to air per minute. RH of the heated air and wet bulb temperature of the heated air.*

AU : Nov.-16, Marks 8

Sol. : Given data :

$$V_1 = 100\ m^3/min$$

$$T_1 = 15\ ℃\ DBT,\ \phi_1 = 80\ \%\ RH$$

$$T_2 = 22\ ℃$$

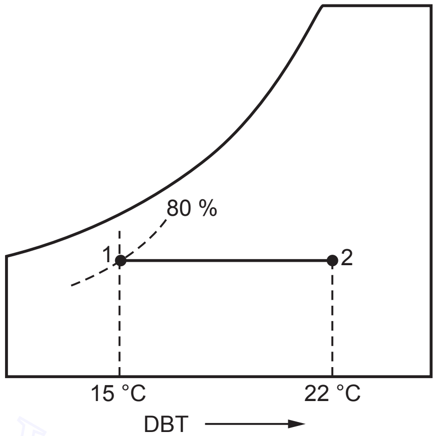


Fig. 5.95

From chart,

$$h_1 = 37 \text{ kJ/kg},$$

$$h_2 = 44 \text{ kJ/kg}$$

$$V_{S_1} = 0.82 \text{ m}^3/\text{kg}$$

To find out, $\dot{m} = ?$

$$\dot{m} = \frac{V}{V_{S_1}} = \frac{100}{0.82} = 121.95 \text{ kg/min.}$$

Heat added (Q)

$$Q = m(h_2 - h_1) = \frac{121.95}{60}(44 - 37)$$

$$Q = 14.22 \text{ kJ/kg}$$

From chart relative humidity and WBT of heated air.

$$Q_2 = 52 \text{ \% and WBT} = 16.7 \text{ °C}$$

5.29 Load Due to Equipments and Appliances

- The equipment and appliances used in the conditioned space may add both sensible as well as latent loads to the conditioned space.
- The sensible load may be in the form of radiation and convection.
- The internal sensible load due to equipment and appliances is given by

$$Q_{s,\text{appliances}} = (\text{Installed wattage}) \cdot (\text{Usage factor}) \cdot \text{CLF}$$

- The installed wattage and usage factor depend on the type of the appliance or equipment.
- For the equipment such as computers, printers etc. the load is in the form of sensible heat transfer and is estimated based on the rated power consumption.

Appliance	Sensible load, W	Latent load, W	Total load, W
Coffee brewer, 0.5 gallons	265	65	330
Coffee warmer, 0.5 gallons	71	27	98
Toaster, 360 slices/h	1500	382	1882
Food warmer / m^2 plate area	1150	1150	2300

Table shows typical load of various types of appliances.

5.30 Load Due to Lightning

- **Lighting adds sensible heat to the conditioned space.**
- The heat transferred from the lighting system consists of radiation and convection.
- The cooling load due to lighting system is given by :

$$Q_{s,\text{ lighting}} = (\text{Installed wattage}) \; (\text{Usage factor}) \; (\text{Ballast factor}) \; \text{CLF}$$

- The usage factor is used when any lamps that are installed but are not switched on at the time at which load calculations are performed.
- The ballast factor is used when the load imposed by ballasts used in fluorescent lights.

A typical ballast factor value of 1.25 is taken for fluorescent lights, while it is equal to 1.0 for incandescent lamps.

5.31 Heat Gain From Products

Chilling load above freezing

- It depends on mass (m), specific heat (C_{pm}), entering temperature (T_1), desired temperature (T_2) and chilling time (t_{ch}) of product.

- It can be calculated as,

$$Q_{ch} = \frac{m\,C_{pm}(T_1 - T_2)}{t_{ch}}$$

Freezing load

- It depends on latent heat (h_{fg}), mass (m) and freezing time (t_F).
- It can be calculated as,

$$Q_F = \frac{m \times h_{fg}}{t_f}$$

Product reaction heat

- Heat is evolved during maturing of food product.

$$Q_R = m \times \text{Evolution of heat per kg / hrs}$$

5.32 Heat Transfer Due to Infiltration

- Heat transfer due to infiltration consists of both sensible and latent components.
- The sensible heat transfer due to infiltration is given by,

$$(Q)_{sen.} = mC_{pm}(T_o - T_i) = V\rho C_{pm}(T_o - T_i)$$

where,

V - infiltration rate (m^3 / sec.)

T_o, T_i - outdoor and indoor DBT.

- The latent heat transfer due to infiltration is

$$(Q)_{Lat} = mh_{fg}(W_o - W_i) = V\rho h_{fg}(W_o - W_i)$$

where,

h_{fg} - latent heat of vaporization.

W_o and W_i - Outdoor and indoor humidity ratio.

- Infiltration rate depends upon wall, windows and doors and prevailing wind direction, speed.
- The infiltration rate by air change method is given by,

$$V_o = \frac{ACH \times V}{3600} m^3 / sec$$

where, ACH - number of air changes per hrs.

V - gross volume of conditioned space in m^3.

- The infiltration rate by crack method is given by,

$$V_o = A \times C \times \Delta P^n \, m^3 / sec.$$

where,

A - Effective leakage area of crack

C - Flow coefficient

ΔP - Difference between outside and inside pressure

n - An exponent

- The value of n depends on nature of flow in crack $0.4 \le n \le 1$.

5.33 Load on System Due to Ventilated Air

- Ventilation is means supply of outside air.
- Ventilation is provided to the conditioned space in order to minimise odour, concentration of smoke and undesirable gases.
- Any air conditioning system consists of cooling coil, supply and return air duct, ventilation and fans.
- The cooling coil has bypass factor X.
- The cooling load on coil due to sensible heat factor of ventilated air is given by,

$$(Q)_{vent} = m_{vent}(1 - X)C_{pm}(T_o - T_i)$$
$$= V_{vent}\rho(1 - X)C_{pm}(T_o - T_i)$$

where,

m_{vent} - Mass of ventilated air.

V_{vent} - Volume flow rate of ventilated air.

X - BPF of coil.

- The latent heat load on coil due to ventilation is given by,

$$(Q)_{vent} = m_{vent}(1 - X)h_{fg}(W_o - W_i)$$
$$= V_{vent}\rho(1 - X)h_{fg}(W_o - W_i)$$

where, W_o, W_i - Outside and inside humidity ratio.

h_{fg} - Latent heat of vaporization.

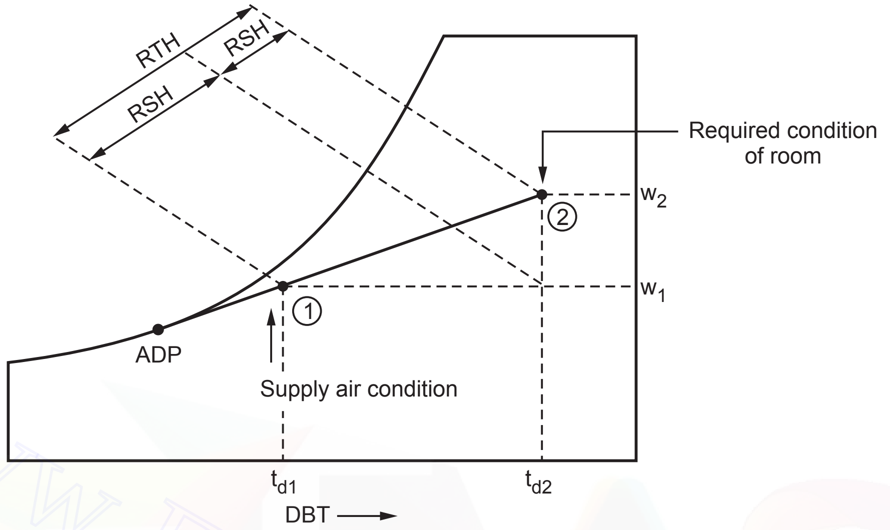


Fig. 5.96

5.34 Room Sensible Heat Factor (RSHF)

- Room sensible heat factor is the ratio of room sensible heat to room total heat.

$$\text{RSHF} = \frac{\text{RSH}}{\text{RTH}} = \frac{\text{RSH}}{\text{RSH} + \text{RLH}}$$

- From the above figure,

Point 1 - represents supply air condition

Point 2 - represents required room design condition

- The line (1-2) is called as room sensible heat factor line (RSHF-line).
- The slope on this line gives the ratio of room sensible heat (RSH) AND Room Latent Heat (RLH).
- The supply air in to the room must have the capacity to absorb room sensible heat load and room latent heat load.
- RSHF line generally used to find out supply air condition.

5.34.1 Steps to Draw RSHF Line

- Calculate the RSHF

$$\text{RSHF} = \frac{\text{RSH}}{\text{RSH} + \text{RLH}}$$

- Mark the point 'X' on Sensible Heat Factor (SHF) scale which is at right side of psychrometric chart.
- Locate the guide point 'Y' (Considering 26 ºC DBT and 50 % RH)
- Join the line XY. This XY line is known as guide line.
- Mark the point 'Z' (Which represents the design condition of room).
- From the point Z draw the line ZZ′ which is parallel to guide line (XY).
- This line ZZ′ represents the RSHF line.

5.35 Grand Sensible Heat Factor (GSHF)

GSHF is defined as the ratio of the total sensible heat to grand total heat which the cooling coil handle.

$$\text{GSHF} = \frac{\text{TSH}}{\text{GTH}}$$

$$\text{GSHF} = \frac{\text{TSH}}{\text{TSH} + \text{TLH}}$$

But,

TSH - Total sensible heat = RSH + Outside air sensible heat

= RSH + OASH

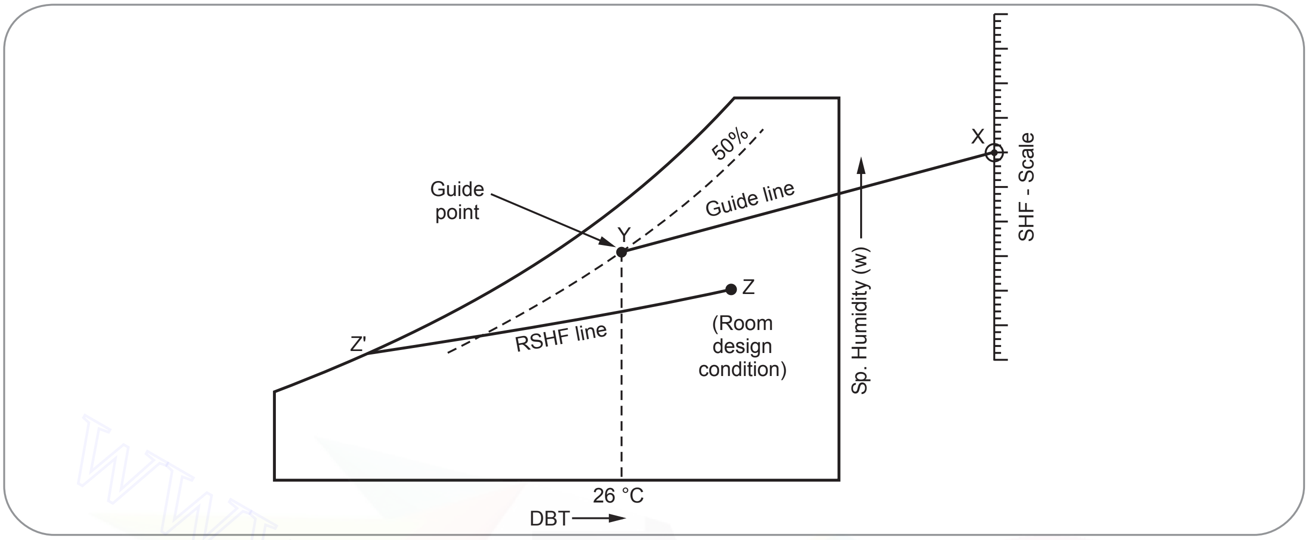


Fig. 5.97

TLH - Total latent heat = RLH + Outside air latent heat

= RLH + OALH

$$\therefore \quad GSHF = \frac{RSH + OASH}{(RSH + OASH) + (RLH + OALH)}$$

GTH - Grand total heat

$$GTH = TSH + TLH$$

$$= RSH + RLH + OATH$$

$$= RSH + RLH + (OASH + OALH)$$

To find	Formula to be used
OASH	$0.02044\ v_1(td_1 - td_2)$
OALH	$50\ v_1(w_1 - w_2)$
OATH	$0.02\ v_1(h_1 - h_2)$

- Point 1 represents outside condition of air.
- Point 2 represent room air condition.
- Point 3 represent mixture condition of air which is entering the cooling coil.
- Point 4 represents leaving condition of air from cooling coil.
- To have economic operation of air conditioning plant the outside fresh air is mixed with recirculated air.
- This air mixture is passed through cooling coil where it is cooled and dehumidified.
- In above figure, this is represented by point 4.
- When point 3 is joined with point 4 it gives one line which is Grand Sensible Heat Factor (GSHF line).
- If this line extended upto the saturation curve it gives Apparatus Dew Point (ADP).
- If we know mixture condition which is entering in cooling coil and Grand Sensible Heat Factor (GSHF) then GSHF line can be drawn in similar way as per the discussion for RSHF line.

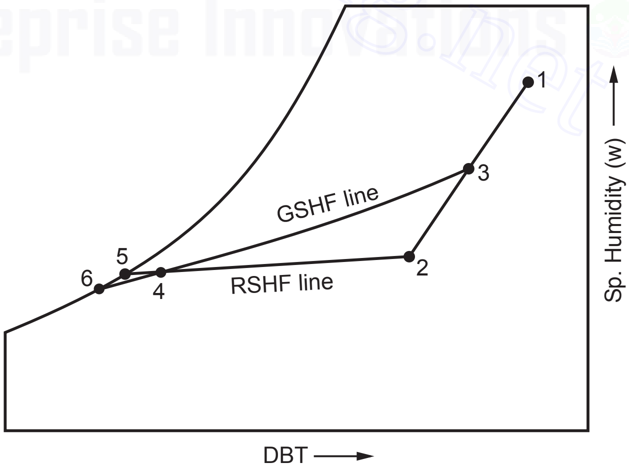


Fig. 5.98

5.36 Effective Room Sensible Heat Factor (ERSHF)

- ERSHF is the ratio of effective room sensible heat to effective room total heat.

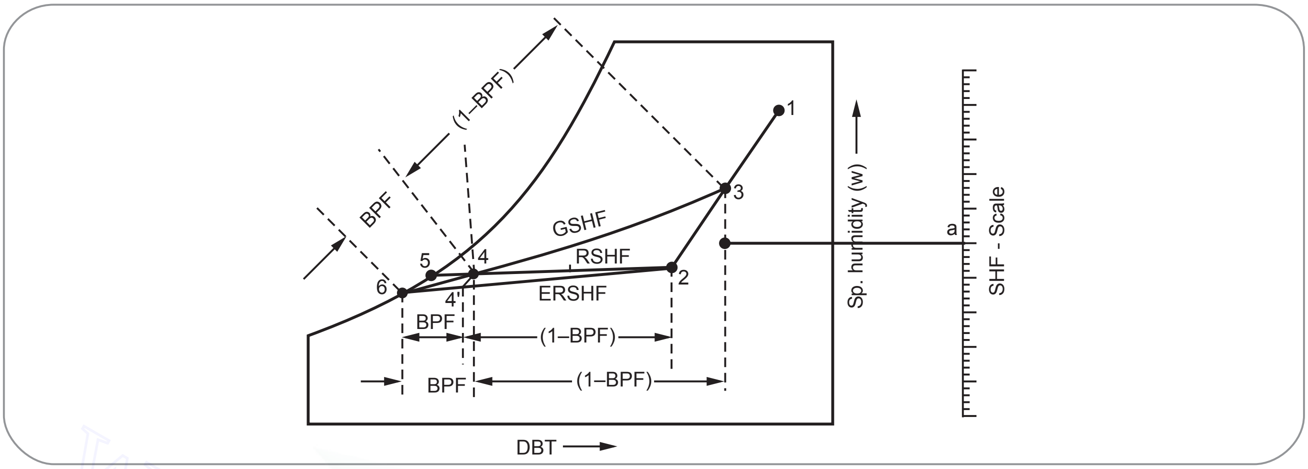


Fig. 5.99

$$\text{ERSHF} = \frac{\text{ERSH}}{\text{ERTH}} = \frac{\text{ERSH}}{\text{ERSH} + \text{ERLH}}$$

Where,

- ERSH - Effective Room Sensible Heat
  = RSH + OASH × BPF
- ERLH - Effective Room Latent Heat
  = RLH + OALH × BPF
- ERTH - Effective Room Total Heat
  = ERSH + ERLH

To find	Formula to be used
ERSH	$RSH + 0.02044\,(td_1 - td_2) \times BPF$
ERLH	$RLH + 50v_1(w_1 - w_2) \times BPF$

- The Effective Room Sensible Heat Factor line (ERSH line) is obtained by joining the point 2 and point 6 (which is on saturation curve i.e. ADP).
- Point 1 represents outdoor condition.
- Point 2 represents room design condition.
- Point 3 represents air mixture condition entering the coil.
- Point 4 represent room supply air condition.
- Point 6 represents ADP.
- From the point 4 draw a line 4 - 4′ which is parallel to line 3 - 2.
- Therefore from triangle 6 - 4 - 4′ and 6 - 3 - 2

We can write,

$$\text{BPF} = \frac{\text{Length } 4-6}{\text{Length } 3-6} = \frac{\text{Length } 4'-6}{\text{Length } 2-6}$$

OR,

$$\text{BPF} = \frac{t_{d4} - \text{ADP}}{t_{d3} - \text{ADP}} = \frac{t_{d4'} - \text{ADP}}{t_{d2} - \text{ADP}}$$

- The mass of dehumidified air can be obtained by

$$m_d = \frac{\text{Room total heat}}{h_2 - h_4}$$

5.37 Solved Numericals on GSHF, RSHF and ERSHF

Ex. 5.73 : *A room has sensible heat gain 15 kW and latent heat gain of 10 kW. The room is to be maintained at 20 °C and 40 % RH. Draw RSHF line.*

Given data : *RSH = 15 kW, RLH = 15 kW, t_{d1} = 20 °C and 40 % RH*

Sol. : Refer Fig. 5.100 (a). (See Fig. 5.100 (a) on next page).

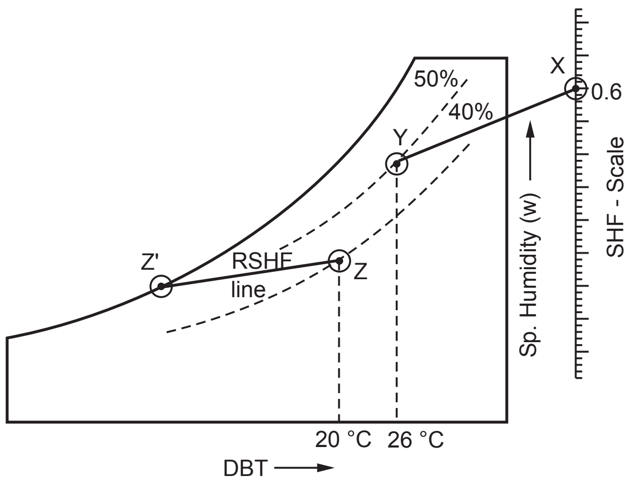


Fig. 5.100

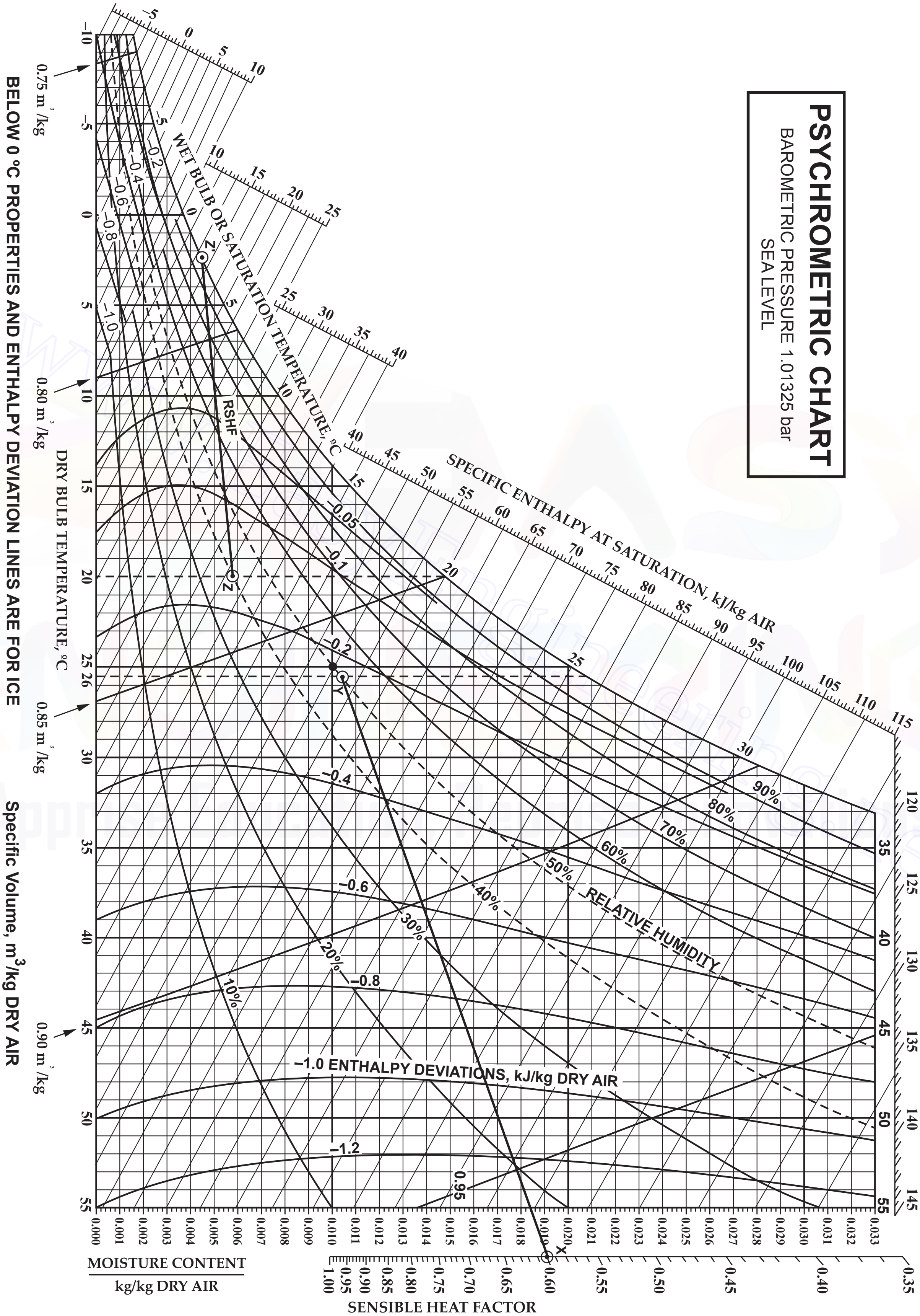

PSYCHROMETRIC CHART
BAROMETRIC PRESSURE 1.01325 bar
SEA LEVEL
WET BULB OR SATURATION TEMPERATURE, °C
SPECIFIC ENTHALPY AT SATURATION, kJ/kg AIR
DRY BULB TEMPERATURE, °C
RELATIVE HUMIDITY
ENTHALPY DEVIATIONS, kJ/kg DRY AIR
RSHF
0.75 m³/kg
0.80 m³/kg
0.85 m³/kg
0.90 m³/kg
BELOW 0 °C PROPERTIES AND ENTHALPY DEVIATION LINES ARE FOR ICE
Specific Volume, m³/kg DRY AIR
MOISTURE CONTENT
kg/kg DRY AIR
SENSIBLE HEAT FACTOR

Fig. 5.100 (a)

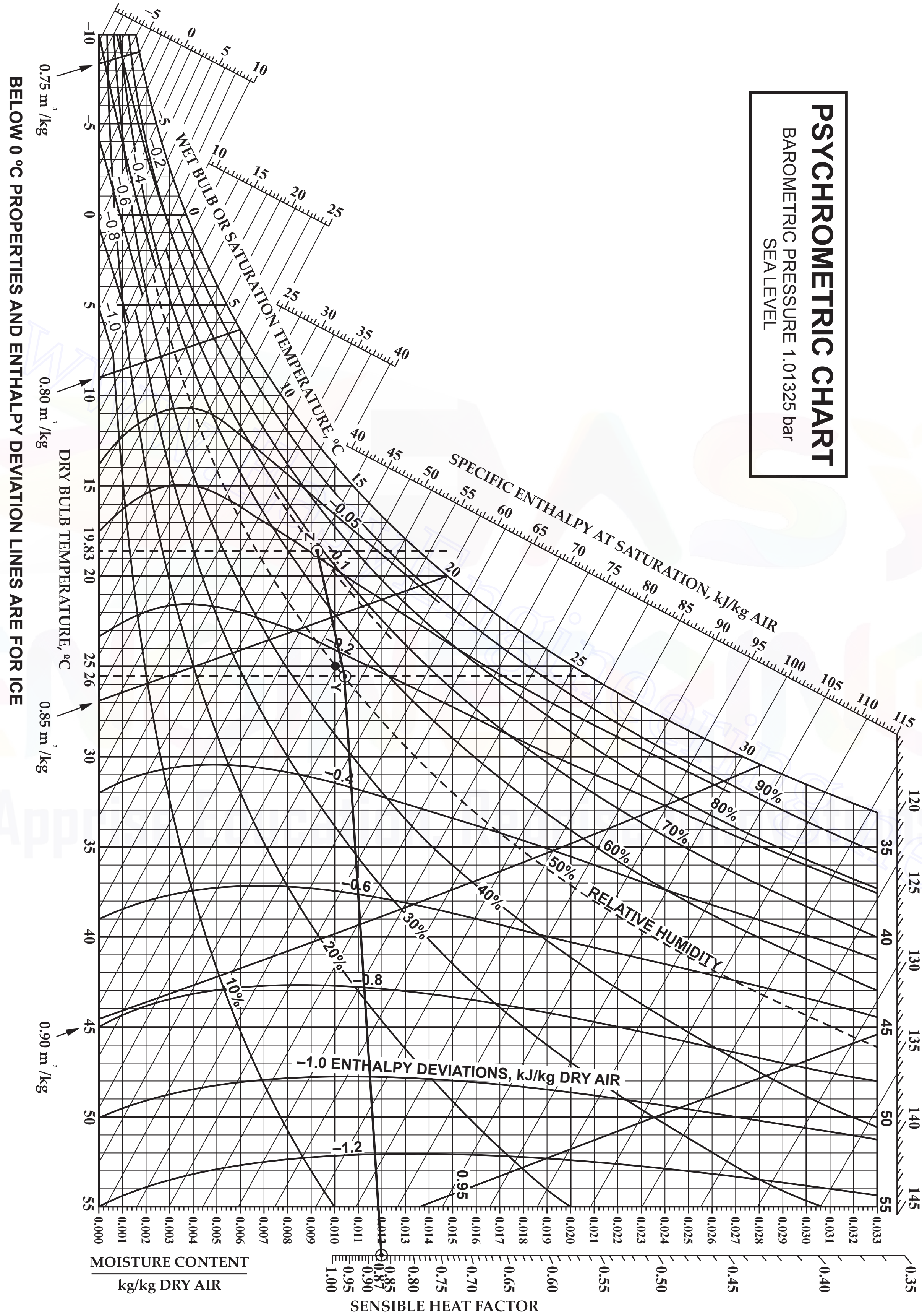

PSYCHROMETRIC CHART
BAROMETRIC PRESSURE 1.01325 bar
SEA LEVEL
WET BULB OR SATURATION TEMPERATURE, °C
SPECIFIC ENTHALPY AT SATURATION, kJ/kg AIR
RELATIVE HUMIDITY
ENTHALPY DEVIATIONS, kJ/kg DRY AIR
DRY BULB TEMPERATURE, °C
BELOW 0 °C PROPERTIES AND ENTHALPY DEVIATION LINES ARE FOR ICE
MOISTURE CONTENT
kg/kg DRY AIR
SENSIBLE HEAT FACTOR

Fig. 5.101 (a)

- Find out the RSHF.

$$\text{RSHF} = \frac{\text{RSH}}{\text{RSH}+\text{RLH}} = \frac{15}{15+10}$$

$$\textbf{RSHF} = \textbf{0.6}$$

- Mark the point X on SHF scale (at 0.6)
- Locate point Y(Considering 26 °C DBT and 50 % RH) i.e. guide point.
- Draw XY line (which is guide line).
- Mark point Z (at 20 °C DBT and 40 % RH) … given data
- From point Z draw a line ZZ′ which is parallel to guide line (XY).
- This ZZ′ line represents RSHF line.

Ex. 5.74 : *A shop has sensible heat gain 24 kW and latent heat gain 3.5 kW and it has to be maintained at 26 °C. DBT and 50 % RH. 190 m^3/min of air is delivered to room. Determine state of supply air.*

Sol. : Refer Fig. 5.101 (a) on previous page.

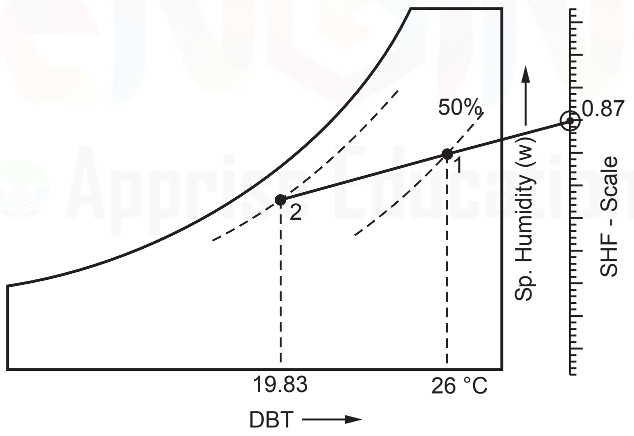


Fig. 5.101

Given data : RSH = 24 kW, RLH = 3.5 kW, $t_{d1} = 26$ °C and 50 % RH, $v = 190\ m^3/min$

$$\text{RSH} = 0.02044\, v(t_{d1} - t_{d2})$$

$$24 = 0.02044 \times 190\,(26 - t_{d2})$$

$$24 = 100.97 - 3.88\ t_{d2}$$

$$t_{d2} = 19.83\ °C$$

Now, $\text{RSHF} = \frac{\text{RSH}}{\text{RSH}+\text{RLH}}\ \frac{24}{24+3.5} = 0.87$

State of supply air

$$\left.\begin{aligned} t_{d1} &= 19.83\ °C \\ \text{RH} &= 68\,\% \\ t_{d1} &= 15.2\ °C \end{aligned}\right\}\ \text{From psychrometry chart}$$

Ex. 5.75 : *Outside design conditions are 40 °C DBT and 30 % RH. Room design conditions are 25 °C DBT and 50 % RH Room sensible heat 50 kW and room latent heat is 10 kW. If outside air quantity is 50 m^3/min and assuming BPF of cooling coil 0.1. Find GSHF and ESHF.*

Sol. : Given data :

Outise condition	Design condition
40 °C DBT	25 °C DBT
30 °C RH	50 % RH

Room Sensible Heat (RSH) = 50 kW

Room Latent Heat (RLH) = 10 kW

Outside air quantity = 50 m^3/min

BPF = 0.1

Locate the point 1 as 40 °C DBT and 30 % RH

Similarly locate point 2 a 30 °C DBT and 50 % RH.

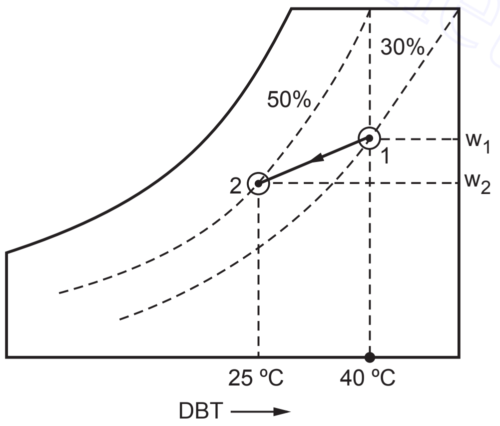


Fig. 5.102

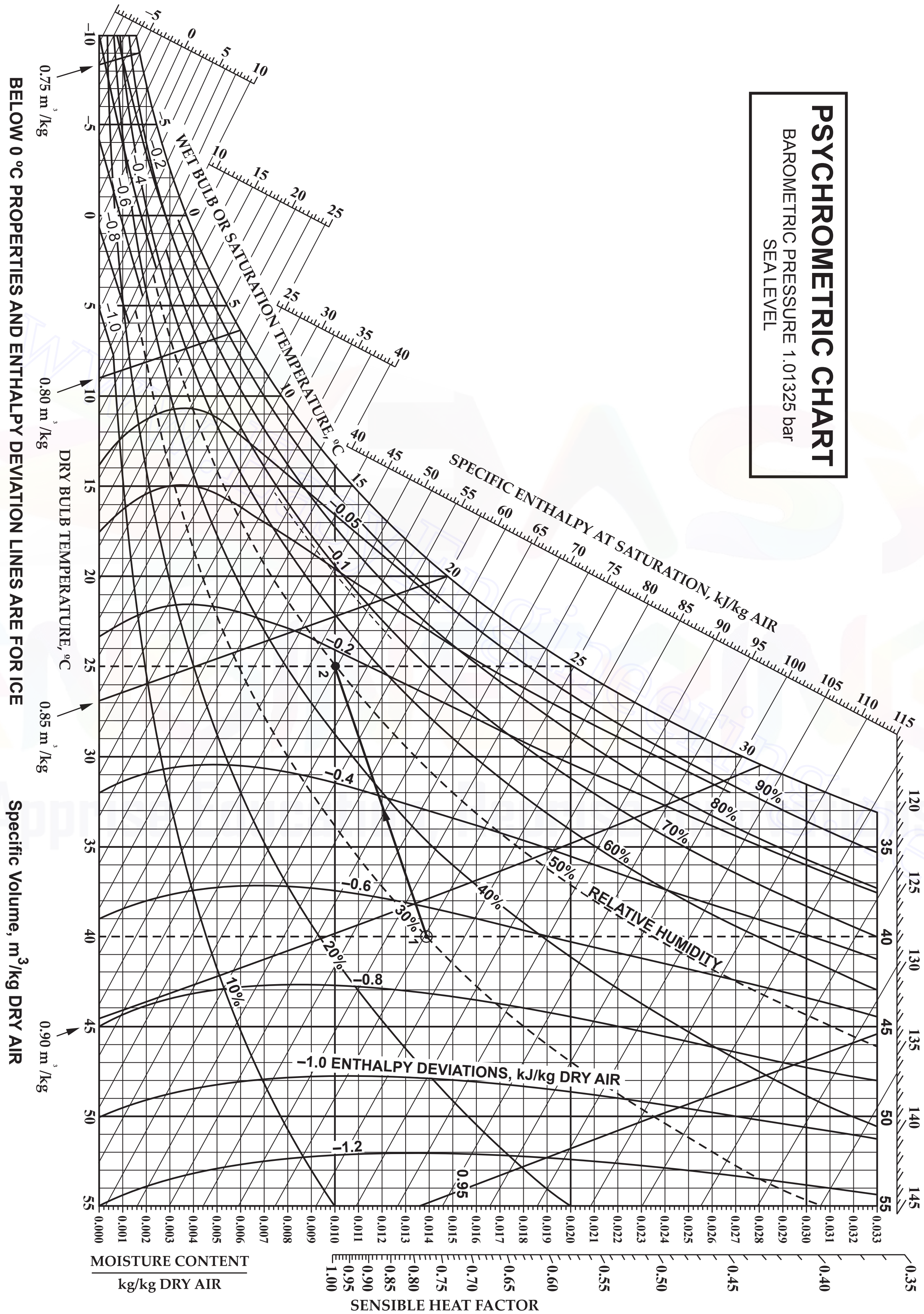

PSYCHROMETRIC CHART
BAROMETRIC PRESSURE 1.01325 bar
SEA LEVEL
WET BULB OR SATURATION TEMPERATURE, °C
SPECIFIC ENTHALPY AT SATURATION, kJ/kg AIR
DRY BULB TEMPERATURE, °C
RELATIVE HUMIDITY
–1.0 ENTHALPY DEVIATIONS, kJ/kg DRY AIR
BELOW 0 °C PROPERTIES AND ENTHALPY DEVIATION LINES ARE FOR ICE
Specific Volume, m3/kg DRY AIR
0.75 m3/kg
0.80 m3/kg
0.85 m3/kg
0.90 m3/kg
MOISTURE CONTENT
kg/kg DRY AIR
SENSIBLE HEAT FACTOR

Fig. 5.102 (a)

- To find outside sensible heat load,

$$OASH = 0.02044\, v_1 (t_{d1} - t_{d2})$$
$$= 0.02044 \times 50 \times (40 - 25)$$
$$= 15.33 \text{ kW}$$

- To find latent heat load,

$$OALH = 50 \times v_1 (w_1 - w_2)$$
$$= 50 \times 50\,(0.014 - 0.010)$$

...[Values of w_1 and w_2 are from psychrometric chart]

$$= 10 \text{ kW}$$

- To find Grand Sensible Heat (GSH)

$$GSH = OASH + RSH$$
$$= 15.33 + 50 = 65.3 \text{ kW}$$

- To find Grand Latent Heat (GLH)

$$GLH = OALH + RLH$$
$$= 10 + 10 = 20 \text{ kW}$$

- To find GSHF

$$GSHF = \frac{GSH}{GSH + GLH}$$
$$= \frac{65.3}{65.3 + 20} = 0.7655$$

- To find effective room heat,

$$ERH = ERSH = RSH + BPF \times OASH$$
$$= 50 + 0.1 \times 15.3$$
$$= 51.53 \text{ kW}$$

- To find effective latent heat,

$$ERLH = RLH + BPF \times OALH$$
$$= 10 + 0.1 \times 10$$
$$= 11 \text{ kW}$$

- To find ESHF,

$$ESHF = \frac{ERSH}{ERSH + ERLH}$$
$$= \frac{51.53}{51.53 + 11} = 0.824$$

Ex. 5.76 : *A room has to be maintained 25 °C DBT and 50 % RH. It has sensible heat gain 20 kW and latent heat gain of 4.8 kW. 190 m^3/min of air is supplied to the room. Estimate state of supply air.*

Sol. : Given data :

$T_{d1} = 25$ °C RH = 50 %

RSH = 20 kW RLH = 4.8 kW

$v = 190\, m^3/min$

Refer Fig. 5.103 (a) on next page.

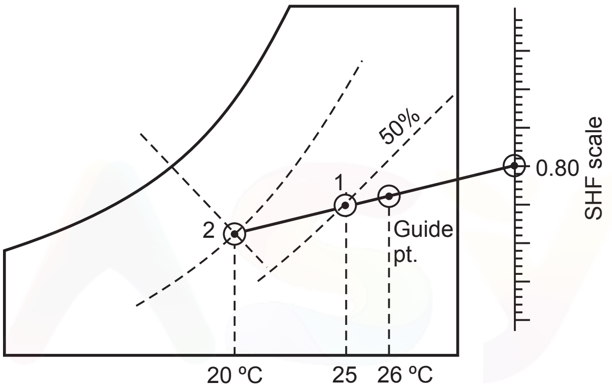


Fig. 5.103

Calculate t_{d2}

$$\therefore \quad RSH = 0.02044\, v_1 (t_{d1} - t_{d2})$$
$$20 = 0.02044 \times 190\,(25 - t_{d2})$$
$$20 = 97.09 - 3.883\, t_{d2}$$
$$t_{d2} = 19.85 \approx 25 \text{ °C}$$

- RSHF

$$RSHF = \frac{RSH}{RSH + RLH} = \frac{20}{20 + 4.8}$$
$$= 0.80$$

- Locate the point 1 as 25 °C DBT and 50 % RH.
- Locate the guide point as 26 °C DBT and 50 % RH.
- Mark the value calculated value of RSHF on SHF-scale and join guide point and point '1'.
- To locate the point '2' extend the line upto $t_{d2} = 20$ °C

From the chart state of supply air is,

$T_{d2} = 20$ °C

$T_{w2} = 16.5$ °C

RH = 68 %

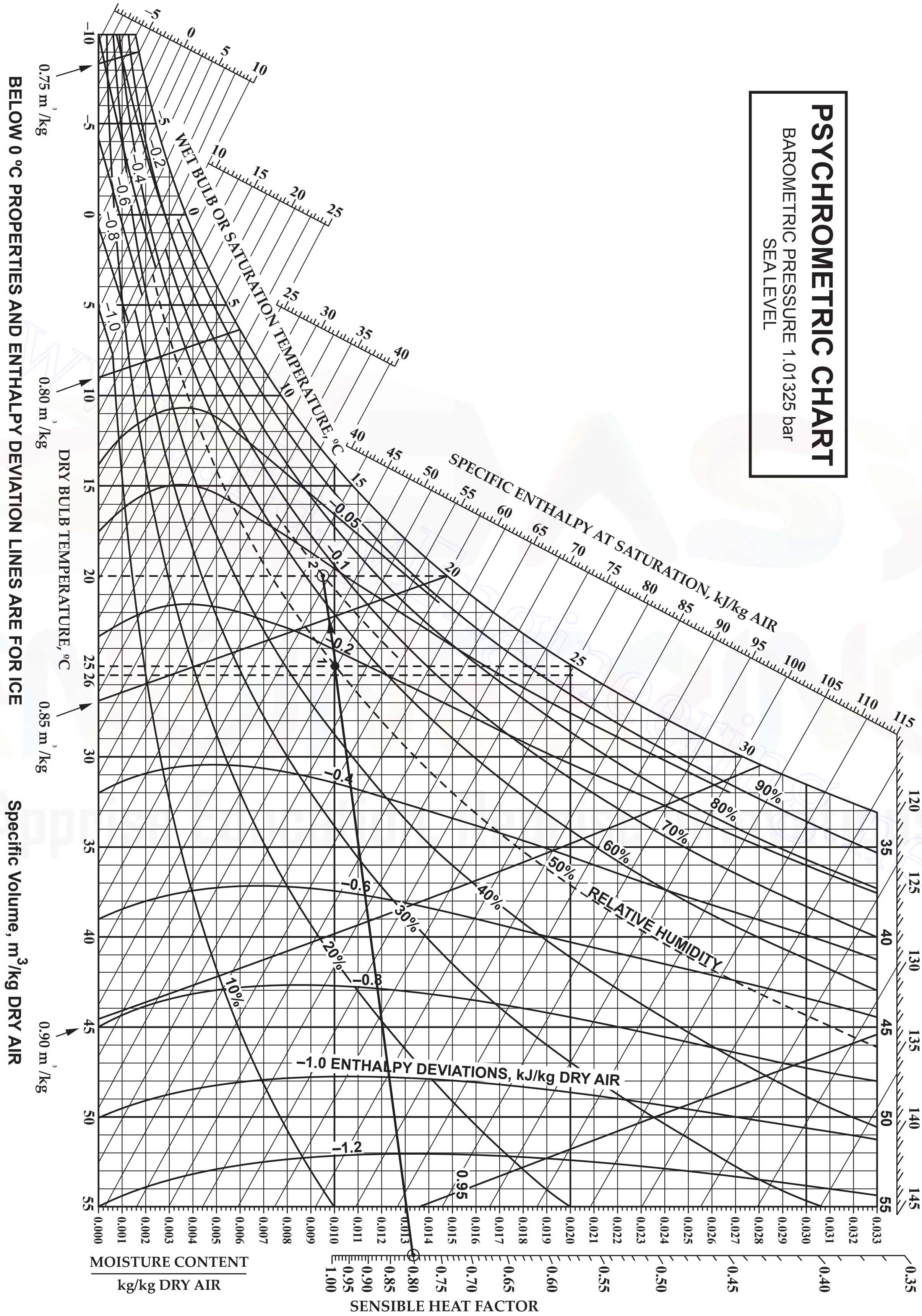

PSYCHROMETRIC CHART
BAROMETRIC PRESSURE 1.01325 bar
SEA LEVEL
WET BULB OR SATURATION TEMPERATURE, °C
SPECIFIC ENTHALPY AT SATURATION, kJ/kg AIR
RELATIVE HUMIDITY
90%
80%
70%
60%
50%
40%
30%
20%
10%
DRY BULB TEMPERATURE, °C
BELOW 0 °C PROPERTIES AND ENTHALPY DEVIATION LINES ARE FOR ICE
Specific Volume, m3/kg DRY AIR
0.75 m3/kg
0.80 m3/kg
0.85 m3/kg
0.90 m3/kg
−1.0 ENTHALPY DEVIATIONS, kJ/kg DRY AIR
MOISTURE CONTENT
kg/kg DRY AIR
SENSIBLE HEAT FACTOR

Fig. 5.103 (a)

Ex. 5.77 : *Following data refers to air conditioned room,*
Room condition - 26.5 DBT and RH - 50 %
Room sensible heat gain = 27 kW
Room sensible heat factor = 0.82
Calculate
i) Room latent heat gain ii) ADP
iii) cmm of air if it is supplied to room at ADP.
iv) cmm of air if it is supplied to room at 16 °C.
v) Specific humidity at 16 °C

Sol. :

Given data

T_{d1} = 26.5 °C, RH = 50 %

RSH = 27 kW RSHF = 0.82

i) To find RLH.,

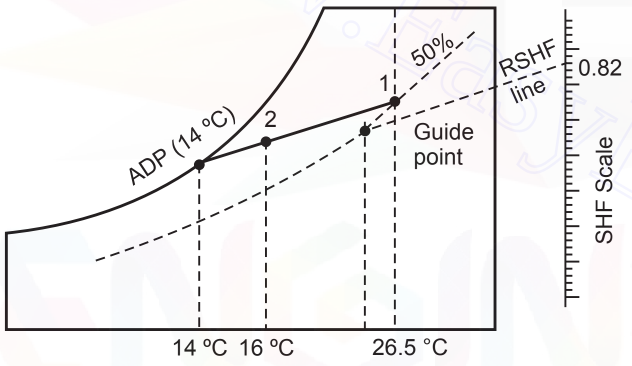


Fig. 5.104

$$RSHF = \frac{RSH}{RSH + RLH}$$

$$0.82 = \frac{27}{27 + RLH}$$

$$RLH = 5.92 \text{ kW}$$

- Mark the point 1 as 26.5 DBT and 50 % RH.
- Mark the guide point as 26 °C DBT and 50 % RH.
- Draw RSHF line (which joint guide point and RSHF value i.e. 0.82).
- Draw a line from point 1 which is parallel to RSHF line where it cuts the saturation curve this point indicates ADP (14 °C)
- Mark the point 2 on this line whose DBT is 17 °C

ii) The ADP = 14 °C from psychrometric chart.

iii) To find cmm of air at ADP

$$cmm = \frac{RSH}{0.0204\,(T_{d1} - ADP)}$$

$$= \frac{27}{0.0204\,(26.5 - 14)}$$

$$= 105.88 \text{ m}^3/\text{min}$$

iv) To find cmm of air at 17 °C

$$cmm = \frac{RSH}{0.0204\,(T_{d1} - T_{d2})}$$

$$= \frac{27}{0.0204\,(26.5 - 16)}$$

$$= 126.05 \text{ m}^3/\text{min}$$

v) Specific humidity

$$w_2 = 0.011 \text{ kg/kg dry air}$$

Ex. 5.78 : *An auditorium is to be maintained at temperature 23 °C and 60 % RH. The sensible heat load 130 kW and 84 kg/hr of moisture has to be removed. Air supplied to auditorium is at 15 °C. Calculate :*
i) Mass of air to be supplied in kg/hr.
ii) ADP, DPT and RH.

Sol. : Given data:

T_{d1} = 23 °C RH = 60 %

RSH = 130 kW T_{d2} = 15 °C

- Find out RLH.

$$RLH = \frac{84}{3600} \times 2447.2$$

($\because$ h_{fg} at 23 °C 2447.2)

$$= 57.10 \text{ kW}$$

- $$RSHF = \frac{RSH}{RSH + RLH}$$

$$= \frac{130}{130 + 57.10}$$

$$= 0.69$$

- Room Total Load (RTL) = RSH + RLH

$$= 130 + 57.10$$

$$= 187.1 \text{ kW}$$

- Locate the point 1 as 23 °C DBT and 60 % RH.

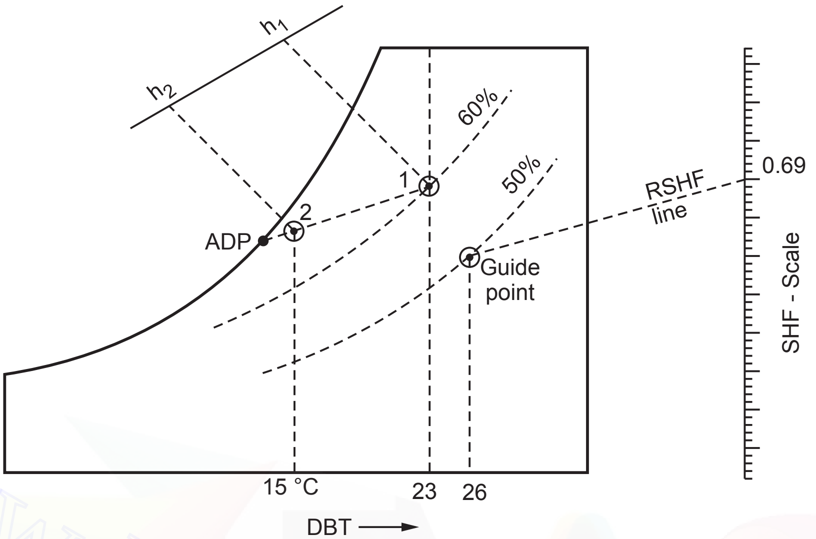


Fig. 5.105

- Locate the guide point at 26 ºC DBT and 50 % RH.
- Draw the line from point 1 which is parallel to RSHF line.
- Extend this line upto saturation curve, it will indicate ADP.
- Locate the point 2 on this line which is at 15 ºC DBT.
- Find out enthalpy h_1 and h_2

h_1 = 52 kJ/kg

h_2 = 38 kJ/kg

i) Mass of air supplied in kg/hr

$$m_a = \frac{RTL}{h_1 - h_2} = \frac{187.1}{(52-38)}$$

$$= 13.36 \text{ kg/sec}$$

$$= 48.09 \times 10^3 \text{ kg/hr}$$

ii) ADP = 12 ºC

DPT = 14 ºC at t_{d2} = 15 ºC

RH = 90 % when t_{d2} = 15 ºC

Ex. 5.79 : *A small office hall of 25 person capacity is provided with summer air conditioning system with following data.*

Outside condition = 34 ºC DBT and 28 ºC WBT

Inside condition = 24 ºC DBT and 50 % RH

Volume of air supplied = 0.4 $m^3/min/person$

Sensible heat load in room = 125600 kJ/hr

Latent heat load in room = 42000 kJ/hr

Find sensible heat factor of room.

Sol. : Given data

Outside condition - 34 ºC DBT and 28 ºC WBT

Inside condition - 24 ºC DBT and 50 % RH

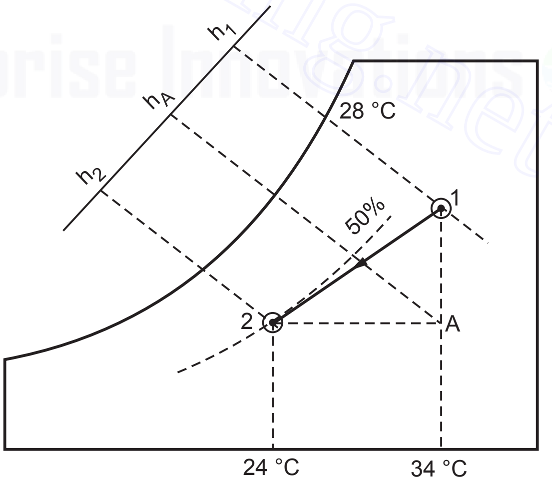


Fig. 5.106

Sol. : Refer Fig. 5.106 (a) on next page.

- Locate point 1 (34 ºC DBT and 28 ºC WBT)

  Find out h_1 = 90 kJ/kg of dry air

- Locate point 2 (24 ºC DBT and 50 % RH)

  Find out h_2 = 48 kJ/kg of dry air

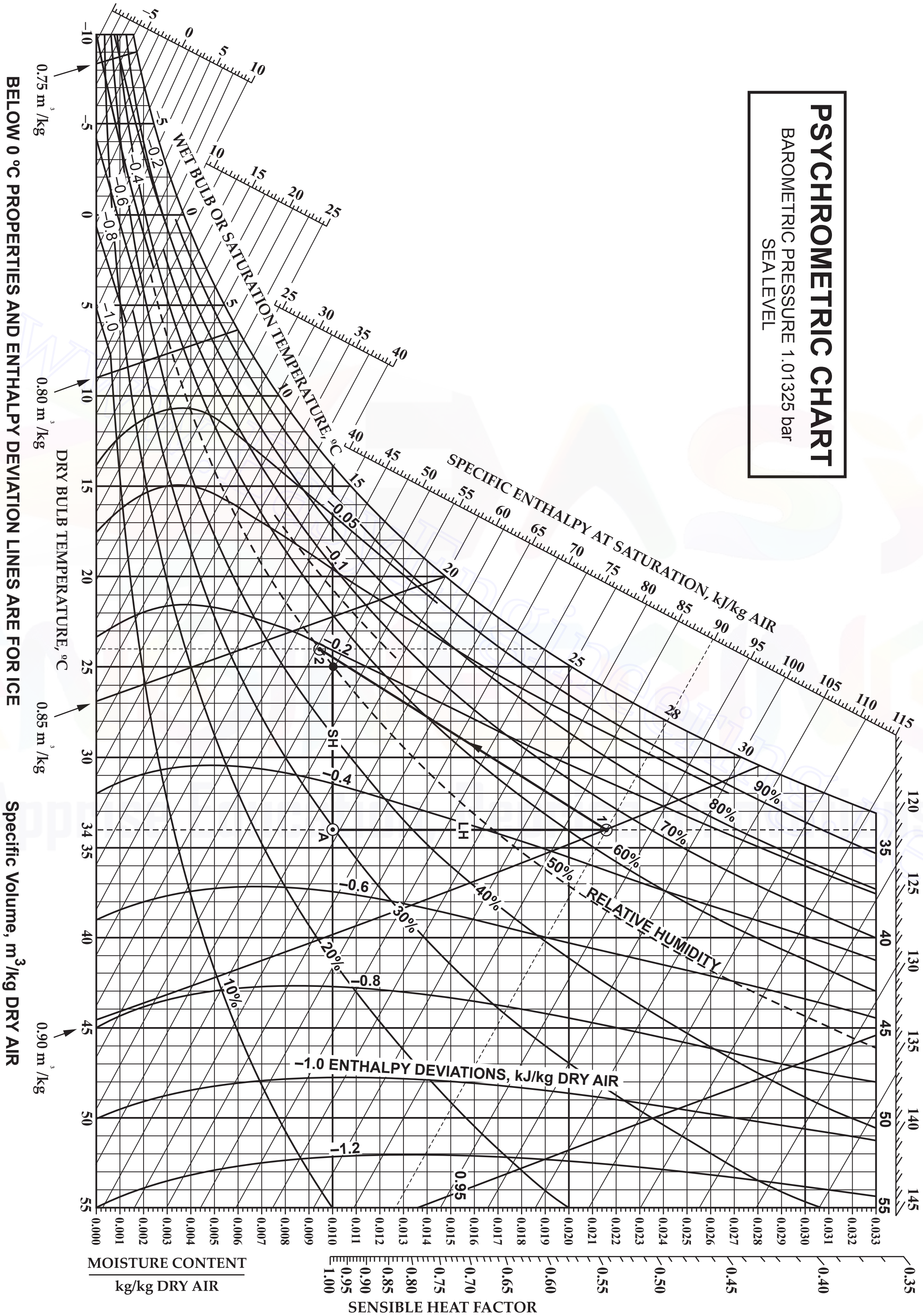

PSYCHROMETRIC CHART
BAROMETRIC PRESSURE 1.01325 bar
SEA LEVEL
WET BULB OR SATURATION TEMPERATURE, °C
SPECIFIC ENTHALPY AT SATURATION, kJ/kg AIR
DRY BULB TEMPERATURE, °C
RELATIVE HUMIDITY
ENTHALPY DEVIATIONS, kJ/kg DRY AIR
BELOW 0 °C PROPERTIES AND ENTHALPY DEVIATION LINES ARE FOR ICE
Specific Volume, m³/kg DRY AIR
MOISTURE CONTENT
kg/kg DRY AIR
SENSIBLE HEAT FACTOR

Fig. 5.106 (a)

- Locate point A by extending lines from point 1 and 2.
- Find out h_A = 58.5 kJ/kg of dry air.
- SH load = 0.4 $m^3/min/person$

Specific volume at point 1 $v_{s1} = 0.901\ m^3/kg$

- SH load of supply air $= \frac{0.4 \times 25 \times 60}{0.901}(h_A - h_2)$

$$= \frac{0.4 \times 25 \times 60}{0.901}(58.5 - 48)$$

$$= 6992.2 \text{ kJ/hr}$$

- LH load of supply air $= \frac{0.4 \times 25 \times 60}{0.901}(h_1 - h_A)$

$$= \frac{0.4 \times 25 \times 60}{0.901} \times (90 - 58.5)$$

$$= 20976 \text{ kJ/hr}$$

- Total sensible heat load = 6992.2 + 125600 = 132592.2

Total latent heat load = 20976 + 42000 = 62976

- SHF $= \frac{TSH}{TSH + TLH} = \frac{132592.2}{132592.2 + 62976} = 0.677\backslash$

5.38 Solved Numericals on Cooling Load Calculation

Ex. 5.80 : *Following conditions are given for a hall to be air conditioned.*

Outdoor condition = 40 °C DBT, 20 °C WBT

Design condition = 20 °C DBT, 60 % RH

Seating capacity of hall = 1300

Amount of outdoor air supply = 0.3 m^3/min per person

If required condition is achieved first by adiabatic humidification and then by cooling calculate.

i) Capacity of cooling coil

ii) Capacity of humidifier

Sol. : Given data :

T_{d1} = 40 °C T_{w1} = 20 °C

Design condition - T_{d3} = 20 °C, RH = 60 %

Capacity of hall - 1300

- Point 1 represents outdoor condition (40 °C DBT and 20 °C WBT).
- Point 3 represents design condition (20 °C DBT and 60% RH)
- Locate the point 2 at intersection of horizontal line from

Point 3 and constant enthalpy line from point 1.

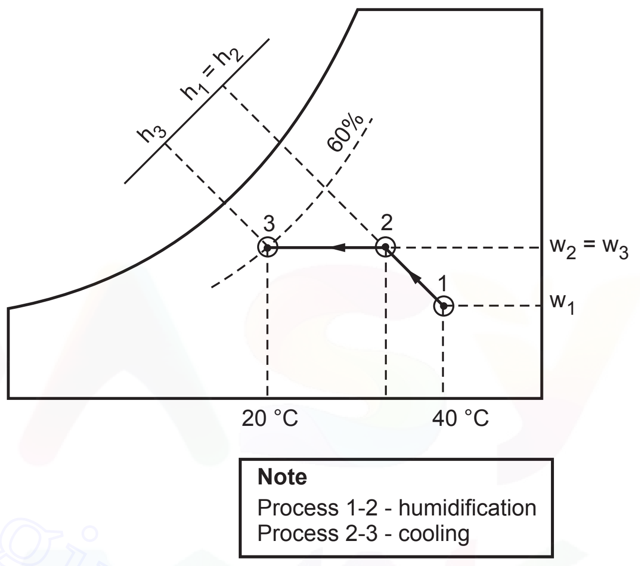


Fig. 5.107

Following values can be taken from psychrometric chart.

$$h_1 = h_2 = 58 \text{ kJ/kg}, \quad h_3 = 42$$

$$w_1 = 0.007 \text{ kg/kg of dry air}$$

$$w_2 = 0.0087 \text{ kg/kg of dry air}$$

$$v_{s1} = 0.89\ m^3/kg$$

- Mass of air supplied,

$$m_a = \frac{v}{v_{s1}} = \frac{1300 \times 0.3}{0.89}$$

$$= 438.20 \text{ kg / min}$$

i) Capacity of cooling coil,

$$Q = m_a(h_2 - h_3)$$

$$= 438.20\ (58 - 42)$$

$$= 7011.2 \text{ kJ / min}$$

ii) Capacity of humidifier

$$m_a = (w_2 - w_1)$$
$$= 438.20(0.0087 - 0.007)$$
$$= 0.744 \text{ kg / min}$$

Ex. 5.81 : *In an air conditioning plant, an air handling unit supplies a total of 4500 m^3/min dry air which comprises by means of 20 % of fresh air at 40 °C DBT and 25 °C WBT and 80 % recirculated air at 24 °C DBT and 50 % RH. The air leaves the cooling coil at 13 °C saturated. Determine.*

i) Total cooling load ii) Room heat gain

Sol. : Given data :

$V_3 = 4500 \ m^3/min$

$T_{d1} = 40$ °C and $T_{w1} = 25$ °C ... 20 % fresh air

$T_{d2} = 25$ °C and RH = 50 % ... 80 % recirculated air

ADP = T_{d4} = 13 %

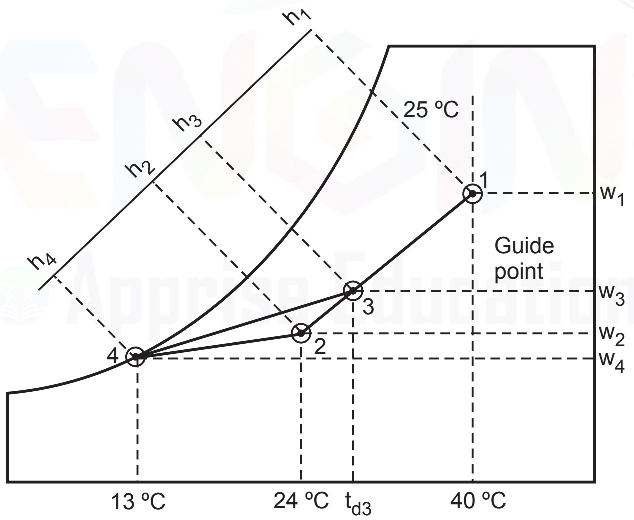


Fig. 5.108

Refer Fig. 5.108 (a) on page 5.119.

- Locate the point 1 (40 °C DBT and 25 °C WBT).
- Locate the point 2 (24 °C DBT and 50 % RH).
- Joint the line 1-2.
- Locate the point 3 on line 1-2 as,

$l(2.3) = 0.2 \times l(2.1)$ ... (∵ 20 % Fresh and 80 % recirculated air)

- Locate the point 4 by drawing vertical line from 13 °C DBT.

It will indicate ADP.

Take the following from values psychromatic chart.

$$h_1 = 77 \text{ kJ/kg}$$
$$w_1 = 0.014 \text{ kg/kg}$$
$$t_{d3} = 26.5 \text{ °C}$$
$$h_2 = 48 \text{ kJ/kg}$$
$$w_2 = 0.0098 \text{ kg/kg}$$
$$v_{s3} = 0.86 \text{ m}^3/\text{kg}$$
$$h_3 = 54 \text{ kJ/kg}$$
$$w_3 = 0.016 \text{ kg/kg}$$

- Mass of air entering the coil,

$$m_{a3} = \frac{v_3}{v_{s3}}$$
$$= \frac{4500}{0.86} = 5232.5 \text{ kg/min}$$

i) Total cooling load,

$$Q = m_{a3}(h_3 - h_4)$$
$$= 5232.5(54 - 36)$$
$$= 94185 \text{ kJ / min}$$

ii) Room heat gain.

20 % fresh air is supplied to room

$$\therefore \quad m_{a1} = 0.2 \times M_{a3}$$
$$= 0.2 \times 5232.5 = 1046.5 \text{ kg/min}$$

$$\text{Fresh cooling load} = m_{a1}(h_1 - h_2)$$
$$= 1046.5(77 - 48)$$
$$= 30348.5 \text{ kJ / min}$$

$$\text{Room heat gain} = \text{Total cooling load} - \text{Fresh air load}$$
$$= 94185 - 30348.5$$
$$= 63836.5 \text{ kJ / min}$$

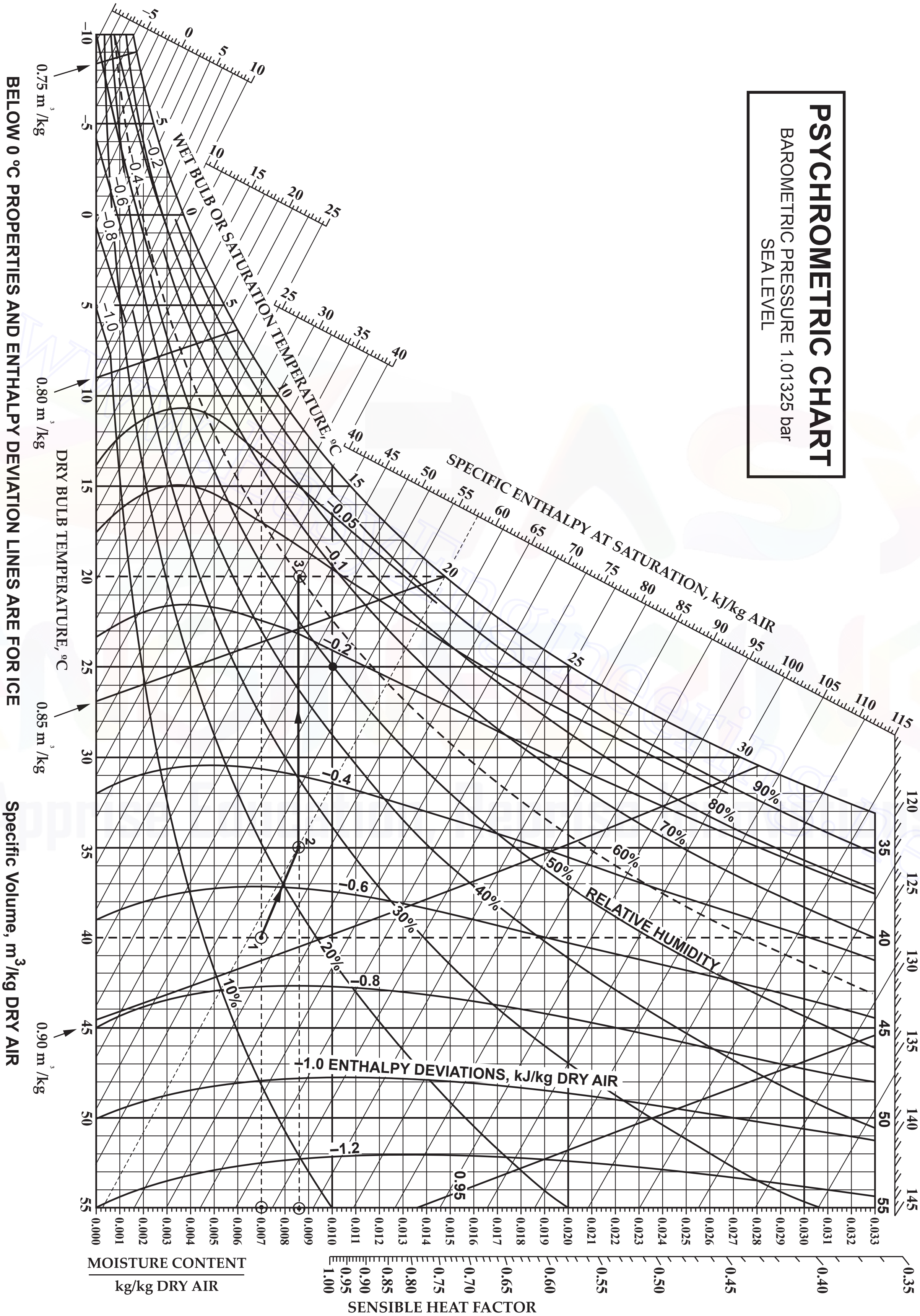
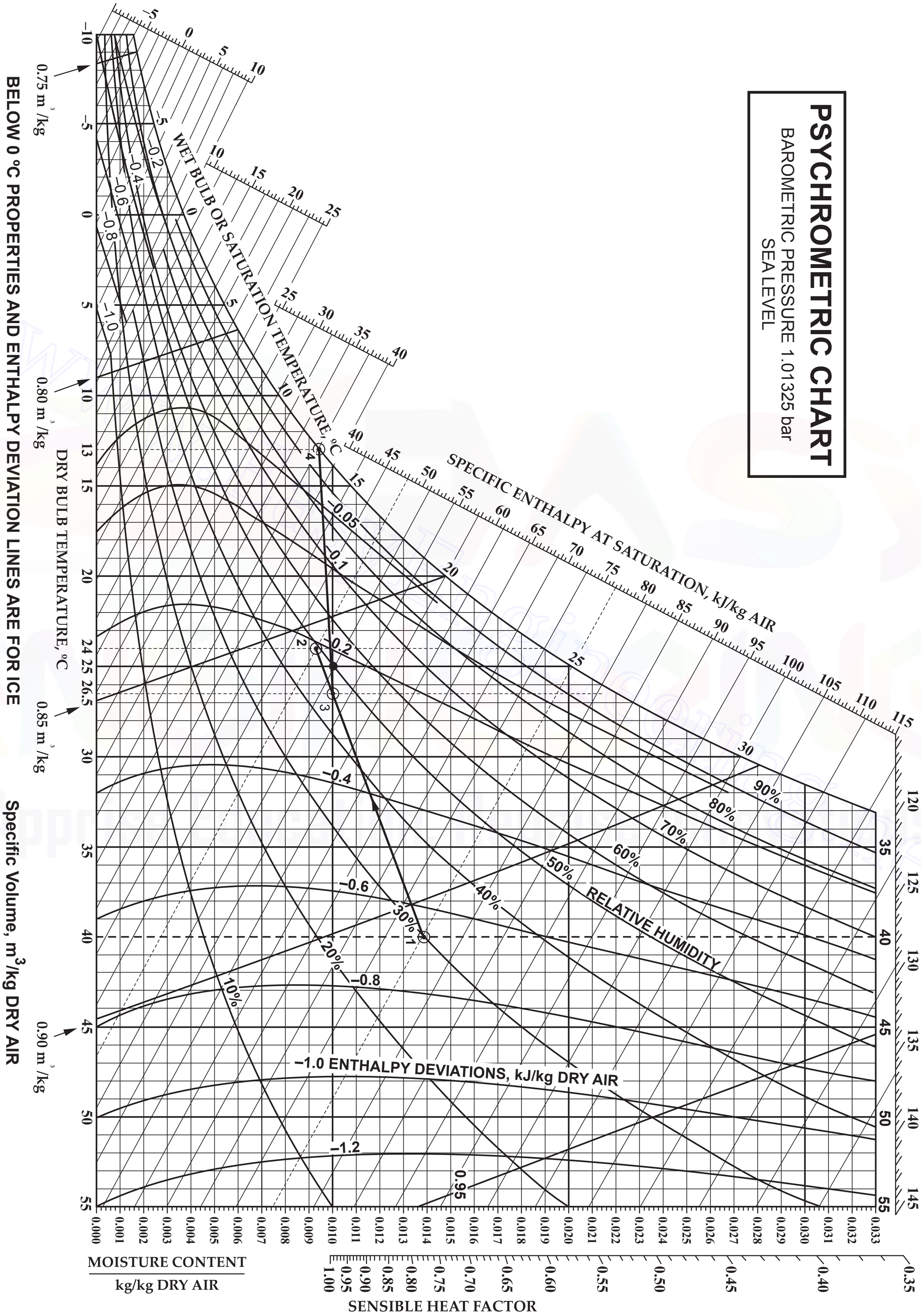

PSYCHROMETRIC CHART
BAROMETRIC PRESSURE 1.01325 bar
SEA LEVEL
WET BULB OR SATURATION TEMPERATURE, °C
SPECIFIC ENTHALPY AT SATURATION, kJ/kg AIR
DRY BULB TEMPERATURE, °C
RELATIVE HUMIDITY
ENTHALPY DEVIATIONS, kJ/kg DRY AIR
BELOW 0 °C PROPERTIES AND ENTHALPY DEVIATION LINES ARE FOR ICE
Specific Volume, m³/kg DRY AIR
0.75 m³/kg
0.80 m³/kg
0.85 m³/kg
0.90 m³/kg
MOISTURE CONTENT
kg/kg DRY AIR
SENSIBLE HEAT FACTOR

Fig. 5.107 (a)



PSYCHROMETRIC CHART
BAROMETRIC PRESSURE 1.01325 bar
SEA LEVEL
WET BULB OR SATURATION TEMPERATURE, °C
SPECIFIC ENTHALPY AT SATURATION, kJ/kg AIR
RELATIVE HUMIDITY
–1.0 ENTHALPY DEVIATIONS, kJ/kg DRY AIR
DRY BULB TEMPERATURE, °C
BELOW 0 °C PROPERTIES AND ENTHALPY DEVIATION LINES ARE FOR ICE
Specific Volume, m3/kg DRY AIR
MOISTURE CONTENT
kg/kg DRY AIR
SENSIBLE HEAT FACTOR

Fig. 5.108 (a)

Ex. 5.82 : *30 cmm of room air at 26 ºC DBT and 50 % RH is mixed with 28 cmm of outside air at 40 ºC DBT and 28 ºC WBT. Determine ventilation load and condition of air after mixing.*

If this above mixture of air passed through an air conditioning equipment. If WBT of air after the equipment is 15 ºC. Calculate heat removed by equipment.

Sol. : Given data

Room air	Outside air
26 ºC DBT	40 ºC DBT
50 % RH	28 ºC WBT
30 cmm	28 cmm

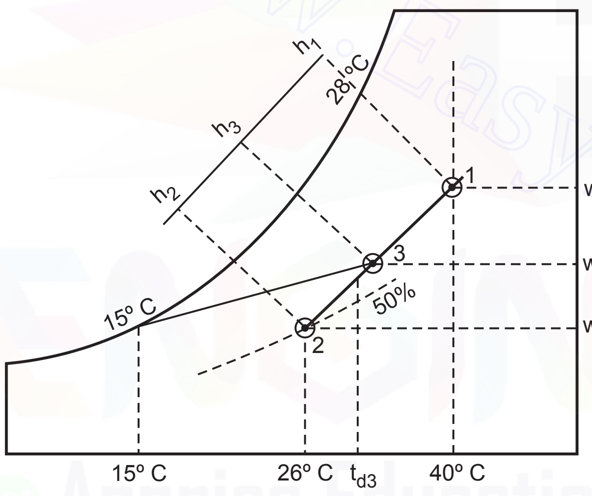


Fig. 5.109

- Point 1 indicates outside air condition (40 ºC DBT and 28 ºC WBT).
- Point 2 indicates room air condition (26 ºC DBT and 50 % RH).
- To locate the point 3 it is required to find temperature at point 3. Point 3 indicates mixture condition.
- Mass flow rate of room air.

$$m_2 = \frac{30}{v_{s2}} = \frac{30}{0.86} = 34.88 \text{ kg/min}$$

- Mass flow of outside air.

$$m_1 = \frac{28}{v_{s1}}$$

$$= \frac{28}{0.905} = 30.99 \text{ kg/min}$$

- To find mass of mixture,

$$m_3 = m_1 + m_2$$

$$= 30.99 + 34.88$$

$$= 65.81 \text{ kg/min}$$

- To find w_3,

$$w_3 = \frac{m_1 w_1 + m_2 w_2}{m_3}$$

$$= \frac{(30.99 \times 0.019) + (34.88 \times 0.011)}{65.81}$$

$$= 0.01477 \text{ kg/kg of dry air.}$$

- To find T_{d3},

$$T_{d3} = \frac{m_1 T_1 + m_2 T_2}{m_3}$$

$$= \frac{(30.99 \times 40) + (34.88 \times 26)}{65.81}$$

$$= 32.61 \text{ ºC}$$

- Now, we have $w_3 = 0.014$ and $T_{d3} = 32.61$ ºC. We can locate the point 3 easily.

$$\text{Ventilation load} = m_1(h_1 - h_2)$$

$$= 30.99(90 - 54)$$

$$= 1115.6 \text{ kJ/min}$$

$$= 18.59 \text{ kW}$$

- For second case

WBT = 15 ºC

Locate the point of $t_{wb} = 15$ ºC on chart and find its enthalpy.

$$h = 40.7$$

- Heat removed by equipment,

$$\text{H.R.} = m_3(h_3 - h)$$

$$= 65.81(68 - 40.7)$$

$$= 1796.6 \text{ kW}$$

Ex. 5.83 : *It is required to maintain air conditioned reading hall at 28 °C DBT and 20 °C WBT. It has sensible heat load of 47 kW and latent heat load of 17 kW. The air supplied from outside atmosphere at 40°C DBT and 28°C WBT is 25 m^3/min. directly into a room through ventilation and infiltration. Outside air to be conditioned is passed through cooling coil whose ADP is 9 °C, 59 % of recirculated air from hall is mixed with conditioned air after cooling coil. Calculate*

i) Condition of air after the coil and before recirculated air mixes with it.

ii) Condition of air entering the reading hall.

iii) Mass of fresh air entering cooler.

iv) BPF v) Refrigerating load on cooling coil.

Sol. : Given data

Reading hall condition	Atmospheric condition
28 °C DBT	40 °C DBT
20 °C WBT	28 °C DBT

RSH = 47 kW RLH = 17 kW

ADP = 9 °C

- Point 1 indicates atmospheric conditions (40 °C DBT and 28 °C WBT).
- Point 4 indicates reading hall condition (28 °C DBT and 20 °C WBT).
- Locate the point A by extending horizontal line from point 4 and extending vertical line from point 1.
- Calculate the value of RSHF.
- Mark this value on SHF scale.
- Locate guide point (as 26 °C DBT and 50 % RH).
- Draw a line from guide point to SHF scale.
- From point 4, draw a line which is parallel to guide line.
- This line is RSHF line.
- Now it is given that 9 °C ADP (Mark ADP on saturation curve).
- Join the point 1 and ADP.
- This line is GSHF line.
- At point 2 GSHF and RSHF lines are intersect.
- This point indicates condition of air leaving cooling coil.
- Locate the point 3 in such a way that,

$l(2 - 3) = 0.59 \times l(2 - 4)$

- Point 3 indicates condition of air entering the hall.

From psychrometric chart, find following values -

$h_1 = 91$ kJ/kg

$h_4 = 58$ kJ/kg

$h_A = 70$ kJ/kg

$v_{s1} = 0.92\ m^3/kg$

- Mass of air in hall

$$m_a = \frac{v_1}{v_{s1}} = \frac{25}{0.92} = 27.17 \text{ kg/min}$$

- Sensible heat load due to infiltrated air,

$$\begin{aligned} Q_s &= m_a(h_A - h_4) \\ &= 27.17(70 - 58) \\ &= 326.04 \text{ kJ / min} \\ &= 5.43 \text{ kW} \end{aligned}$$

- Latent head load,

$$\begin{aligned} Q_L &= m_a(h_1 - h_A) \\ &= 27.17(91 - 70) \\ &= 570.57 \text{ kJ / min} \\ &= 9.50 \text{ kW} \end{aligned}$$

- Total room sensible heat load = 47 + 5.43

= 52.43 kW

- Total room latent heat load = 17 + 9.50

= 26.5 kW

- $$\begin{aligned} RSHF &= \frac{RSH}{RSH + RLH} \\ &= \frac{52.43}{52.43 + 26.5} \\ &= 0.664 \end{aligned}$$

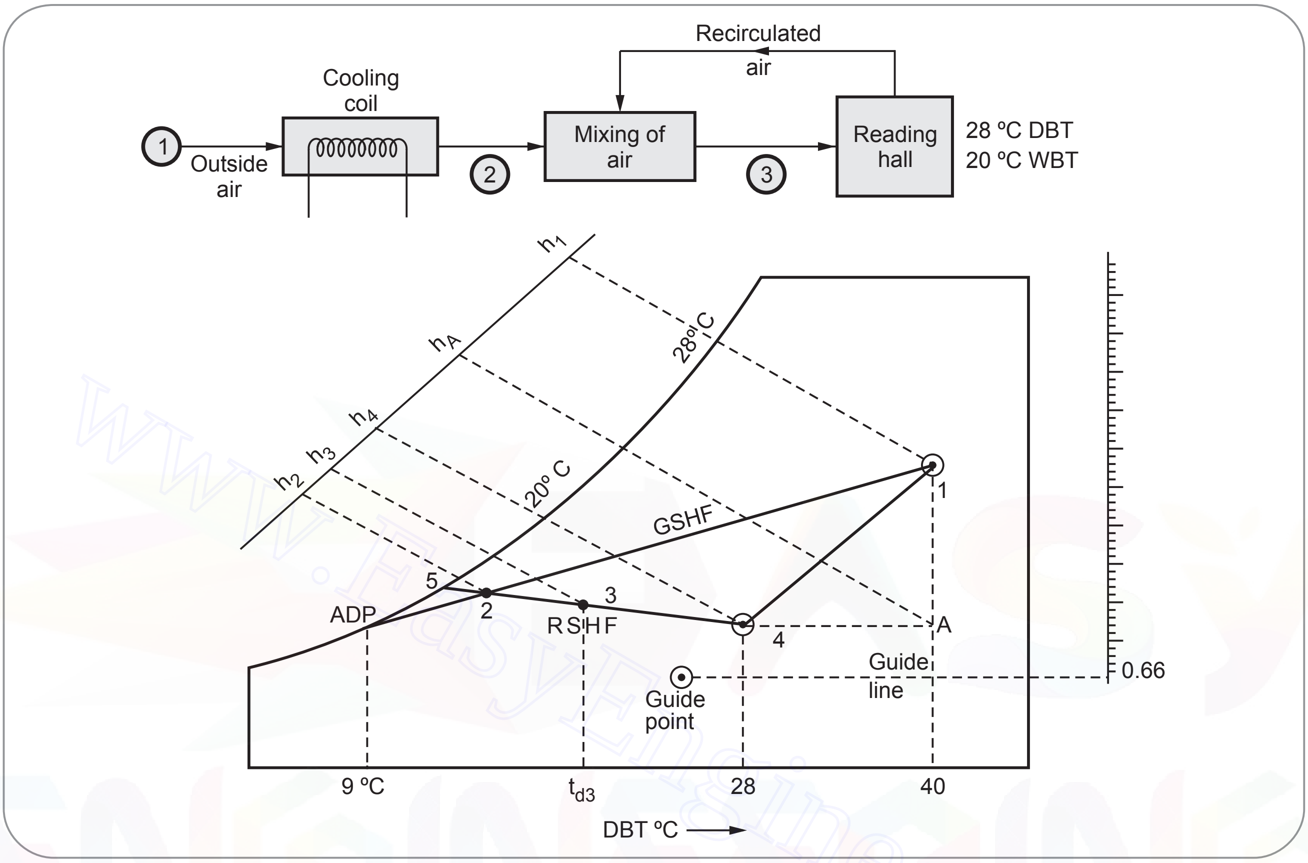


Fig. 5.110

i) Condition of air after the coil and before recirculated air mixes with it,

From psychrometric chart,

$T_{db2} = 13\,°C$

$T_{w2} = 12.1\,°C$

RH = 96 %

ii) Condition of air entering the hall

From psychrometric chart,

$T_{db3} = 23\,°C$

$T_{w3} = 18\,°C$

RH = 65 %

iii) Mass of fresh air entering the cooler,

$$m = \frac{RSH + RLH}{(h_4 - h_2)}$$

$$= \frac{52.43 + 26.5}{(58 - 34)}$$

$$= 3.28 \text{ kg/min}$$

iv) $$BPF = \frac{T_{db2} - ADP}{T_{db1} - ADP}$$

$$= \frac{13 - 9}{40 - 9} = 0.129$$

v) Refrigerating load on cooling coil,

$$= \text{Mass of fresh air} \times (h_1 - h_2)$$

$$= 3.28(91 - 34)$$

$$= 186.9 \text{ kJ/min}$$

Refer Fig. 5.110 (a) on next page.

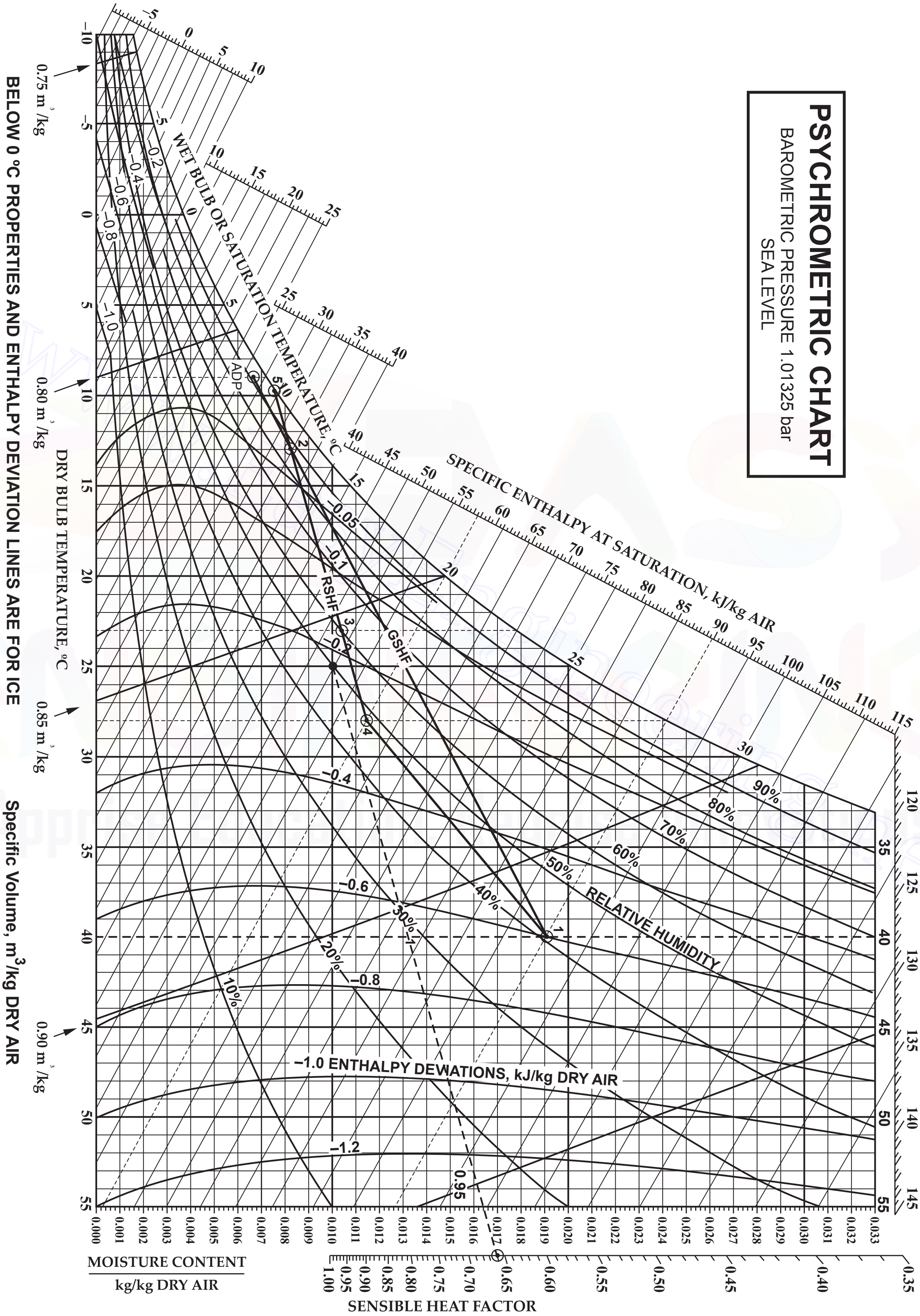

PSYCHROMETRIC CHART
BAROMETRIC PRESSURE 1.01325 bar
SEA LEVEL
WET BULB OR SATURATION TEMPERATURE, °C
SPECIFIC ENTHALPY AT SATURATION, kJ/kg AIR
RELATIVE HUMIDITY
DRY BULB TEMPERATURE, °C
BELOW 0 °C PROPERTIES AND ENTHALPY DEVIATION LINES ARE FOR ICE
Specific Volume, m3/kg DRY AIR
−1.0 ENTHALPY DEWATIONS, kJ/kg DRY AIR
RSHF
GSHF
ADP
MOISTURE CONTENT
kg/kg DRY AIR
SENSIBLE HEAT FACTOR

Fig. 5.110 (a)

Ex. 5.84 : *Moist air at standard atmospheric pressure is passed over a cooling coil. The inlet state - DBT 30 °C, RH 50 % and exit state - DBT 15 °C, RH 90 %. Show the process on psychrometric chart. Determine the amount of heat and moisture removed per kg of dry air.*

Sol. : Given data :

Inlet condition = 30 ºC DBT and 50 % RH

Exit condition = 15 ºC DBT and 90 % RH

- First of all, plot inlet air condition i.e. point '1' at the intersection of 30 ºC DBT and 50 % RH.

From psychrometric chart, we get

$$h_1 = 64.5 \text{ kJ/kg} ;$$

$$h_2 = 41 \text{ kJ/kg}$$

$$w_1 = 0.0128 \text{ kg/kg d.a.} ;$$

$$w_2 = 0.0098 \text{ kg/kg d.a.}$$

i) Amount of heat removed per kg of d.a.

$$Q = h_1 - h_2$$

$$Q = 64.5 - 41 = \mathbf{23.5 \text{ kJ/kg d.a.}} \quad \text{... Ans.}$$

ii) Moisture removed per kg of d.a.

$$\Delta w = w_1 - w_2 = \mathbf{0.0032 \text{ kg/kg d.a.}} \text{ ... Ans.}$$

Ex. 5.85 : *An air conditioning plant is required to supply 50 m^3 of air per minute at a DBT of 22° C and 50 % RH. The atmospheric condition is 32° C with 65 % R.H. Determine the mass of moisture removed and capacity of cooling coil, if the required effect is obtained by dehumidification and sensible cooling coil, if the required effect is obtained by dehumidification and sensible cooling process. Also calculate sensible heat factor.*

AU : Nov.-15, Marks 16

Sol. : Given data :

$$V_1 = 50 \text{ m}^3$$

$$T_1 = 22 \text{ °C DBT}, \quad \phi_1 = 50 \text{ \% RH}$$

$$T_2 = 32 \text{ °C DBT}, \quad \phi_2 = 65 \text{ \% RH}$$

To find out $\dot{m}$

$$\dot{m} = \frac{V}{V_{s1}} = \frac{50}{0.84} = 59.52 \text{ kg/min}$$

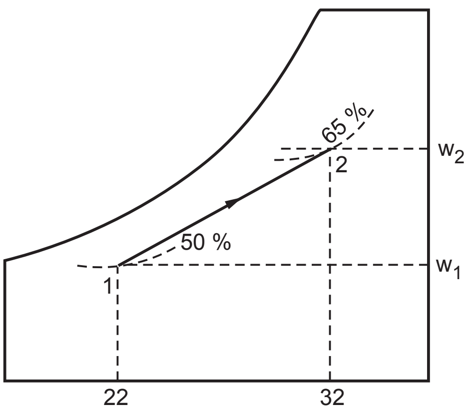


Fig. 5.111

Amount of moisture removed,

$$Q = \dot{m}(\omega_2 - \omega_1)$$

$$= \frac{59.52}{60}(0.020 - 0.0085)$$

$$= 0.0114 \text{ kJ/kg of d.a.}$$

$$\text{Capacity} = m(h_2 - h_1)$$

$$= \frac{59.52}{60}(84 - 43)$$

$$= 40.67 \text{ kJ/kg}$$

5.39 Cooling Tower

- Cooling towers are a very important part of many chemical plants.
- The primary task of a cooling tower is to reject heat into the atmosphere.
- They represent a relatively inexpensive and dependable means of removing low-grade heat from cooling water.
- The make-up water source is used to replenish water lost to evaporation.
- Hot water from heat exchangers is sent to the cooling tower.

5.40 Types of Cooling Tower

- Cooling towers fall into two main categories :
  1. Natural draft 2. Mechanical draft.
- Natural draft towers use very large concrete chimneys to introduce air through the media.
- Due to the large size of these towers, they are generally used for water flow rates above 45,000 m^3/hr.
- These types of towers are used only by utility power stations.
- Mechanical draft towers utilize large fans to force or suck air through circulated water.
- The water falls downward over fill surfaces, which help increase the contact time between the water and the air - this helps maximise heat transfer between the two.
- Cooling rates of Mechanical draft towers depend upon their fan diameter and speed of operation. Since, the mechanical draft cooling towers are much more widely used.

5.41 Mechanical Draft Tower

- Mechanical draft towers are available in the following airflow arrangements :
  1. Counter flows induced draft.
  2. Counter flow forced draft.
  3. Cross flow induced draft.

5.41.1 Counter Flow Induced Draft

- In counter flow induced draft hot water enters at the top, while the air is introduced at the bottom and exits at the top.
- Both forced and induced draft fans are used.

5.41.2 Cross Flow Induced Draft

- In cross flow induced draft towers the water enters at the top and passes over the fill. The air, however, is introduced at the side either on one side (single-flow tower) or opposite sides (double-flow tower).
- An induced draft fan draws the air across the wetted fill and expels it through the top of the structure.

(See Fig. 5.113 on next page)

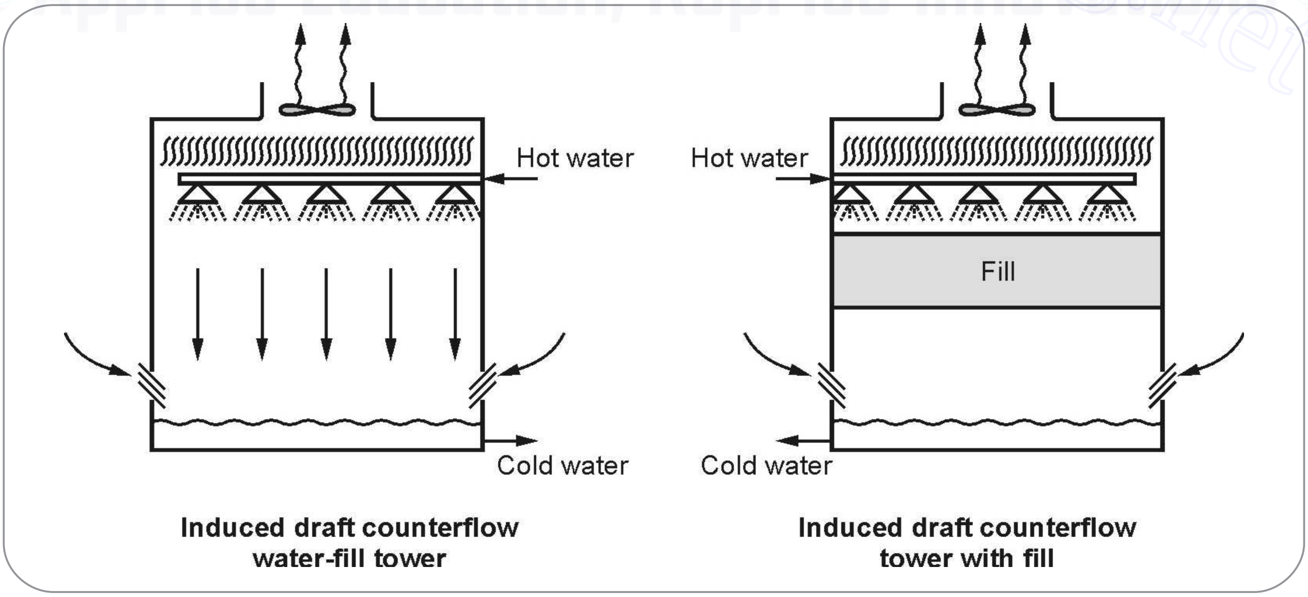


Fig. 5.112 : Counter flow induced draft

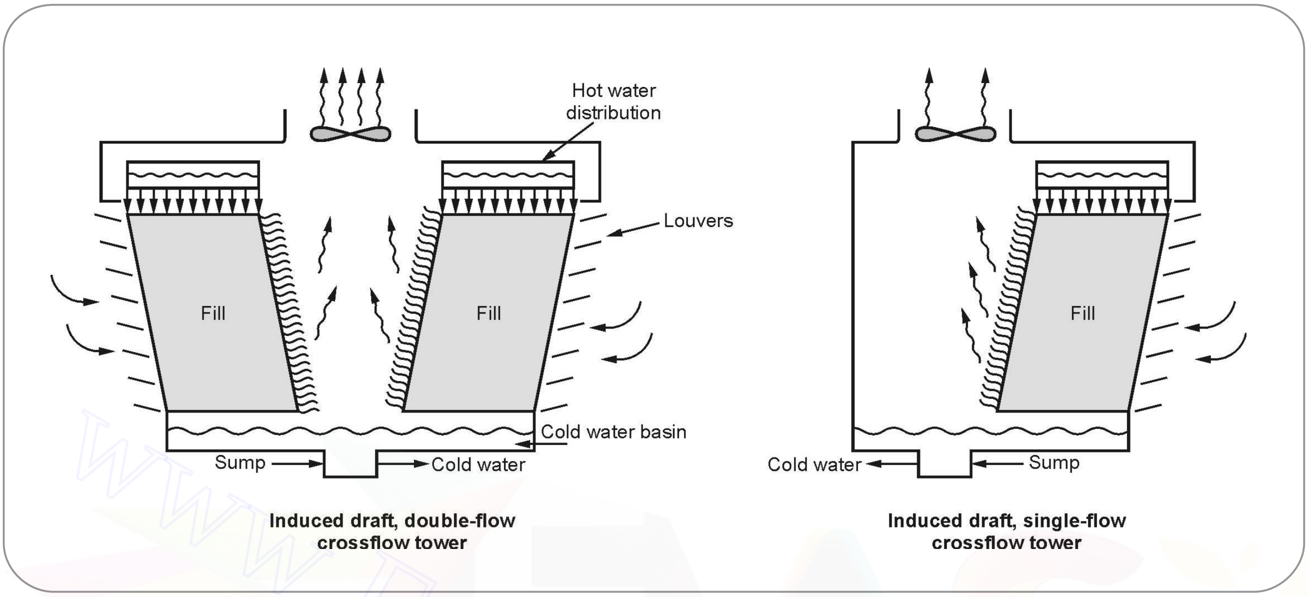


Fig. 5.113 : Cross flow induced draft

5.41.3 Counter Flow Forced Draft

- It is suited for high air resistance due to centrifugal blower fan.
- Fans are relatively quite.

5.42 Two Marks Questions with Answers

Q.1 Short note on Bell-Coleman cycle. (Refer section 5.10)

Q.2 Define COP and Refrigerating effect. (Refer section 5.4)

Q.3 What are the major components of VCC ? (Refer section 5.5)

Q.4 Draw P-h and T-S diagram for superheating. (Refer section 5.12 and Fig. 5.12)

Q.5 What is dry and wet compression ? (Refer sections 5.6.1 and 5.6.2)

Q.6 What is ideal VCC ? (Refer section 5.5)

Q.7 What is the significance of psychrometric chart ? (Refer section 5.20)

Q.8 Define sensible heating and sensible cooling. (Refer sections 5.25.1 and 5.25.2)

Q.9 What do you mean by dew point temperature ? (Refer section 5.24))

Q.10 What is humidity ratio and relative humidity ? (Refer section 5.24)

Q.11 Write the carrier equation. (Refer section 5.22.1)

Q.12 Define degree of saturation with equation. (Refer section 5.22.4)

Q.13 Draw the process heating with humidification on psychrometric chart. (Refer section 5.25.8)

Q.14 Define dry bulb and wet bulb temperature. (Refer section 5.21)

Q.15 What is saturated air ? (Refer section 5.21)

Q.16 Define vapour pressure. (Refer section 5.21)

Q.17 List the main components used in VCC

Ans. : Compressor, condenser, expansion valve, evaporator.

Q.18 Capillary tubes are used in _____.

Ans. : small units

Q.19 List the various types of compressor.

Ans. : 1) Reciprocating

2) Centrifugal

3) Rotary

Q.20 When volume flow rate of refrigerant is large, which compressor is used?

Ans. : Centrifugal

Q.21 Why is multistage compression with intercooling adopted ?

Ans. : By using a single stage with high pressure ratio decreases volumetric efficiency, high pressure ratio with dry compression gives high compressor discharge temperature.

Q.22 Why is ammonia used in food refrigeration?

Ans. : High COP

Low cost

Lower energy cost

Q.23 Define psychrometry.

Ans. : The science which deals with the study of behaviour of moist air (mixture of dry air and water vapour) is known as psychrometry.

Q.24 What is humidification and dehumidification ?

Ans. : The addition of water vapour into air is humidification and the removal of water vapour from air is dehumidification.

Q.25 Define specific humidity.

Ans. : It is defined as the ratio of the mass of water vapour (ms) in a given volume to the mass of dry air in a given volume (ma).

Q.26 Differentiate absolute humidity and relative humidity.

Ans. : Absolute humidity is the mass of water vapour present in one kg of dry air. Relative humidity is the ratio of the actual mass of water vapour present in one kg of dry air at the given temperature to the maximum mass of water vapour it can with hold at the same temperature. Absolute humidity is expressed in terms of kg/kg of dry air. Relative humidity is expressed in terms of percentage.

Q.27 What is effective temperature ?

Ans. : The effective temperature is a measure of feeling warmth or cold to the human body in response to the air temperature, moisture content and air motion. If the air at different DBT and RH condition carries the same amount of heat as the heat carried by the air at temperature T and 100% RH, then the temperature T is known as effective temperature.

Q.28 Define Relative humidity.

Ans. : It is defined as the ratio of partial pressure of water vapour (pw) in a mixture to the saturation pressure (ps) of pure water at the same temperature of mixture.

Q.29 Define degree of saturation.

Ans. : It is the ratio of the actual specific humidity and the saturated specific humidity at the same temperature of the mixture.

Q.30 What is meant by adiabatic saturation temperature (or) thermodynamic wet bulb temperature ?

Ans. : It is the temperature at which the outlet air can be brought into saturation state by passing through the water in the long insulated duct (adiabatic) by the evaporation of water due to latent heat of vaporization.

Q.31 What is dew point temperature?

Ans. : How it is related to dry bulb and wet bulb temperature at the saturation condition? The temperature at which the vapour starts condensing is called dew point temperature. It is also equal to the saturation temperature at the partial pressure of water vapour in the mixture. The dew point temperature is an indication of specific humidity. For saturated air, the dry bulb, wet bulb and dew point temperature are all same.

Q.32 What is meant by dry bulb temperature (DBT) ?

Ans. : The temperature recorded by the thermometer with a dry bulb. The dry bulb thermometer cannot affect by the moisture present in the air. It is the measure of sensible heat of the air.

Q.33 What is meant by wet bulb temperature (WBT) ?

Ans. : It is the temperature recorded by a thermometer whose bulb is covered with cotton wick (wet) saturated with water. The wet bulb temperature may be the measure of enthalpy of air. WBT is the lowest temperature recorded by moistened bulb.

Q.34 Define dew point depression.

Ans. : It is the difference between dry bulb temperature and dew point temperature of air vapour mixture.

Q.35 What is psychrometer ?

Ans. : Psychrometer is an instrument which measures both dry bulb temperature and wet bulb temperature.

Q.36 What is psychrometric chart ?

Ans. : It is the graphical plot with specific humidity and partial pressure of water vapour in y axis and dry bulb temperature along x axis. The specific volume of mixture, wet bulb temperature, relative humidity and enthalpy are the properties appeared in the psychrometric chart.

Q.37 Define sensible heat and latent heat.

Ans. : Sensible heat is the heat that changes the temperature of the substance when added to it or when abstracted from it. Latent heat is the heat that does not affect the temperature but change of state occurred by adding the heat or by abstracting the heat.

Q.38 What are the important psychometric processes ?

Ans. :

- Sensible heating and sensible cooling
- Cooling and dehumidification
- Heating and humidification
- Mixing of air streams
- Chemical dehumidification
- Adiabatic evaporative cooling.

Q.39 Define coefficient of volume expansion.

Ans. : The coefficient of volume expansion is defined as the change in volume with the change in temperature per unit volume keeping the pressure constant.

Q.40 Define bypass factor (BPF) of a coil.

Ans. : The ratio of the amount of air which does not contact the cooling coil (amount of bypassing air) to the amount of supply air is called BPF.

Q.41 What factors affect by pass factor ?

Ans. :

- Pitch of fines
- Number of coil tubes
- Air velocity over the coil
- Direction of air flow.

Q.42 What is meant by adiabatic mixing ?

Ans. : The process of mixing two or more stream of air without any heat transfer to the surrounding is known as adiabatic mixing. It is happened in air conditioning system.

Q.43 What is the difference between air conditioning and refrigeration ?

Ans. : Refrigeration is the process of providing and maintaining the temperature in space below atmospheric temperature. Air conditioning is the process of supplying sufficient volume of clean air containing a specific amount of water vapour and maintaining the predetermined atmospheric condition with in a selected enclosure.

Q.44 Define Dalton's law of partial pressure.

Ans. : The total pressure exerted by air and water vapour mixture is equal to the barometric pressure.

Q.45 What are the effect of superheat and sub cooling on the vapour compression cycle ?

Ans. : Superheating increases the refrigeration effect and COP may be increased or decreased. But sub cooling always increase the COP of the refrigeration and also decrease the mass flow rate of refrigerant.

Q.46 What are the properties of good refrigerant ?

Ans. : An ideal refrigerant should possess the following desirable properties.

1. The refrigerant should have low freezing point.
2. It must have high critical pressure and temperature to avoid large power requirements.
3. It should have low-specific volume to reduce the size of the compressor.
4. It should be nonflammable, non-explosive, non-toxic and non-corrosive.

Q.47 Name the various components used in simple vapour absorption system.

Ans. :

1. Absorber
2. Pump
3. Generator
4. Condenser.
5. Throttle valve.
6. Evaporator.

Q.48 Define refrigerant.

Ans. : Any substance capable of absorbing heat from another required substance can be used as refrigerant.

Q.49 List the practical applications of vapor refrigeration system.

Ans. : On a small scale, there is Electrolux refrigerator

Large air conditioning plants having cooling capacity much greater than 50 tons.

Q.50 Which energy is used in vapor absorption system ?

Ans. : It is cheapest and easily available heat energy which operates the vapor absorption refrigeration system be used as refrigerant.

Q.51 Piping material of vapor absorption system ?

Ans. : It is the steel pipes because ammonia is highly corrosive to copper pipes.

Q.52 In vapour compression refrigeration system where is the location of oil ?

Ans. : Separator is placed between Compressor and Condenser.

Q.53 Air refrigerator works on

Ans. : Bell Coleman cycle

Q.54 What is meant by perfect inter-cooling ?

Ans. : If the temperature of air leaving the intercooler is equal to the original inlet temperature the inter-cooling is known as perfect inter-cooling. By having inter-cooling, we can approach the isothermal process. So the isothermal efficiency will be increased by perfect inter-cooling.

Q.55 Distinguish between summer air conditioning and winter air conditioning.

Ans. : In summer air conditioning the air gains both sensible and latent heat. Hence the conditioning of air is done by both cooling and dehumidification. In winter air conditioning, heating and humidification is done to the air.

Q.56 Define RSHF line.

Ans. : It is Room Sensible Heat Factor (RSHF) line. This line is drawn parallel to the base line in the psychrometric chart.

Q.57 Show the simple vapour compression cycle on pressure - enthalpy diagram.
(Refer section 5.5) **AU : May-16**

Q.58 List out the basic elements of an air conditioning system.
(Refer section 5.27) **AU : Dec.-17**

Q.59 Define RSHF and GSHF.
(Refer section 5.34 and 5.25) **AU : Dec.-17, Marks 16**

Q.60 Compare vapour compression and absorption systems. (Refer section 5.18) **AU : May-18**

Q.61 Define the term Air-conditioning.
(Refer section 5.27) **AU : May-18**

Review Questions

1. *Explain with neat sketch simple VCC,*
2. *With block diagram explain heat engine, heat pump and refrigerator.*
3. *Explain the working of simple vapour absorption system.*
4. *Explain Li-Br absorption system.*
5. *What is thermo electric refrigeration ?*

5.43 University Questions with Answers

May - 2016

Q.1 *Describe the following refrigeration systems with layout : (i) Ammonia water system, (ii) Lithium - bromide water system.*
(Refer sections 5.12 and 5.15) **[8]**

Q.2 *Describe the working principle of a centralised air conditioning system and enumerate the need for it.* **(Refer section 5.27)** **[12]**

Q.3 *List the loads that contribute to the overall cooling load.* **(Refer section 5.25)** **[4]**

Dec. - 2017

Q.4 *Explain the working of Vapoir - Compression system with neat sketch.* **(Refer section 5.5)** **[13]**

May - 2018

Q.5 *Explain the working of vapour absorption refrigeration cycle with a neat schematic layout.* **(Refer sections 5.12 and 5.13)** **[13]**

❑❑❑